Effect of Temperature Gradients in Thermionic Plasmas

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Treatments of drift wave velocities in Cs and K plasmas up to now have neglected the radial electric field in the plasma. It is shown that this field greatly affects such measurements, but can be made negligible by reducing the cathode temperature gradient.

EXPERIMENTS in cesium and potassium plasmas up to now have been performed in Q devices in which the cathode temperature varies as much as ±50 to ±100°K across the surface. We wish to point out that such temperature gradients ∇T can have, and in the past probably have had, a serious effect on experimental results on drift waves and diffusion. Four possible effects of ∇T are considered: (1) effect on the density and potential distributions, (2) effect of radial drifts on diffusion measurements, (3) effect on drift wave velocities, and (4) possible new instabilities.

For simplicity we consider an electron-rich plasma between identical grounded hot cathodes in which end plate recombination is the dominant loss process. The electron and ion balance equations in equilibrium then give

$$j_T = n\bar{v}_e e^{-e\phi/KT}, \tag{1}$$

$$n = (Cj_0 j_T/T)^{\frac{1}{2}}, \tag{2}$$

where

$$j_T = AT^2 e^{-e\phi_w/KT} \quad \text{(Richardson current)}, \tag{3}$$

$$C \equiv (1-\beta)^{-1}(2\pi m_i/K)^{\frac{1}{2}}(2\pi m_e/K)^{\frac{1}{2}}. \tag{4}$$

Here j_0 is the ion input flux due to ionization of the neutral beam; β is the end plate re-ionization probability, whose temperature dependence we neglect; and the rest is self-explanatory. We assume $T_i = T_e = T_{\text{cath}} = T$ along each line of force. Taking the gradient of Eq. (2) with the help of Eq. (3), we find

$$\nabla \ln n = \tfrac{1}{2}[\nabla \ln j_0 + (1+\eta_w)\nabla \ln T], \tag{5}$$

where $\eta_w \equiv e\phi_w/KT$ is the work function normalized to the local temperature. Solving Eq. (1) for ϕ and taking the gradient with the help of Eq. (5), we find

$$\nabla \ln \phi = (2\eta)^{-1}\nabla \ln j_0 + [1-(2+\eta_w)/2\eta]\nabla \ln T, \tag{6}$$

where $\eta \doteq e\phi_w/KT$ is the plasma potential normalized to the local temperature.

(1) *Effect on n and* ϕ. From Eq. (5) it is seen that a nonuniformity in cathode temperature has a much larger effect on the density profile than a nonuniformity in neutral flux, because η_w is of order 25. If n is to be uniform to 10%, T must be uniform to 0.8%. Similarly, if $\nabla j_0 = 0$, the dominant term in Eq. (6) gives

$$\nabla\phi \approx -\tfrac{1}{2}\phi_w \nabla \ln T. \tag{7}$$

A transverse electric field is imbedded in the plasma by small gradients in cathode temperature. Again, ϕ_w appears as a large multiplying factor.

(2) *Effect on diffusion measurements.* A temperature gradient $\nabla T = T'\hat{\theta}$ in the azimuthal direction will give rise to a radial E/B drift of guiding centers, according to Eq. (7). This is of magnitude

$$|v_r| \approx \tfrac{1}{2}\eta_w\left(\frac{KT}{eB}\right)\left(\frac{T'}{T}\right).$$

This is to be compared with the Bohm diffusion velocity

$$|v_B| = \frac{1}{16}\left(\frac{KT}{eB}\right)\left(\frac{d}{dr}\right)\ln n.$$

Guiding centers drift along the isotherms in the cathode. If an isotherm crosses the collector with which one is trying to measure the Bohm diffusion rate, an anomalously large rate will result unless $|v_r| \ll |v_B|$. This requires

$$\left(\frac{1}{r}\right)\left(\frac{\partial T}{\partial\theta}\right) \ll \frac{T}{400\Lambda},$$

where Λ is the radial density scale length. If ∇T has the same scale length, we require $\Delta T/T \ll \frac{1}{4}\%$, which is almost impossible to achieve. In practice, the ∇T drift will be so large that plasma on the isotherms intersecting the collector will be quickly drained and will be replenished by diffusion from the interior. The diffusion rate is then correctly obtained as long as the isotherms are roughly circular; this requires only that $\partial T/\partial r \gg \partial T/r\,\partial\theta$.

(3) *Drift wave velocities.* The universal-type instabilities, in the $E = 0$ frame, have a perpendicular wave velocity close to but slightly less than the electron diamagnetic drift velocity v_{De}. If $\nabla T = 0$, Eq. (1) requires that a radial electric field E_r exist wherever a radial density gradient does. This E_r gives an E/B drift which is exactly equal and opposite to v_{De}, so that in the laboratory frame drift waves ideally should have only a very small velocity $v \ll v_{De}$ in the ion drift direction. On the other hand, centrifugal instabilities with $k_\parallel \approx 0$ travel in the ion direction and appear even faster in the lab frame. Previous reports[1-3] of drift waves traveling in the electron direction can be correct only if a radial temperature gradient existed in the experi-

ments. To calculate this effect, we write (top sign for ions, bottom for electrons):

$$\mathbf{v}_D = \frac{\mathbf{E}\times\mathbf{B}}{B^2} \mp \frac{\nabla p \times \mathbf{B}}{enB^2}$$

$$= -\frac{\nabla\phi\times\mathbf{B}}{B^2} \mp \frac{KT}{en}\frac{\nabla n\times\mathbf{B}}{B^2} \mp \frac{K}{e}\frac{\nabla T\times\mathbf{B}}{B^2}. \quad (8)$$

With the use of Eqs. (5) and (6), this becomes

$$\mathbf{v}_{De} = \left(\frac{KT}{eB}\right)(\eta_w - \eta + \tfrac{5}{2})(\nabla T \times \hat{z}/T), \quad (9)$$

$$\mathbf{v}_{Di} = -\left(\frac{KT}{eB}\right)[(\nabla j_0 \times \hat{z}/j_0) + (\eta + \tfrac{1}{2})(\nabla T \times \hat{z}/T)], \quad (10)$$

where $\hat{z}$ is a unit vector in the direction of B.

If $\omega/k_\perp = v_{De}$, we see that a drift wave in the lab frame has a phase velocity only if $\nabla T \neq 0$. This velocity is in the electron drift direction if $\partial T/\partial r < 0$. If $\partial T/\partial r \geq 0$, drift and centrifugal instabilities should appear as waves traveling in the ion direction. The frequency of a wave is often used to identify it with a drift wave; Eq. (9) shows that this can be done correctly only if the temperature gradient is accurately measured.[4] As an estimate of what constitutes a negligible temperature gradient, we set v_{De} (lab) $\ll v_{De}$ ($E = 0$ frame). Using Eq. (9) for v_{De} (lab), we find that $T'/T \ll 1/30\ \Lambda$ is required, or $\Delta T/T \ll 3\%$. If $\Delta T/T = 0.3\%$, then a variation of less than 8°K at 2700°K can be tolerated.

(4) ∇T *instabilities*. It is well known in the theory of universal instabilities that the plasma is unstable for $\partial \ln T/\partial \ln n < 0$ and for $\partial \ln T/\partial \ln n > 2$. Such gradients are unlikely in a Q device, but perhaps these instabilities can be deliberately excited by inverting the temperature gradient. Since electron emission varies greatly with temperature, there is a limit to the magnitude of the gradient that one can achieve. On the other hand, it is possible that the stability limits are different in the resistive case than in the collisionless case implied above. Note that the centrifugal instability[5] competes with the ∇T instability except in a small range of $\partial \ln T/\partial \ln n$.

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[4] This statement and the accuracy of Eq. (9) have been verified experimentally in detail by H. Hendel (to be published).
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