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# The Cross-Coupled Pair—Part I

An elegant circuit is one that realizes a function efficiently. A beautiful circuit is one that stands the test of time. The cross-coupled pair (XCP) is such a topology: it has evolved for 95 years and adapted itself to various device technologies, supply voltages, and operation speeds. In this and future columns, we analyze this circuit's properties and study its applications in both analog and digital design.

## Brief History

The XCP was introduced in June 1919 in two independent papers published within four days of each other. Authored by Abraham and Bloch [1] and Eccles and Jordan [2], both papers exploited the XCP to create a multivibrator. (Abraham and Jordan coined this term to emphasize the harmonically rich output of the circuit.) These papers are difficult to find but Abraham and Bloch show a multivibrator circuit (Figure 1) in another paper that they published in December 1919 [3]. [For readers not familiar with vacuum tubes, terminals F, G, and P (filament, grille, and plaque, respectively, in French) are somewhat similar to the source, gate, and drain of a field-effect transistor, respectively.] Of course, concepts such as positive feedback and regeneration were well understood at the time. In the 1920s, van der Pol analyzed the multivibrator as a "relaxation" oscillator [4].

The XCP's utility as a bistable (memory) element was also recognized by Eccles and Jordan in another paper in December 1919 [5]. The "Eccles-Jordan flipflop" (what we call a regenerative latch today) was thus born. The ENIAC, the first general-purpose computing machine, incorporated this structure for storage [6]. After the invention of the bipolar transistor in the 1940s, the XCP naturally began to play similar roles in semiconductor circuits.

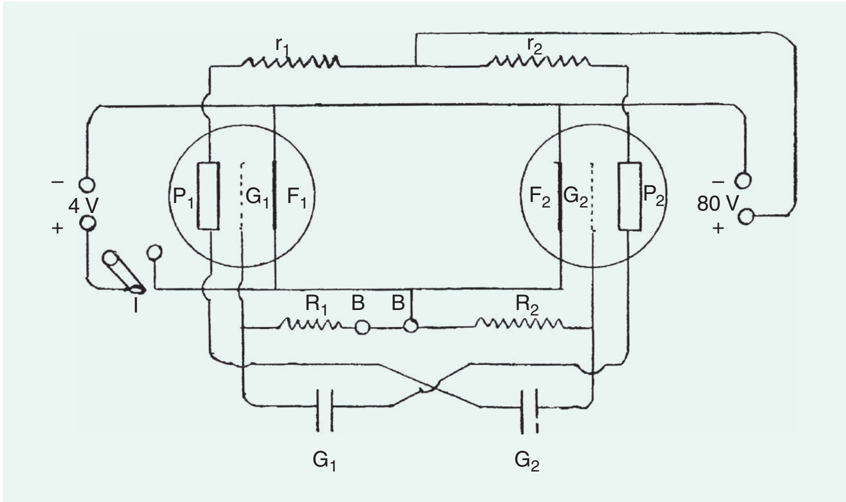


**FIGURE 1:** A multivibrator using cross-coupled triodes reported in 1919.

*The XCP was introduced in June 1919 in two independent papers published within four days of each other.*

In addition to serving as a memory cell, the XCP also emerged in two distinct types of digital systems, namely, as a regenerative component in emitter-coupled logic (ECL) circuits, shown in Figure 2(a) [7], [8], and as a sense amplifier in memories, shown in Figure 2(b) [9], [10].

The latter eventually morphed into comparators for use in analog design [11]. The use of the XCP as a negative-$G_m$ cell in semiconductor LC oscillators can be traced back to [12] [Figure 2(c)].

## Small-Signal Properties

If the XCP begins in or near equilibrium (with its drain voltages equal or close to each other), it behaves in the small-signal regime. Owing to the internal positive feedback, the pair can operate as an impedance "negator." Shown in Figure 3, the XCP produces an impedance of $Z_{in1} = -Z_1 - 2/g_m$ between the drains or $Z_{in2} = -Z_2 + 2/g_m$ between the sources.

Two special cases of Figure 3(a) are of particular interest. First, if $Z_1 = 0$, the pair exhibits a negative resistance equal to $-2/g_m$, serving numerous applications, from amplifiers to oscillators. Interestingly, the real value of $-2/g_m$ remains unchanged even if all of the circuit's capacitances are taken

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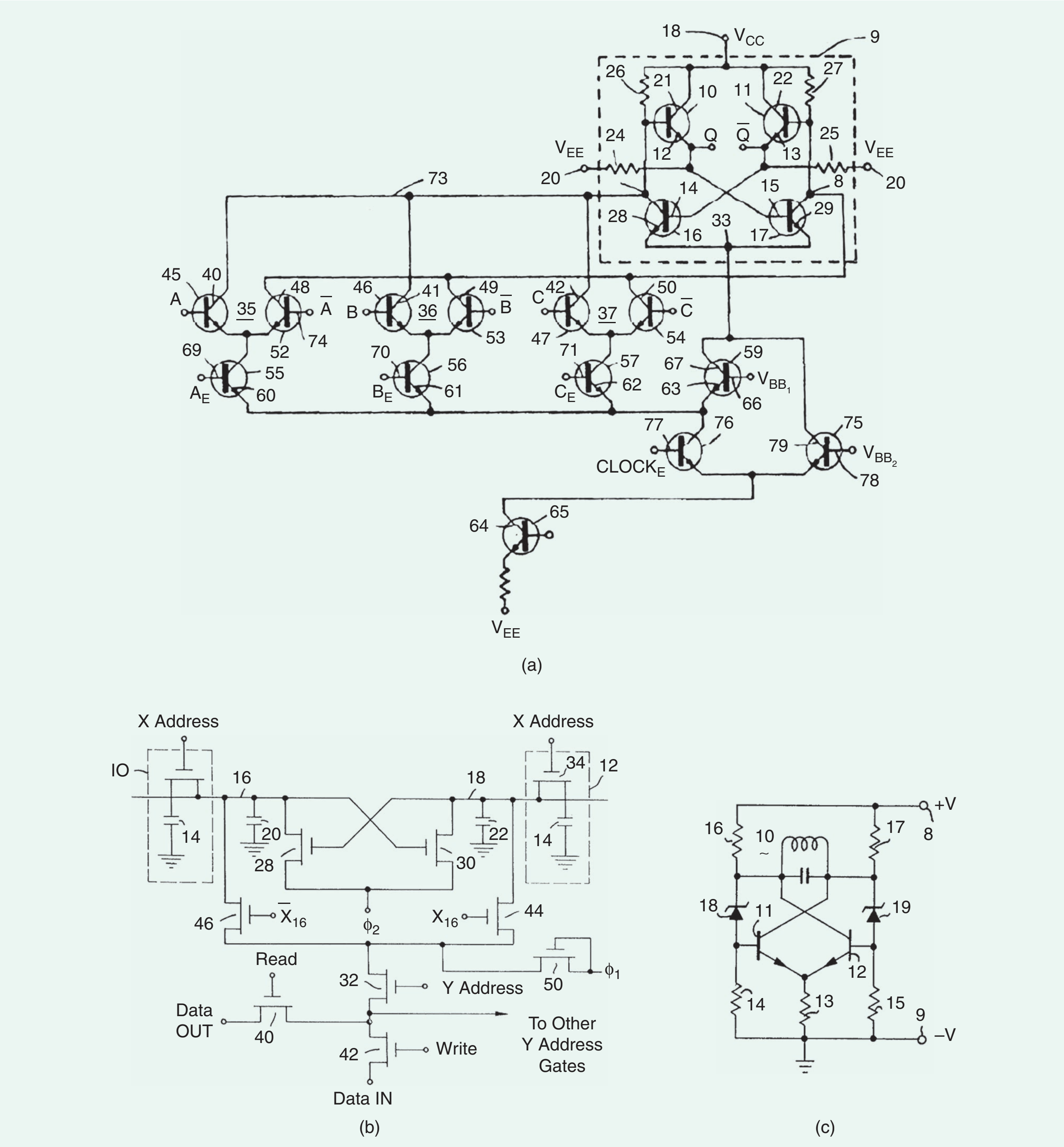


**FIGURE 2:** The early use of an XCP in (a) an ECL circuit, (b) a sense amplifier for memories, and (c) an LC oscillator.

into account; but in the presence of the gate resistance, it degrades to $[-g_m/2 + R_G C_{GS}\omega^2/2]^{-1}$ [13]. Second, if $Z_1$ is a capacitor, $Z_{in1}$ contains a negative capacitance, allowing the cancellation of positive capacitance at the drains. It can be shown that the input-referred noise voltage of the XCP with $Z_1 = 0$ is equal to $8kT\gamma/g_m$ (per unit bandwidth).

## Large-Signal Properties

The XCP can operate as a bistable element with zero static power dissipation, a versatile attribute exploited in memories and digital circuits. Consider, for example, the resistively loaded differential buffer shown in Figure 4(a), which draws a static current even with rail-to-rail inputs. Replacing the loads with a PMOS XCP as illustrated in Figure 4(b), we obtain an arrangement that consumes no static power while operating with rail-to-rail inputs and outputs. This topology also proves superior to two inverters in that the PMOS devices do not load the input. Moreover, the circuit can act as a dynamic reset-set (RS) latch. Of course, such concepts were not feasible in the vacuum-tube, bipolar, or GaAs predecessors of the XCP.

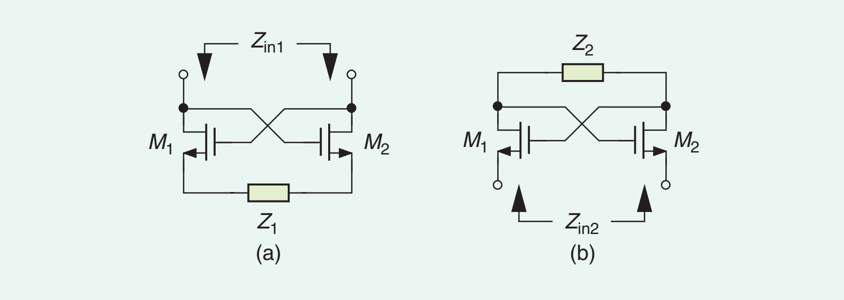


**FIGURE 3:** An XCP as an impedance negator.

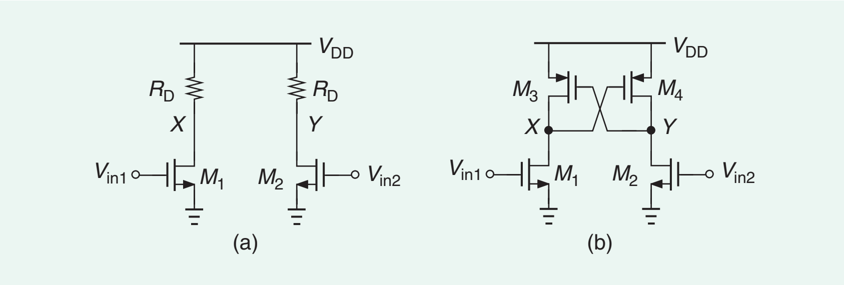


**FIGURE 4:** A differential buffer using (a) resistive loads and (b) the XCP.

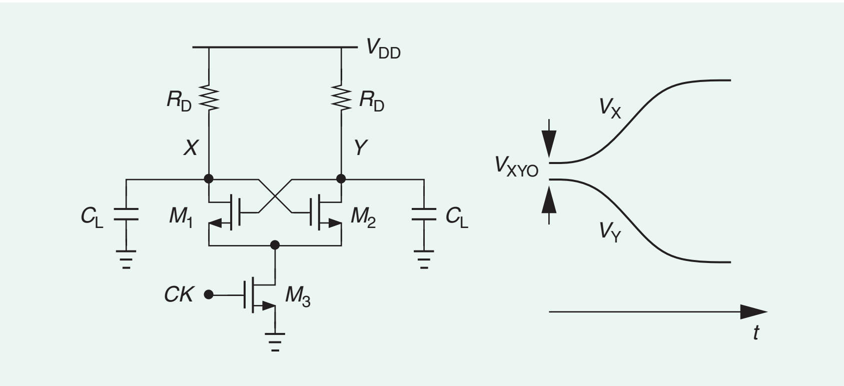


**FIGURE 5:** The regeneration behavior of XCP.

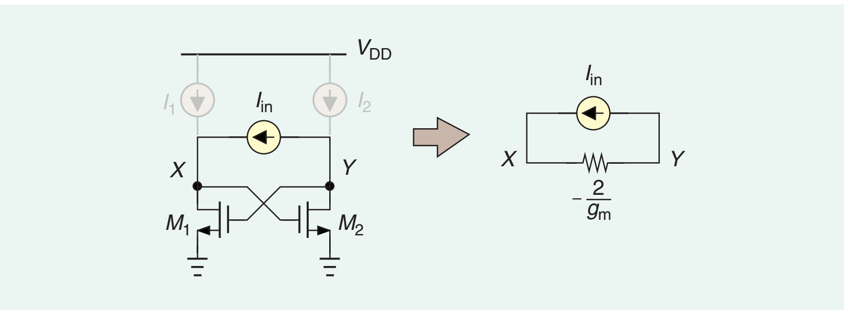


**FIGURE 6:** The XCP operation from two perspectives.

## Hysteresis Versus Amplification

The bistable pair in Figure 4(b) creates hysteresis in the circuit's input–output characteristic. If, for example, $M_1$ turns on while $V_X$ is high, $V_{in1}$ must rise enough for $M_1$ to overcome $M_3$, initiate regeneration around the loop, and change the state. The regeneration continues until $M_3$ turns off and $M_1$ enters the deep triode region, after which $V_{in1} - V_{in2}$ must become quite negative before the state is changed again. Due to the hysteresis in the circuit, only a large input swing can change the state.

A remarkable inflection point occurred in the late 1960s, when it was realized that the XCP could be *clocked*. The profound observation was that regeneration can begin only when needed, and, therefore, the circuit can amplify even small differences. Shown in Figure 5 is an example where $M_1$ and $M_2$ amplify an initial imbalance between $V_X$ and $V_Y$, $V_{XY0}$, according to

$$V_{XY}(t) = V_{XY0} \exp \frac{t}{\tau_{\text{reg}}}, \tag{1}$$

with $\tau_{\text{reg}} = R_L C_L/(g_m R_L - 1)$ denoting the small-signal regeneration time constant. This "synchronous amplification" property soon emerged in sense amplifiers for memory design.

***If the XCP begins in or near equilibrium (with its drain voltages equal or close to each other), it behaves in the small-signal regime.***

Equation (1) suggests that the XCP can provide infinite gain, another remarkable advantage over unclocked (asynchronous) amplifiers. The circuit's ability to regenerate small differences to logical levels proved useful in analog comparators but it also brought forth the problem of metastability. After all, the infinite gain accrues only if the circuit is given infinite time.

In the next column, we study digital applications of the XCP.

## Questions for the Reader

The foregoing overview raises a number of interesting questions:

1) Is negative capacitance the same as positive inductance?
2) Can the cancellation of positive capacitance by negative capacitance be a *resonance* effect?
3) Why is the circuit in Figure 4(b) a *dynamic* latch?

4) In Figure 6, $M_1$ and $M_2$ are biased and balanced by $I_1$ and $I_2$ $(I_1 = I_2)$. At $t = 0$, $I_{\rm in}$ jumps from zero to a small positive value, $I_0$. We intuitively expect that $V_X$ rises and $V_Y$ falls. However, viewing the XCP as a resistance equal to $-2/g_m$, we obtain $V_{XY} = (-2/g_m) I_0 u(t)$, concluding that $V_X$ should descend and $V_Y$ should ascend! How do we explain the discrepancy between these two results?

We will answer these questions in the next issue. You can share your thoughts by e-mailing me at razavi@ee.ucla.edu.

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