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COMMENTS ON THE ELECTRODYNAMICS OF PLASMAS*

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One of the more interesting problems concerning the foundations of the electrodynamics of continuous media is how to perform the proper averaging of microscopic variables to obtain macroscopic observables. These considerations can range from quantum-statistical mechanical ensemble-averaging techniques [1] to finite Larmor radius [2] corrections of macroscopic MHD to even obtaining the most relevant velocity [3] associated with a plasma differential element.

Recently an attempt [4] was made to reformulate the electrodynamics of plasma media in terms of the average guiding-centre drift current instead of the usual average convection current and certain interesting consequences resulted.

Unfortunately, the analysis was in error. The points in question will be discussed below.

Karlovitz [4] uses as his foundation of plasma electrodynamics a Poynting theorem [5] in the form:

$$\nabla \cdot (\vec{E} \times \vec{B}/\mu_0) = -\vec{E} \cdot \partial_t \vec{D} - \partial_t (B^2/2\mu_0) - (\vec{J}_g + \nabla \times \vec{M}) \cdot \vec{E} - (\vec{M}/\mu_0) \cdot \nabla \times \vec{E} \qquad (1)$$

which he obtains from the usual Poynting theorem $\nabla \cdot (\vec{E} \times \vec{H}) = -\vec{E} \cdot \vec{J}_f - \vec{H} \cdot \partial_t \vec{B} - \vec{E} \cdot \partial_t D$ by the substitution $\vec{J}_f = \vec{J}_g + \nabla \times \vec{M}$, the replacement $\vec{H} \rightarrow \vec{B}/\mu_0$, and the addition of $-(\vec{M}/\mu_0) \cdot \nabla \times \vec{E}$ to the right-hand side.

In Eq. (1) $\vec{J}_f$ represents the average conduction current, $\vec{J}_g$ the average guiding-centre drift current and all other variables have their usual (rationalized MKSQ) meanings.

He next makes the substitution $\vec{J}_f = \vec{J}_g + \nabla \times \vec{M}$ and the replacement $\vec{H} \rightarrow \vec{B}/\mu_0$ in the usual Maxwell $\partial_t \vec{D}$ equation to obtain

$$\nabla \times (\vec{B}/\mu_0) = \vec{J}_g + \nabla \times \vec{M} + \partial_t \vec{D} \qquad (2)$$

From Eqs (1) and (2) he invokes a consistency requirement to obtain the chief result of his paper, a new induction law,

$$-\partial_t \vec{B} = (1 + \vec{M} \cdot \vec{B}/B^2) \nabla \times \vec{E} \qquad (3)$$

This modified induction law, coupled with the diamagnetic properties of plasmas, would have numerous interesting properties.

The chief justification which Karlovitz uses for his ad-hoc choice of Eq. (1) is the insistence of <u>both</u> $\vec{E} \cdot \nabla \times \vec{M}$ and $\vec{M} \cdot \nabla \times \vec{E}$ work terms appearing. However, a straightforward, proper substitution of the above variables into the usual Poynting theorem gives

$$\nabla \cdot (\vec{E} \times \vec{H}) = -(\vec{J}_g + \nabla \times \vec{M}) \cdot \vec{E} - \vec{M} \cdot \nabla \times \vec{E} - \partial_t (B^2/2\mu_0) - \vec{E} \cdot \partial_t \vec{D} \qquad (4)$$

which meets this rather arbitrary requirement and keeps the standard form of Maxwell's equations intact. A most useful attribute of the Minkowski formulation of electrodynamics is the form-invariance of the equations for <u>all</u> moving, deforming media, with the medium properties entering into constitutive relations for the field vectors $\vec{E}, \vec{D}, \vec{B}, \vec{H}, \vec{J}$. Other formulations of moving-medium electrodynamics which do not have this property are discussed by Penfield and Haus [6].

In closing, one should note that the modified induction law, Eq. (3), is itself inconsistent with the rest of Maxwell's equations as they would require that

$$(\nabla \times \vec{E}) \cdot \nabla (\vec{M} \cdot \vec{B}/B^2) \equiv 0 \qquad (5)$$

which in general is <u>not</u> satisfied since $\vec{M}$ and $\vec{B}$ are independent.

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(Letter received 28 July 1969)

* Supported in part by the US Atomic Energy Commission

LOW-FREQUENCY PLASMA STABILIZATION BY FEEDBACK-CONTROLLED NEUTRAL BEAMS

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A number of feedback stabilization experiments have been carried out with electrostatic probes as sensors and also as control elements [1, 2]. Recently, such experiments have proved successful in suppressing collisional drift instabilities [3]. The theory of electrostatic feedback stabilization of drift waves has been formulated for several particular schemes [3 - 6].

The application of feedback control by means of material probes or electrodes is limited to plasmas of moderate density and temperature. We shall consider here a basically different approach, using neutral-beam injection to provide feedback-controlled volume sources of particle and momentum density. (The perturbations can be sensed by optical or microwave beams.) As a simple illustration, we discuss the stabilization of the resistive drift wave.

We consider a neutral beam incident on a plasma sheet $\partial n_0/\partial x = -n_0/\lambda$ lying in the yz-plane. The beam current I_b per unit area can be modulated arbitrarily in y, z, and t, as required for the feedback stabilization. The neutral beam is ionized in proportion to the zero-order density n_0 (since the beam amplitude is itself a first-order quantity in the stability analysis). There results the plasma source term $S = I_b n_0 \sigma_i v_e |V_x|^{-1}$ and the associated momentum source $M\vec{V}S$, where $\vec{V}$ is the beam velocity, v_e is the plasma electron velocity, and we assume $v_e \gg V \sim v_i$. Owing to charge exchange, there can be an additional momentum source $M\vec{V}S_c$, where $S_c = I_b n_0 \sigma_c$, but for simplicity we neglect this contribution.

The neglect of the x-dependence in S requires special comment. For the parameters appropriate to a thermonuclear reactor, a monoenergetic neutral beam is absorbed in a range of order λ about a penetration depth x_0 which turns out to be a convenient function of beam energy. By feedback-controlling the beam energy composition, one could in principle realize an arbitrary x-dependence of S. For purposes of controlling plasma transport in practice, we are, however, interested mainly in stabilizing long-wavelength modes with $k_x \sim \lambda^{-1}$ in the vicinity of some point x_0 where the plasma density gradient is maximal. For simplicity, we will therefore consider only the x-independent stabilization problem.

STABILIZATION OF RESISTIVE DRIFT WAVES

We shall use the standard two-fluid equations for the plasma [7], adding sources of density and momentum:

$$Mn\left(\frac{\partial \vec{v}_i}{\partial t} + \vec{v}_i \cdot \nabla \vec{v}_i\right) = en\left(-\nabla\phi + \vec{v}_i \times \vec{B}\right) - KT_i \nabla n - \nabla \cdot \pi + M\vec{V}S \qquad (1)$$

$$0 = -en(-\nabla\phi + \vec{v}_e \times \vec{B}) - KT_e \nabla n - n^2 e^2 \eta \vec{v}_e \qquad (2)$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v}_i) = \frac{\partial n}{\partial t} + \nabla \cdot (n\vec{v}_e) = S \qquad (3)$$

In equilibrium, we assume $\vec{B} = B\hat{\vec{z}}$, and define $\omega^* \equiv kv_{de}$, $\omega_i \equiv kv_{di}$, where $\vec{v}_{de} = \hat{\vec{y}}KT_e/eB\lambda$ and $\vec{v}_{di} = -\hat{\vec{y}}KT_i/eB\lambda$ are the electron and ion diamagnetic drift velocities. The resistivity η is characterized by $\omega_s \equiv (k_z/k)^2(B^2/Mn_0\eta)$. The ion cyclotron frequency is $\omega_c = eB/M$. The perturbed quantities are assumed to vary as $\exp i(ky + k_z z - \omega t)$, with the x-dependence neglected. This is strictly valid only for $k_x \gg k_y \gg \lambda^{-1}$, but the approximation is still good for the case of maximum interest here: $k_x \gtrsim k_y \gtrsim \lambda^{-1}$. For simplicity, we set $V_y = 0$. The "gain" of the feedback system is indicated by the magnitude of $\omega_f \equiv S/n_1$. The phase is given by $\theta = \arg(\omega_f/\omega^*)$.

Equations (1-3) are then linearized and combined in the usual fashion to give a quadratic equation for ω:

$$\omega(\omega - \omega_i - i\omega_f) + i\omega_s\left(\omega - \omega^* - i\omega_f + \frac{kV_x}{\omega_c}\omega_f\right) + \frac{V_x}{\lambda}\omega_f = 0 \qquad (4)$$

Here we have neglected ion viscosity and assumed $|\omega^*|$, $|\omega_i| \ll |\omega_c|$, $|k_z| \ll |k|$, and other standard approximations for the simple resistive drift wave, as discussed in Ref. [7] for the analogous dispersion relation with $\omega_f = 0$.

When $\omega_f = 0$ and $\omega_s \gg |\omega^*|$, Eq. (4) has the usual two solutions $\omega_a \approx \omega^* + i\omega^*(\omega^* - \omega_i)/\omega_s \equiv \omega^* + i\gamma$ and $\omega_b \approx \omega_i - \omega^* - i\omega_s$. Our objective is to use the neutral beam to stabilize ω_a without destabilizing ω_b. We find three basically different stabilizing mechanisms (characterized by their phases θ), which become important in various limits.

(I) <u>Density smoothing.</u> For $|V_x| \ll |\omega^*\lambda|$, the marginal stability requirement in (4) is $\omega_f^{(I)} = \omega_s(\omega - \omega^*)/\omega$; and for $\omega_s \gg |\omega^*| \gg |\omega_f|$ this becomes simply $\omega_f^{(I)} = -\gamma$. Thus, the phase shift θ is 180°, corresponding to a smoothing of the density perturbation by the neutral-beam injected plasma.

(II) <u>Simulation of minimum B.</u> For $|V_x| \gg |\omega^*\lambda|$, the pressure exerted by the momentum of the neutral beam becomes important. If the phase is chosen as $\theta = 0°$, then the beam pressure against the crest of the density perturbation is somewhat analogous to the effect of a favourable gradient of the zero-order magnetic field [8]. The marginal stability condition is $\omega_f^{(II)} = -\omega^*(\omega^* - \omega_i)\lambda/V_x$.

(III) <u>De-energization of the wave.</u> Still in the limit $|V_x| \gg |\omega^*\lambda|$, we find an alternative stabilizing mechanism at a phase of $\theta = -90°$. This mechanism works best for very small natural rates γ, corresponding to $\omega_s^2|k\lambda| \gg |\omega_c\omega^*|$, and then requires a gain $\omega_f^{(III)} = i\gamma\omega_c/kV_x$ for marginal stability. The momentum of the neutral beam exerts pressure against the region where the plasma is moving outward in x, and thus reduces the energy of the wave.

Comparing the effectiveness of these three approaches, we note that the gain required for stabilization by density smoothing (I) is rather large. If we define a time $\tau_R \equiv C\gamma^{-1}(n_0/n_1)$ for replacement of the plasma by the feedback injection, then $\tau_R^{(I)}$ corresponds to $C^{(I)} = 1$, which means that the improvement over the natural loss time depends only on the smallness of the achievable n_1/n_0. Momentum control can be more effective than density control if we assume $V_x \sim v_i$. Then $C^{(II)} = (\gamma/\omega^*)(ka_i)^{-1}$, so that the mechanism (II) has a relatively long plasma replacement time $\tau_R^{(II)}$ in the case of many plasma ion gyroradii $a_i = v_i/\omega_c$. The alternative momentum control mechanism (III) has $C^{(III)} = ka_i$, which is better than (II) only for very small (γ/ω^*) and is never better than (I).

NUMERICAL EXAMPLES

For any given set of parameters, several of the basic stabilizing mechanisms may be effective simultaneously, and the optimum phase angle (that requiring the lowest gain) will lie at intermediate values. In the present section we assume $-\omega_i = \omega^* > 0$ and use the illustrative parameters $b \equiv \frac{1}{2}(ka_i)^2 = 2\times 10^{-2}$ or 10^{-6}, $\omega_s/\omega^* = 10$ or 200, and $V_x < 0 < \lambda$. In Fig. 1, the momentum control is dominant ($P \equiv |V_x|/\lambda\omega^* = 7\times 10^2$). The growth rate $\Gamma = \mathrm{Im}(\omega)/\omega^*$ is given as a function of the phase angle θ for several values of gain, characterized by $|\omega_f/\omega^*|$.

The transition between stabilization by density control (I) and momentum control (II or III) is illustrated in Figs 2 and 3. For low P, the density control mechanism at $\theta \sim 180°$ is dominant. For large values of P, the most important mechanism is (II), except in the case having both small growth rate ($\omega_s/\omega^* = 200$) and not too small gyroradius

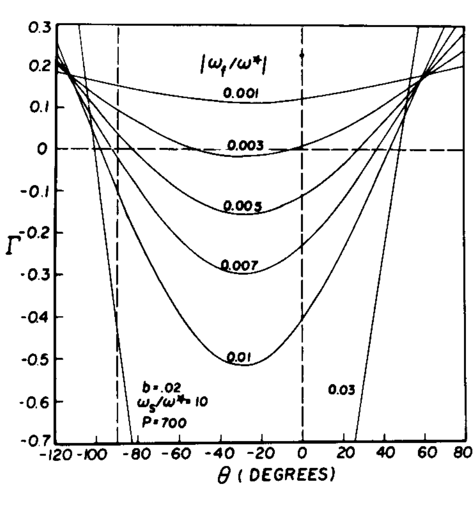

FIG. 1. Normalized growth rate Γ as a function of feedback phase θ for various values of feedback gain.

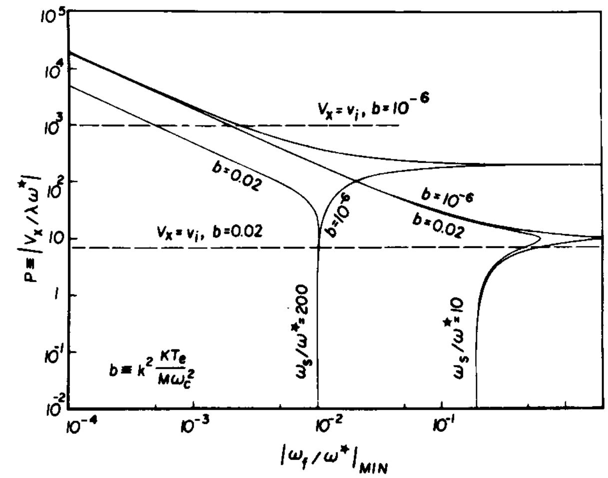

FIG. 2. Minimum gain required for stabilization as a function of normalized beam momentum P.

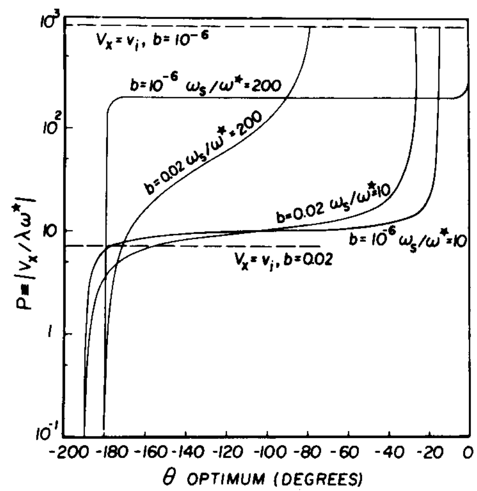

FIG. 3. Optimum phase angle θ as a function of beam momentum P for various combinations of operating parameters.

($b = 2\times 10^{-2}$), where (III) dominates. We note the strong interference between (I) and (II), but not between (I) and (III), at intermediate values of P.

For practical purposes, it is most natural to use a beam velocity $V_x \sim v_i$, and so a given value of b implies a particular operating point $P \sim b^{-\frac{1}{2}}$ in Figs 2 and 3. For $b = 2\times 10^{-2}$, the point P = 7 falls into the range dominated by mechanism (I); while for $b = 10^{-6}$, the point $P = 10^3$ falls into the range dominated by (II). The mechanism (III) never becomes important. We note that the optimum system parameters depend on the perturbation wave number k; but only $k \sim \lambda^{-1}$ is of principal importance for plasma transport.

DISCUSSION

In short-mean-free-path MHD-stable plasmas, it is fairly well established that drift waves can make a dominant contribution to anomalous plasma transport [9]. A marked effect of the collisional drift-wave amplitude on the plasma transport rate has indeed been demonstrated directly by electrode-based feedback experiments in a Q machine [3].

A basic problem facing plasma confinement in closed systems is that in the long-mean-free-path MHD-stable regime there does not yet exist an experimentally confirmed theoretical identification of the dominant modes causing plasma transport. The favoured theoretical candidates are trapped-particle instabilities of the magnetic or electrostatic types. These modes tend to have long wavelengths ($k \sim \lambda^{-1}$), as well as low frequencies and growth rates ($\omega \sim \gamma \sim \tau_{Bohm}^{-1}$), so that they show the general characteristics of "convective cells" [10]. We shall limit ourselves here to a qualitative discussion of the feedback problem based on our short-mean-free-path results.

When the plasma electrons have many collision times in an instability growth time, then quasi-static potential irregularities can exist within magnetic surfaces only by virtue of irregularities in the ion density. In this sense, the density-smoothing feedback mechanism (I) should be a panacea for low-frequency modes or "convective cells." Since the momentum-control mechanism (II) simulates minimum-B stabilization, it is also likely to have widespread effectivenes. The mechanism (III), which is aimed at the direct de-energization of instabilities, is the least likely to be generally effective, since it depends on the nature of the driving mechanism.

In applying neutral-beam feedback techniques for practical purposes, for example to a thermonuclear reactor, one must verify that both the resultant cross-field loss time τ_L and the time τ_R of plasma replacement are sufficiently long compared with the desired confinement time τ. If only MHD stability has been assured by the magnetic confinement geometry, the natural instability growth time and plasma loss time may be as short as τ_{Bohm}. For an anomalous transport scaling like $(n_1/n_0)^2$, application of the feedback technique would then lead to the conditions $\tau < \tau_L \sim (1/16)(n_0/n_1)^2 \tau_{Bohm}$, $\tau < \tau_R \sim C(n_0/n_1)\tau_{Bohm}$. Even with the density-control mechanism (I), where $C^{(I)} \sim 1$, this arrangement could be practical for $\tau \lesssim 10^2 \tau_{Bohm}$. With the momentum-control mechanism (II), the factor $C^{(II)}$ could be several orders of magnitude larger, and then the neutral beam current required for feedback is well below the injection currents that are sometimes envisaged for neutral-beam generation of the main reactor plasma.

Turning to some other practical points, we noted earlier that the beam break-up fraction for reactor parameters is conveniently of order unity. The frequency response of the beam is ample for any mode spectrum at or below the drift frequency. Spatial response can be provided for all mode numbers around the torus minor circumference; but the beam injection would have to be localized in a number of discrete stations around the major circumference. Favourable gradients of magnetic field strength could be introduced in the regions between neutral beam stations.

In present-day experiments, the feedback control of local plasma production (neutral beam injection or perhaps controlled ionization of a thermal gas feed) represents a rather advanced and costly technique relative to plasma self-stabilization in appropriate magnetic field configurations. Even if the latter approach should prove to be fully effective, we note that in a fusion reactor the cost considerations will be reversed, with the provision of complex stabilizing magnetic fields representing a large incremental factor in reactor size, whereas feedback control of non-MHD plasma instabilities may impose a relatively small additional burden.

ACKNOWLEDGEMENT

We should like to acknowledge helpful discussions with Drs M.N. Rosenbluth, P.H. Rutherford, and J.B. Taylor, and with the authors of Ref. [3]. The work was done under the auspices of the US Atomic Energy Commission, Contract AT(30-1)-1238.

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