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Translational Circuits

Translational circuits are periodically driven, time-variant systems and encompass topologies such as "commutated networks" and "N-path filters." These systems cause frequency "translation" (shift) of impedances and transfer functions, a useful property that has led to myriad new and interesting concepts in RF design. In this article, we study the operation principles of these circuits.

Early History

The use of time-variant systems (e.g., mixers) for the translation of transfer functions dates back to the late 1940s and early 1950s. In their 1948 paper [1], Busignies and Dishal propose the arrangement shown in Figure 1(a), where a mechanical wheel holding a number of capacitors rotates at a rate of f turns per second, allowing each capacitor to charge to the input and, half a cycle later, discharge at the output. The input–output transfer function thus appears as in Figure 1(b), revealing a "comb" filter response with band-pass peaks located at the harmonics of f.

In 1953, Le Page et al. [2] presented the implementation depicted in Figure 2, which employs switches A and B to alternately connect each capacitor to the input and to the output. Also in 1953, Smith [3] provided a comprehensive and intuitive treatment of these circuits. Shown in Figure 3(a) are Smith's original drawings, suggesting that a first-order low-pass RC response can be translated to a center frequency of f_R if the capacitor is replaced by an array of N commutated capacitors that connect to the output node one at a time at a rate of f_R. (This action is realized today by N switches driven by nonoverlapping phases of a clock.)

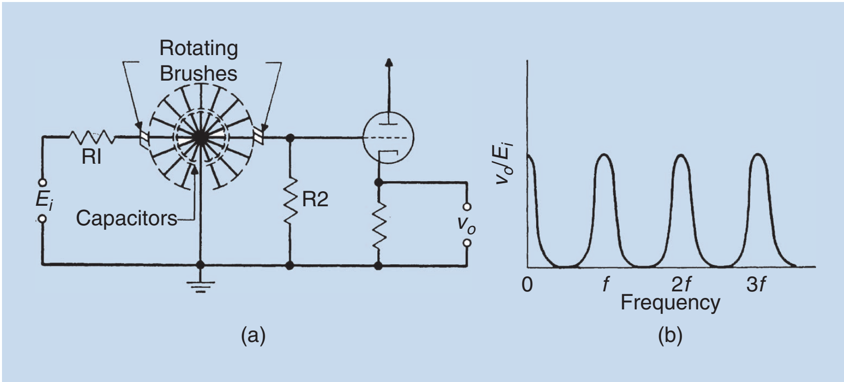

FIGURE 1: (a) A 1948 electromechanical system and (b) its transfer function.

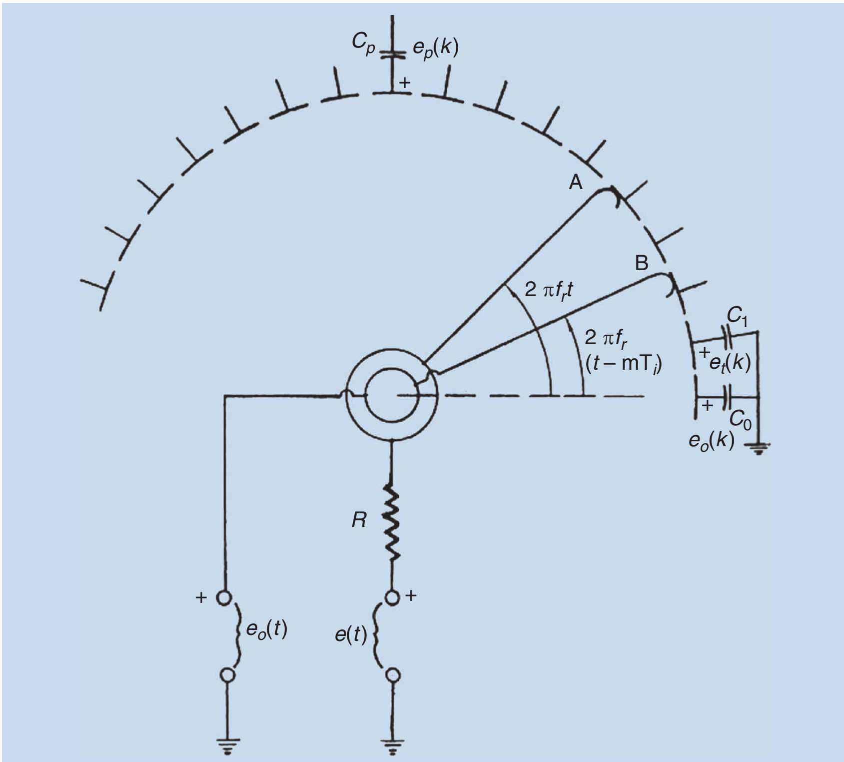

FIGURE 2: A 1953 electromechanical system with translational behavior.

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Notably, Smith also predicted the 3-dB bandwidth of the resulting band-pass response to be $1/(\pi N R_S C)$, where R_S denotes the source resistance. This equation proves useful for estimating the bandwidth in terms of the total commutated capacitance. Smith extended the idea to a first-order high-pass response as well, arriving at the notch filter illustrated in Figure 3(b).

The foregoing topologies exemplify time-variant circuits, lending themselves to a general model proposed by Franks and Sandberg in 1960 [4] and shown in Figure 4. Each path mixes the input signal with different phases of the local oscillator (LO), translates the spectrum to baseband, subjects the downconverted signals to a desired transfer function, and mixes the results with the LO phases again so as to return the (shaped) spectrum to its original center frequency. The term "*N*-path filters" was evidently coined by Franks and Sandberg to refer to these circuits. An important difference between this abstraction and Smith's circuits is that the former assumes unilateral stages whereas the latter entail "transparency" between the input and the output, a useful property for impedance translation.

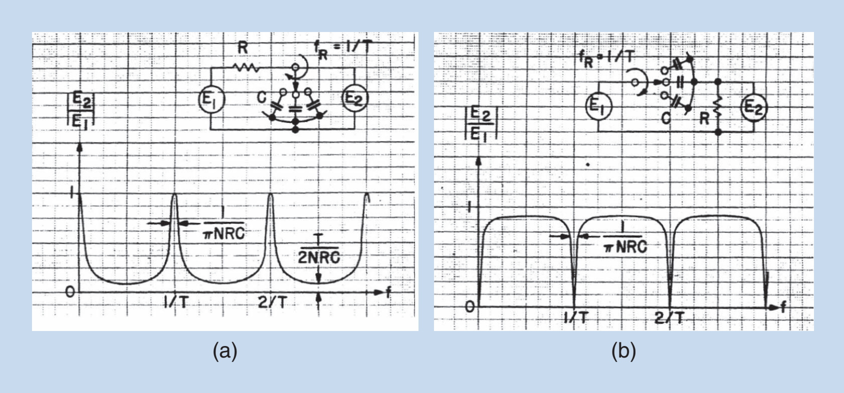

FIGURE 3: Smith's (a) band-pass and (b) notch filter implementations.

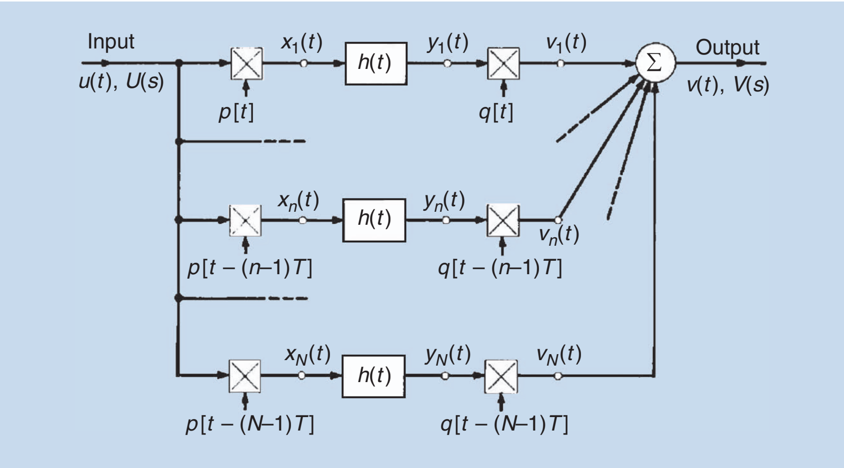

FIGURE 4: A general model of translational circuits proposed by Franks and Sandberg.

Impedance Translation by Partial Commutation

Translational circuits can shift an impedance to a well-defined center frequency. For example, the impedance of a capacitor, $1/(j\omega C)$, can be translated to a center frequency of ω_{LO}, taking on the form $1/[j(\omega-\omega_{LO})C]$. In other words, the impedance of the new network goes to infinity at $\omega=\omega_{LO}$ rather than at $\omega=0$. In this section, we investigate how the translation occurs. Due to switching activities, commutated networks can produce a nonsinusoidal voltage in response to a sinusoidal current. To find the impedance, therefore, we must compute the first harmonic of the voltage.

Consider the parallel RC branch shown in Figure 5(a), assuming $I_{in}(t)=I_0\sin\omega_{in}t$ and $R_1C_1 \gg T_{in}=(2\pi/\omega_{in})$. In this case, most of the input current prefers to flow through C_1, generating $V_{AB}\approx(1/C_1)\int I_{in}(t)\,dt$. Now, let us switch C_1 periodically and study a special case where the input frequency

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is *equal* to the LO frequency [Figure 5(b)]. We wish to study the impedance seen between A and B, Z_{AB}. With the phase relationship shown here, we observe that the positive half cycles of I_{in} flow primarily through C_1, and the negative half cycles entirely through R_1. That is, C_1 receives a half-wave rectified version of I_{in}, thereby accumulating positive charge. The circuit reaches the steady stage when, during each positive half cycle, the charge delivered by I_{in} to C_1 is equal to the charge drawn by R_1 from C_1. The key point here is that the output voltage swing can become arbitrarily large if R_1 has an arbitrarily high value—even though C_1 periodically switches into the circuit.

This analysis culminates in the following observation: a sinusoidal current of frequency $f_{in}=\omega_{in}/(2\pi)=f_{LO}$ can produce a very large voltage swing at f_{in} between nodes A and B in Figure 5(b), revealing that the equivalent impedance of the switched

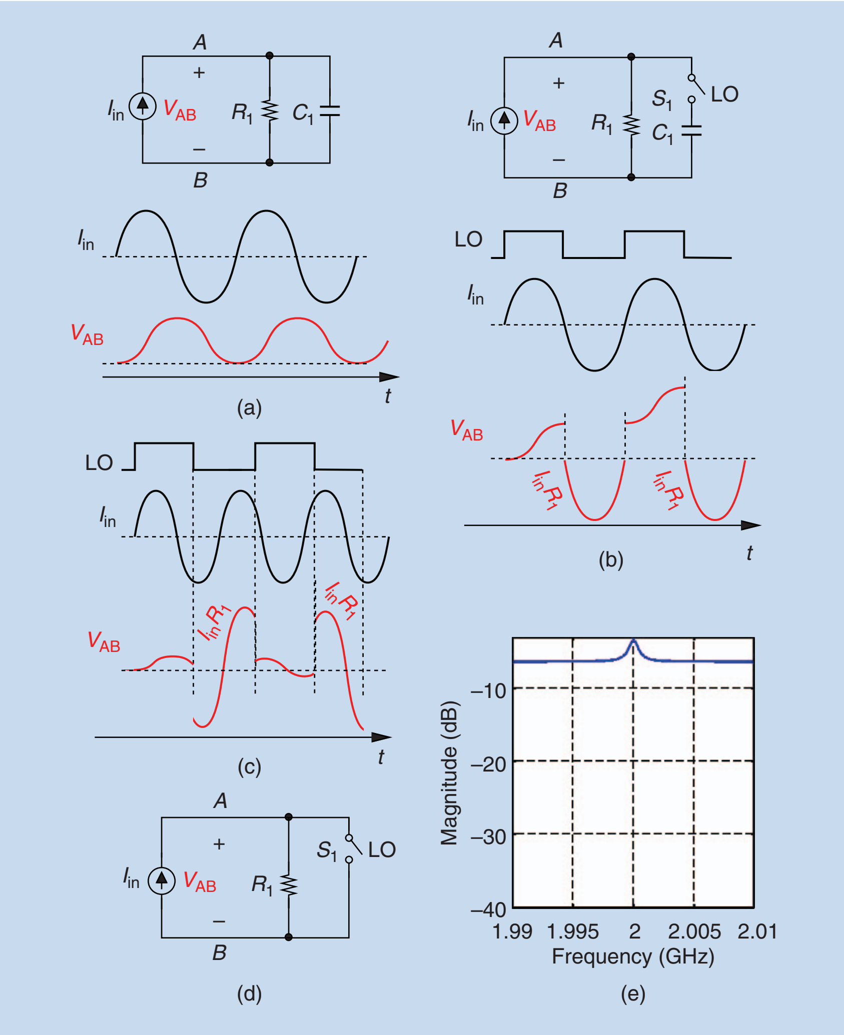

FIGURE 5: (a) A simple RC circuit driven by a sinusoidal current, (b) waveforms when C_1 is commutated, (c) waveforms when $f_{in} \neq f_{LO}$, (d) a simplified circuit if $|f_{in} - f_{LO}|$ is relatively large, and (e) the magnitude of impedance versus frequency.

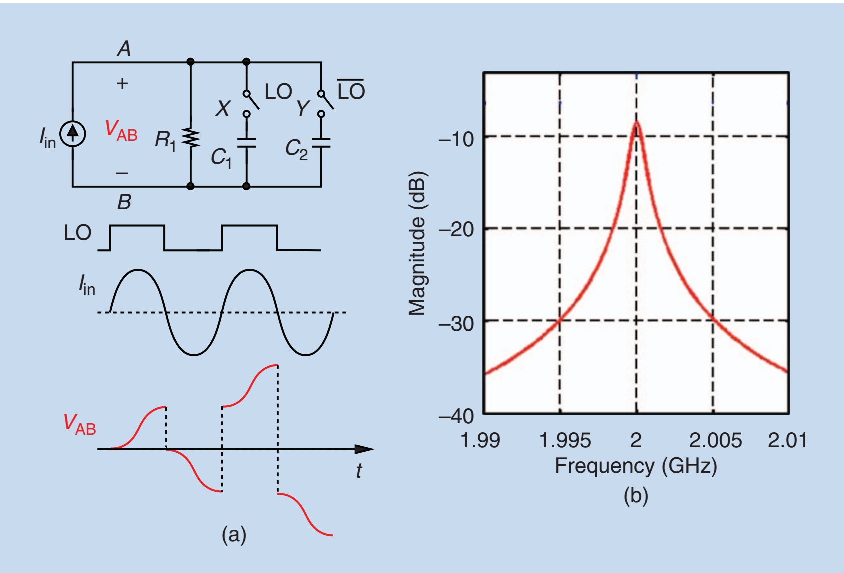

FIGURE 6: (a) An RC circuit using two commutated capacitors and (b) the magnitude of impedance versus frequency.

capacitor is very high at this frequency. This situation stands in contrast to that in Figure 5(a), where C_1 yields a low impedance at f_{in}. We therefore conclude that the impedance of the capacitor is translated (upconverted) to a center frequency of f_{LO}, emerging as $1/[j(\omega - \omega_{LO})C_1]$. Of course, this property arises from the bilateral nature of the switch: the voltage across the capacitor is mixed with the LO and up-converted as it manifests itself in V_{AB}. We call this circuit's operation "partial commutation" to emphasize that I_{in} does not see a capacitance for half of the LO period.

What happens if f_{in} in Figure 5(b) departs from f_{LO}? The synchronicity between I_{in} and the LO no longer holds, C_1 receives both positive and negative charge from I_{in}, and the capacitor voltage grows to a lesser extent [Figure 5(c)]. If $|f_{in} - f_{LO}|$ is large enough, we can say that, when S_1 is on, the swing in V_{AB} is approximately proportional to $1/(C_1\omega_{in})$ (by virtue of integration), and when S_1 is off, the swing is proportional to R_1, a much greater value because $R_1C_1 \gg T_{in}$. We can thus approximate C_1 by a short circuit and reduce the circuit to that in Figure 5(d). Here, $V_{AB}(t)$ is simply equal to $I_{in}(t)S(t)$, where $S(t)$ denotes a square wave toggling between zero and one. Of interest to us is the input impedance in the vicinity of f_{in}; so we must seek the component of V_{AB} at this frequency. Since $S(t)$ has a dc value of 0.5, the amplitude of V_{AB} at f_{in} is given by $0.5I_0R_1$. We conclude that the input impedance falls from (approximately) R_1 at $f_{in} = f_{LO}$ to $0.5R_1$ for f_{in} somewhat far from f_{LO}. The simulated plot in Figure 5(e), where $f_{LO} = 2$ GHz, confirms this result, revealing very little selectivity in this input impedance.

Impedance Translation by Full Commutation

How can we improve the selectivity of the impedance plotted in Figure 5(e)? As noted above, the component resulting from $I_{in}R_1$ overwhelms the up-converted capacitance as f_{in} substantially departs from f_{LO}. This means that we must not permit much of I_{in} to flow through R_1 even when C_1 is switched out. This is accomplished by adding one more capacitive branch that is controlled by $\overline{LO}$ [Figure 6(a)], thus creating "full commutation." Assuming that $C_1 = C_2 = C$, $R_1 C_1 \gg T_{LO}$, and $f_{in} = f_{LO}$, we observe that V_{AB} is equal to V_X for one half cycle and to V_Y for the other half, growing in amplitude in both positive and negative directions. That is, V_{AB} is defined by capacitive dynamics on top and bottom, with both its positive and negative amplitudes decreasing as $|f_{in} - f_{LO}|$ increases. Plotted in Figure 6(b), the impedance continues to fall, exhibiting greater selectivity than that seen in Figure 5(e).

We should make two remarks. First, the condition $R_1 C \gg T_{LO}$ is necessary for significant up-conversion of the capacitive impedance. Each time a switch turns on, the corresponding capacitor impresses its voltage upon the output; the longer this voltage lasts, the more pronounced is the effect of the capacitor. Plotted in Figure 7 is the output voltage for the extreme cases $R_1 C \gg T_{LO}$ and $R_1 C \ll T_{LO}$, revealing a spectrally rich signal in the former that stems from substantial up-conversion of the capacitor voltages. In the latter, on the other hand, each capacitor rapidly loses its charge when it is switched in, allowing V_{AB} to assume the shape of I_{in}. The key point here is that the commutated network need not be driven by a "high" source resistance so long as $R_1 C \gg T_{LO}$.

Second, the translation of the capacitive impedance in Figure 6(a) also occurs at the higher harmonics of the LO. For example, an input of the form $I_{in} = I_0 \sin(3\omega_{LO} t)$ leads to accumulation of charge on C_1 and C_2 and a tall waveform for V_{AB}.

Translation of Transfer Functions

Equipped with the commutated capacitors of Figure 6(a), we can now explore methods of translating transfer functions. We surmise that

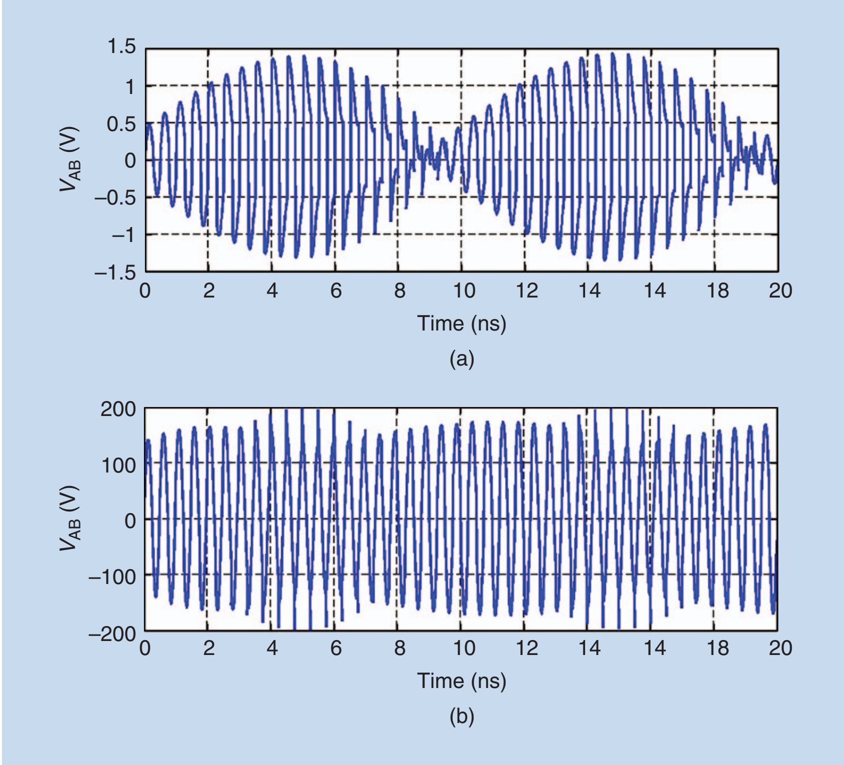

FIGURE 7: The voltage across a commutated network with (a) $R_1 C \gg T_{in}$ and (b) $R_1 C \ll T_{in}$.

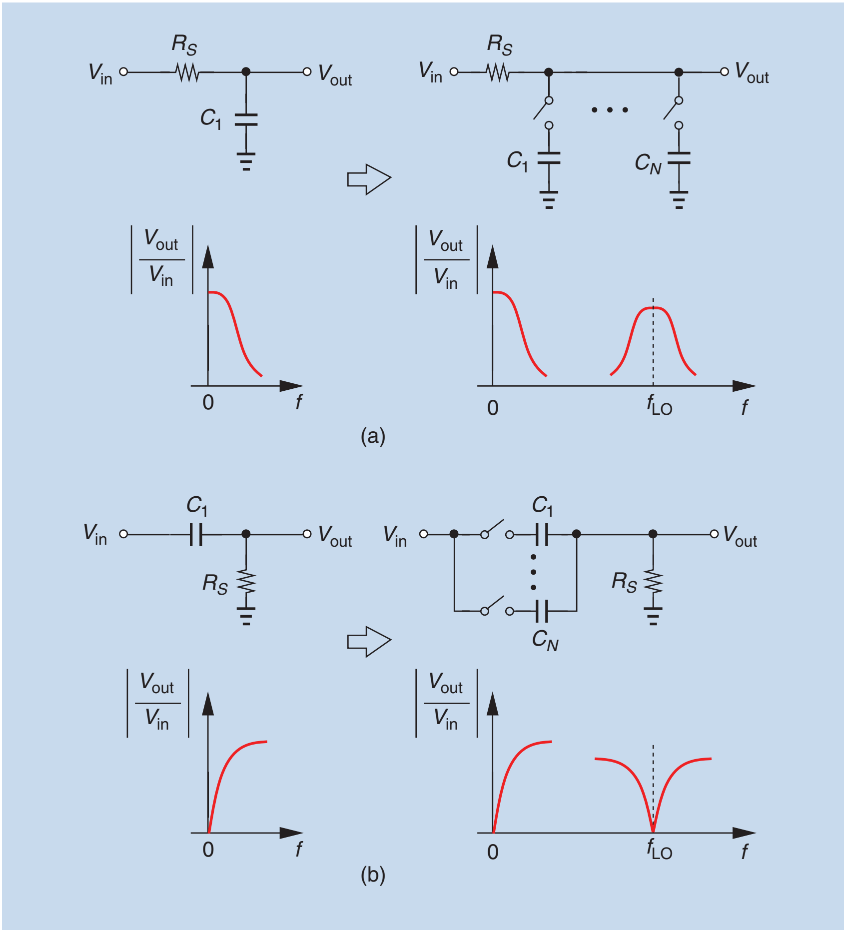

FIGURE 8: (a) Low-pass to band-pass and (b) high-pass to notch transformations.

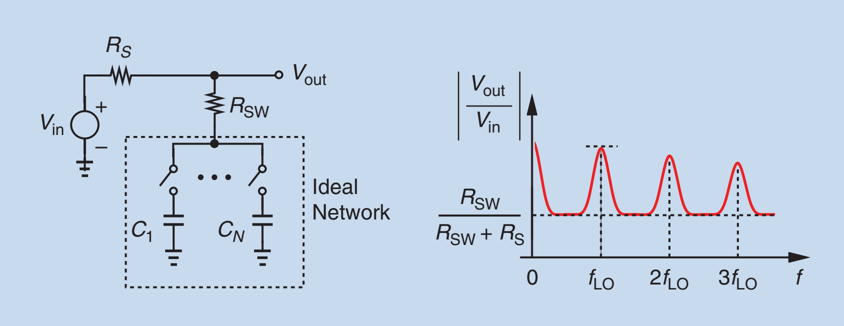

FIGURE 9: The effect of switch resistance on the transfer function.

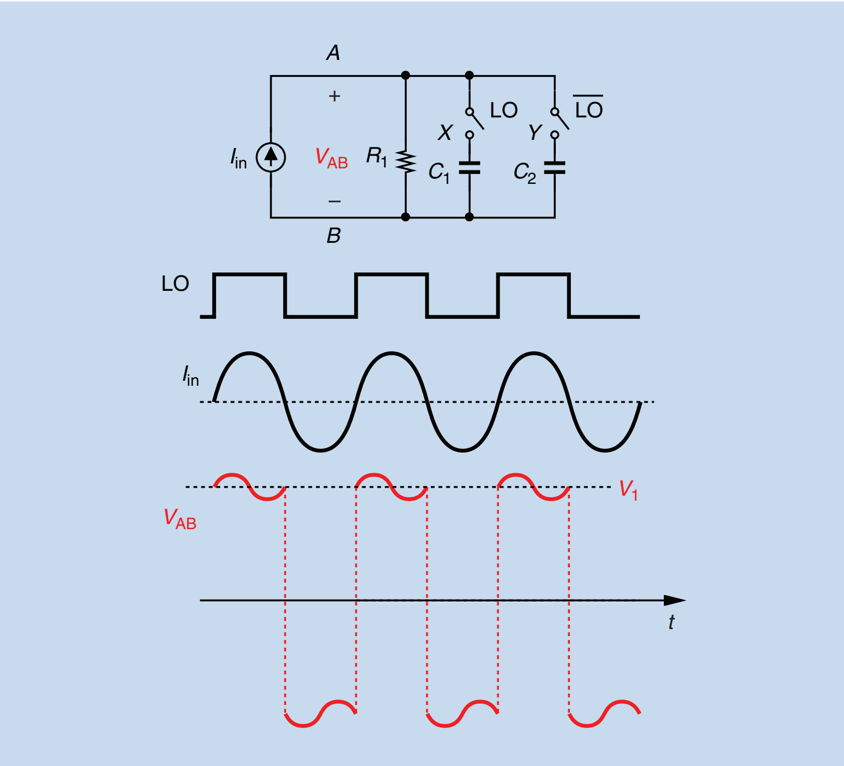

FIGURE 10: Steady-state waveforms in a commutated circuit.

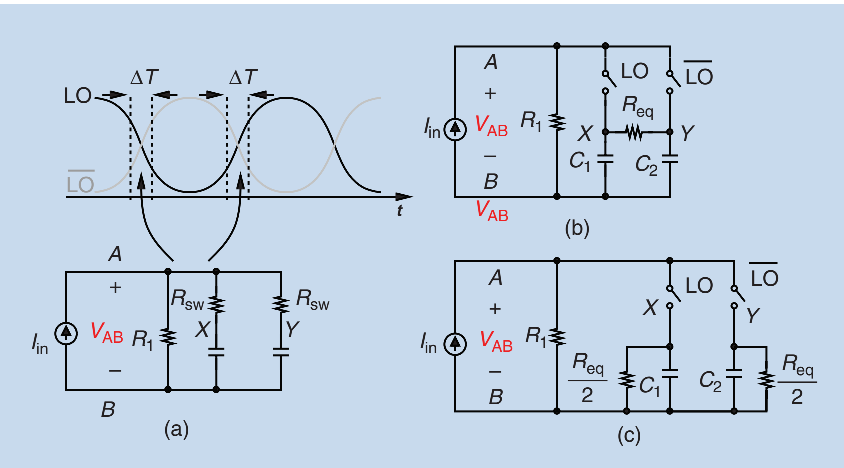

FIGURE 11: (a) The overlap time between LO and $\overline{LO}$, (b) the equivalent resistance for differential discharge, and (c) a single-ended equivalent.

replacing continuous-time capacitive branches in a network with their commutated counterparts permits such a translation.

Applying this procedure to a first-order RC filter yields the circuit in Figure 8(a) and a band-pass response around f_{LO} (and its harmonics). Similarly, as depicted in Figure 8(b), a high-pass section can be converted to a notch filter centered at f_{LO}. In both cases, $R_S C$ must be much greater than T_{LO}.

Recall from Figure 3 that Smith predicts the bandwidth of the response in Figure 8(a) to be $1/(N\pi R_S C)$. We can thus identify three critical attributes of this circuit: 1) a band-pass response with an arbitrarily narrow bandwidth centered around an arbitrarily high LO frequency, and this attribute represents an RF filter with an arbitrarily high quality factor Q; 2) a response with a well-defined, precise center frequency, f_{LO}; and 3) a precisely programmable center frequency. Absent in time-invariant RF filters, this unique combination enables accurate channel selection in RF receivers.

Smith's expression, $1/(N\pi R_S C)$, is intriguing in that it implies a reduction in the bandwidth as the number of commutated capacitors increases even though only one capacitor is tied to the output at a given point in time. This puzzle can be solved if we recognize in Figure 8(a) that the "dwell" time of the capacitors, i.e., the pulsewidth of the LO phases, decreases as N increases. Consequently, each capacitor has less time to interact with I_{in} and R_S and experiences a smaller voltage change—as if its value were larger.

Second-Order Effects

A number of phenomena degrade the performance of commutated networks and require attention. These include the on-resistance of the switches, R_{sw}, the translations of the source impedance, and the finite rise and fall times of the LO.

The on-resistance of the switches in Figure 8(a) can be factored out as illustrated in Figure 9(a), suggesting that the rejection at relatively large values of $|f_{in} - f_{LO}|$ is limited to $R_{sw}/(R_S + R_{sw})$ because the ideal commutated network itself exhibits a negligibly small impedance. To minimize R_{sw}, the width of the switches must be increased, demanding a higher power consumption in the LO path.

The magnitude of Z_{AB} at $f_{in} = f_{LO}$ in Figure 6(b) is about 7 dB lower than the source resistance. Why does this happen? Let us plot the steady-state output voltage as shown in Figure 10, noting that the voltage on, say, C_1 begins and ends at the same value, V_1, in every other half of the LO cycle. If $I_{in} = I_0 \sin \omega_{LO} t$ and the discharge of C_1 through R_1 is expressed as $V_1 \exp[-t/(R_1 C_1)]$, one can prove that $V_1 \approx 2R_1 I_0/\pi$ provided that $R_1 C_1 \gg T_{LO}$. That is, $V_{AB}(t)$ can be approximated by a square wave having a peak-to-peak amplitude of $4R_1 I_0/\pi$. It is remarkable that the circuit generates a square-wave voltage in response to a sinusoidal current—as if it contained harmonically scaled resonators at f_{LO}, $3f_{LO}$, etc. Since the first harmonic of V_{AB} has a peak amplitude of $(2R_1 I_0/\pi)(4/\pi) = (8/\pi^2)R_1 I_0$, we conclude that the impedance at $f_{in} = f_{LO}$ is equal to $(8/\pi^2)R_1 \approx 0.81 R_1$, about 1.8 dB lower than R_1.

The other 5-dB reduction in the peak impedance in Figure 6(b) arises from the overlap between the LO phases. Creating a temporary resistive path between the top terminals of C_1 and C_2, the overlap may simply occur because LO and $\overline{LO}$ have finite rise and fall times, keeping the two switches on simultaneously twice per period [Figure 11(a)]. The resulting differential discharge of C_1 and C_2 can be attributed to an equivalent resistance [Figure 11(b)] given by

$$R_{eq} = 2R_{sw} \frac{1}{2} \frac{T_{LO}}{\Delta T}, \qquad (1)$$

where $2R_{sw}$ is prorated according to the fraction of the period during which the switches are on, and the factor of 1/2 accounts for two such events per LO cycle. As illustrated in Figure 11(c), this effect translates to a resistance of $R_{eq}/2$ in parallel with each capacitor and hence parallel with Z_{AB}.

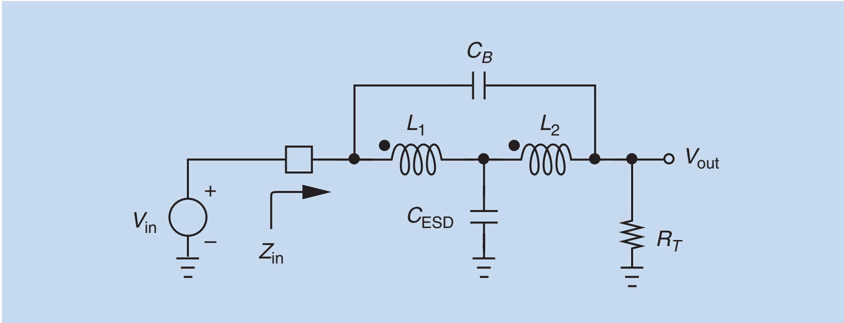

FIGURE 12: A T-coil network at input.

Questions for the Reader

1) The commutated capacitors of Figure 6(a) are placed at the antenna port of a GSM receiver so as to attenuate by 20 dB a 0-dBm blocker at 20-MHz offset. What issues does such a circuit face?
2) Does V_0 in Figure 10 change if the circuit contains four capacitive branches that are driven by 25% duty-cycle LO phases?

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Answers to Last Issue's Questions

1) Use a power dissipation argument to determine the transfer function of the circuit shown in Figure 12.

We know that $Z_{in} = R_T$ at all frequencies. That is, the power delivered by V_{in} to the circuit is equal to $V_{in,rms}^2/R_T$. For a lossless T-coil network, all of this power is delivered to R_T, generating an output equal to the input. The transfer function is thus equal to one.

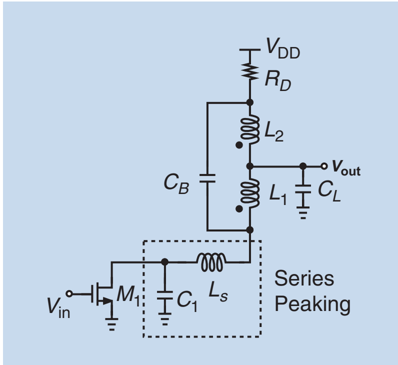

FIGURE 13: A T-coil network at output.

2) In Figure 13(a), how should L_S be chosen if the damping factor of the series peaking network must remain around $\sqrt{2}/2$?

Since the resistance seen by L_S on the right is equal to R_D, we reduce the circuit to the series combination of C_1, L_S, and R_D. For this network to have a damping factor equal to $\sqrt{2}/2$, we have $\zeta = (R_D/2)\sqrt{C_1/L_S} = \sqrt{2}/2$ and hence $L_S = R_D^2 C_1/2$.

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