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I. Introduction

In experiments on nonlinear effects induced by high-powered CO_2-laser pulses impinging on a magnetically confined plasma target, the temperature of the plasma is raised by resistive heating. Typical targets are θ-pinches and magnetically confined arcs. Since the thresholds for nonlinear phenomena depend on T_e, we wish to calculate the change in T_e caused by the laser beam itself. We consider pulses short enough that the ions do not have time to be heated by the electrons, and B fields strong enough that $\omega_c{}^2\tau^2$ is much larger than 1 for electrons. The temperature is limited in the steady state by heat conduction transverse to $\underline{B}$. We find that axially directed gigawatt beams can heat the plasma to an equilibrium temperature of several tens of eV independently of density, but that in practice this equilibrium temperature is not reached because of the finite duration of the laser pulse and possibly also because of inelastic collisions with partially ionized ions.

II. The High-Frequency Absorption Coefficient

We first evaluate the inverse bremsstrahlung absorption coefficient α according to the new corrected formula given by Johnston and Dawson[1]:

$$\alpha = \frac{7.8 \times 10^{-9}\ Zn_e{}^2\ \ln \Lambda(\nu)}{\nu^2 T_{eV}^{3/2}(1-\omega_p{}^2/\omega^2)^{1/2}}\ \text{cm}^{-1} , \qquad (1)$$

where $\omega = 2\pi\nu$ is the laser frequency, equal to 1.78×10^{14} for CO_2. For $\omega > \omega_p$, Λ depends on ω; for CO_2, it is given approximately by

$$\Lambda = 5\ T_{eV} \quad . \tag{2}$$

This assumes that $T_e \lesssim 30$ eV, so that quantum effects can be neglected in evaluating the minimum impact parameter.

A more convenient quantity to remember is the electron-ion collision frequency ν_{ei}, related to α by

$$\nu_{ei} = c\alpha\omega^2/\omega_p{}^2 \quad , \tag{3}$$

Eq. (3) comes from the dispersion relation for light waves with collisional damping:

$$\epsilon = \frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p{}^2}{\omega(\omega+i\nu_{ei})} \quad , \tag{4}$$

$$k = \frac{\omega}{c}\left(1 - \frac{\omega_p{}^2/\omega^2}{1+i\nu_{ei}/\omega}\right)^{1/2} \quad .$$

The imaginary part of k is, for $\omega_p{}^2 << \omega^2$, $\nu_{ei} << \omega$,

$$\mathrm{Im}\ k = \frac{1}{2}\alpha = \frac{1}{2}\frac{\nu_{ei}}{c}\frac{\omega_p{}^2}{\omega^2} \quad , \tag{5}$$

which yields Eq. (3). The factor $\frac{1}{2}$ in Eq. (5) appears because α is the absorption coefficient for intensity rather than amplitude. Eqs. (1) and (3) give, for $\omega^2 >> \omega_p{}^2$,

$$\nu_{ei} = 2.94 \times 10^{-6}\ \frac{n_e \ln \Lambda}{T_{eV}^{3/2}} \quad , \tag{6}$$

in agreement with Heald and Wharton[2]. For $T \sim 5$ eV, $\ln \Lambda$ is 3.2, according to Eq. (2). The collision rate for CO_2 and $n = 10^{16}$ cm^{-3} is then

$\nu_{ei}/\omega \approx 5 \times 10^{-5}$.

III. Heat Conductivity

The equation of continuity for electron heat flow is

$$\frac{\partial}{\partial t}\left(\frac{3}{2} n\kappa T_e\right) + \underline{\nabla} \cdot \underline{q}_e = Q \qquad \text{ergs/cm}^3\text{/sec} \quad , \tag{7}$$

where Q is the heat source and $\underline{q}_e$ the electron heat flux. For flow parallel to $\underline{B}$, $\underline{q}_e$ is given by[3]

$$q_{\parallel} = -C_e \frac{n\kappa T_e}{m\nu_e} \nabla_{\parallel} \kappa T_e \equiv -K_{\parallel}\nabla_{\parallel}T_e \qquad \text{ergs/cm}^2\text{/sec} \quad , \tag{8}$$

where κT_e is in ergs, T_e is in °K, $K_{\parallel}$ is the parallel heat conductivity in ergs/cm/sec/°K, and C_e is a numerical constant computed to be 3.1616. In parallel conduction, only electron-ion collisions matter, and we may replace ν_e in Eq. (8) by the ν_{ei} of Eq. (6). The value of $K_{\parallel}$ is then

$$K_{\parallel} = 2.6 \times 10^5 \; T_{eV}^{5/2}/\ln \Lambda \qquad \text{ergs/sec/cm/°K} \tag{9}$$

The density has cancelled out because both the energy density and the frictional drag are proportional to n.

For conduction perpendicular to B, K is reduced by the usual factor $1+\omega_c^2\tau^2$, the last term being the square of the ratio of cyclotron frequency to collision frequency or of mean free path to Larmor radius.

$$K_{\perp} = K_{\parallel}/(1+\omega_c^2/\nu_e^2) \quad . \tag{10}$$

However, electron-electron collisions also contribute to cross-field conductivity;

and for ν_e in Eq. (10), both explicitly and in evaluating $K_{\parallel}$, one must substitute the total collision frequency $\nu_{ei}+\nu_{ee}$. For ν_{ee} one may use the reciprocal of Spitzer's[4] self-collision time t_{ee}:

$$\nu_{ee} = \frac{n \ \ln \Lambda}{11.4 \ A^{1/2} T_e^{\ 3/2}} = 2.9 \times 10^{-6} \frac{n \ \ln \Lambda}{T_{eV}^{3/2}} \quad . \tag{11}$$

This has the same apparent value as ν_{ei} in Eq. (6), but in Sec. III $\ln \Lambda$ is to have its ordinary value instead of the high-frequency one, as in Sec. II. For instance, at $n = 10^{15}$ cm^{-3} and $\kappa T_e = 5$ eV, $\ln \Lambda$ is 8.6.

Although both ν_{ei} and ν_{ee} contribute to transverse heat conductivity, the physical mechanisms are different. In an electron-ion collision, little energy is transferred to the ion, but the electron migrates across B, carrying its energy with it. When an electron collides with another electron, there is no net particle diffusion, but the energy can be transferred to an electron with its guiding center on a different line of force.

At B = 4kG, $n = 5 \times 10^{15}$ cm^{-3}, and $T_e = 5$ eV, $\omega_c^{\ 2}/\nu_e^{\ 2}$ has the value 8. Thus, for lower densities and higher fields and temperatures we may assume $\omega_c^{\ 2}/\nu_e^{\ 2} \gg 1$ and neglect the 1 in the denominator of Eq. (10). Taking half the value of $K_{\parallel}$ given in Eq. (9) because $\nu_e = \nu_{ei} + \nu_{ee} \simeq 2\nu_{ei}$, we then find

$$K_{\perp} = 1.4 \times 10^{6} \frac{n_{16}^{2} \ \ln \Lambda}{T_{eV}^{1/2} B_{kG}^{2}} \qquad \text{ergs/sec/cm/°K} \quad , \tag{12}$$

where n_{16} is the density in units of 10^{16} cm^{-3}. We assume n and B to be uniform and T_{eV} to vary spatially; $K_{\perp}$ is not constant. Eqs. (7) and (8) give for the heat flow

$$\frac{3}{2} n\kappa \frac{\partial T}{\partial t} - \nabla_{\parallel} \cdot (K_{\parallel}\nabla_{\parallel}T) - \nabla_{\perp} \cdot (K_{\perp}\nabla_{\perp}T) = Q \quad . \tag{13}$$

IV. Heating with Axial Beams

When the laser beam is incident along the axis of a cylindrical plasma in a magnetic field $\underline{B}$, the focal region is a long, thin filament aligned with $\underline{B}$. In this case, heat conduction is radial, and Eq. (13) becomes

$$\frac{3}{2} n\kappa \frac{\partial T}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}\left(rK_{\perp} \frac{\partial T}{\partial r}\right) = Q. \quad . \tag{14}$$

To evaluate Q, we note that the beam intensity after traversing a heating region of focal depth L is

$$I = I_0 e^{-\alpha L} \qquad \text{ergs/cm}^2\text{/sec} \quad . \tag{15}$$

In an underdense plasma where $\alpha L << 1$, the energy lost is

$$I_0 - I = I_0(1 - e^{-\alpha L}) \approx I_0 \alpha L \quad . \tag{16}$$

Hence the heat input to the plasma is

$$Q = (I_0 - I)/L = \alpha I_0 \qquad \text{ergs/cm}^3\text{/sec} \quad . \tag{17}$$

From Eq. (1) for the case of CO_2 lasers and $\omega_p{}^2 << \omega^2$, we have

$$Q = 9.7 \times 10^{-4} Z n_{16}^2 \; \ell n \; \Lambda(\nu) I \; T_{eV}^{-3/2} \qquad \text{ergs/cm}^3\text{/sec} \quad . \tag{18}$$

Let us first calculate the equilibrium temperature in the steady state in which the heat input is balanced by radial conduction. Dropping the time-dependent term in Eq. (14), changing from T° to T_{eV}, and assuming that n and B, and therefore $K_\perp T_{eV}^{1/2}$, are constant, we have

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\, T_{eV}^{-1/2}\frac{\partial T_{eV}}{\partial r}\right) = -Q/(11{,}600)K_\perp T_{eV}^{1/2} \ . \qquad (19)$$

From Eqs. (12) and (18), we obtain

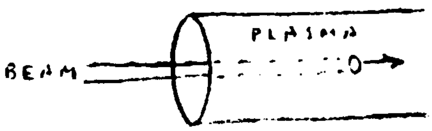
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\, T_{eV}^{-1/2}\frac{\partial T_{eV}}{\partial r}\right) = -0.6\,\frac{Z I_{MW} B_{kG}^2}{T_{eV}^{3/2}}\,\frac{\ln \Lambda(\nu)}{\ln \Lambda} \ , \qquad (20)$$

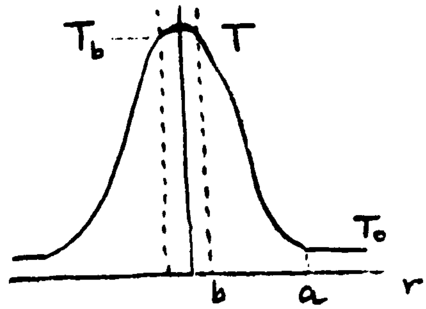
where I_{MW} is the laser beam intensity in MW/cm^2. The ratio of $\ln \Lambda$ values for high and low frequencies has a value of about $\frac{1}{2}$. We thus have the nonlinear differential equation

$$T^{3/2}\, r^{-1}(r\, T^{-1/2} T^*)^* + C = 0 \ , \qquad (21)$$

where $C \simeq 0.3\, I_{MW} B_{kG}^2$ for a singly ionized plasma, if T is in eV. The density does not appear in this equation because both the energy absorption and the thermal conductivity are proportional to n^2. The temperature in steady state, therefore, does not depend on the density.

It is not generally possible to solve for the temperature distribution within the focal spot, since neither the density nor the intensity distribution is known accurately there. Because of the small radius of the focal region, we may assume that T inside is only slightly larger than T_b, the temperature at the boundary of the focal region.





In the exterior region, T satisfies the equation

$$rT^{-1/2}T' = A = \text{const.} \quad , \tag{22}$$

or

$$\frac{dT}{T^{1/2}} = \frac{A}{r}\,dr \quad . \tag{23}$$

Integrating from the beam boundary r = b to a radius r = a where the temperature is at the initial value T_0, we have

$$T_0^{1/2} - T_b^{1/2} = \frac{1}{2} A \,\ell n(a/b) \quad . \tag{24}$$

The constant A is found from the condition that the total heat input is equal to the heat flux across the cylinder at r = b:

$$\pi b^2 Q = 2\pi b q = -2\pi b \; K_\perp T' \quad , \tag{25}$$

or

$$-T' = [bQ/2K_\perp]_{r=b} \quad {}^\circ K/cm \quad . \tag{26}$$

Eq. (22) then becomes

$$A = -b^2 Q/2K_\perp T_b^{1/2} \quad . \quad ({}^\circ K)^{1/2} \tag{27}$$

Substituting from Eqs. (12) and (18) and converting to eV, we find

$$A = -32b^2 \frac{\overline{I}_{MW} B_{kG}^2}{T_{eV}^{3/2}} \frac{Z \ln \Lambda(\nu)}{\ln \Lambda} \quad . \tag{28}$$

Eq. (24) then gives the steady-state temperature:

$$[T_b^{3/2}(T_b^{1/2} - T_0^{1/2})]_{eV} = 0.15\ b^2\ \overline{I}_{MW} B_{kG}^2 \frac{Z \ln \Lambda(\nu)}{\ln \Lambda} \ln \frac{a}{b} \quad . \tag{29}$$

Since the incident power is $P_{MW} = \pi b^2 \overline{I}_{MW}$ and $Z \ln \Lambda(\nu)/\ln \Lambda \sim 0.5$ for $Z = 1$, this becomes

$$[T_b^{3/2}(T_b^{1/2} - T_0^{1/2})]_{eV} = .024\ P_{MW} B_{kG}^2 \ln(a/b) \quad . \tag{30}$$

The temperature depends on the beam power, not on its intensity. The radius a is determined by the dimensions of the plasma or by the volume of plasma that can be heated during the pulse. For the particular case $T_b^{1/2} \gg T_0^{1/2}$, $B_{kG} = 4$, and $a/b = 10$, we have

$$T_{eV}^2 = 0.87\ P_{MW} \quad . \tag{31}$$

We next investigate the minimum rise time of the temperature under the condition that the heat conduction is negligible. Eq. (14) then becomes

$$\frac{3}{2} n\kappa \frac{\partial T}{\partial t} = Q \tag{32}$$

or, from Eq. (18),

$$\frac{\partial T_{eV}}{\partial t} = \frac{2}{3} \frac{Q}{1.6 \times 10^4 n_{16}} = 4 \times 10^{-8}\ Z n_{16} I \ln \Lambda(\nu)\ T_{eV}^{-3/2} \quad , \tag{33}$$

$$\frac{\partial}{\partial t} \left(\frac{2}{5} T_{eV}^{5/2} \right) = 0.4\ n_{16} I_W Z \ln \Lambda(\nu) \tag{34}$$

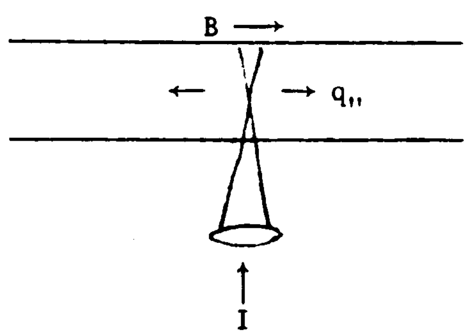
$$T_{eV}^{5/2} - T_0^{5/2} \approx 3\, n_{16} I_W\, t \quad . \tag{35}$$

Here we have taken Z = 1 and $\ln \Lambda(\nu) = 3$, since $\ln \Lambda(\nu)$ varies from 2 to 5 as T_{eV} varies from 1.5 to 30. Eq. (35) shows a sensitivity to density and spot size, which Eq. (31) does not.

As a numerical example, consider 25-ns, 1 GW beam focussed to 1 mm diameter in a plasma of initial temperature T_0 = 2 eV and density n_{16} = 1 in a 4 kG magnetic field. For a/b = 10, Eq. (30) yields T_b = 34 eV, while Eq. (35) yields 39 eV. Since the latter does not greatly exceed the former, there is insufficient time to reach the equilibrium temperature, and the nonlinear heat equation Eq. (14) must be solved. We can, however, estimate the time-limited temperature by considering the source Q to be spread out over a volume larger than the focal region. As a pessimistic limit, let the beam be spread over the area πa^2 instead of πb^2. Q is then reduced by 10^2, and T_{eV} by $(10^2)^{2/5} = 6.3$. The final temperature is then 6 eV instead of 39 eV. Since T_{eV} varies as $[\ln(a/r)]^2$ outside the beam, the temperature inside will be considerably above 6 eV but below 34 eV.

V. Heating with Transverse Beams

If the laser beam is incident along a radius of the plasma, both parallel and perpendicular heat conduction will occur. If B is large



enough, parallel conduction will dominate, and we may neglect the third term in Eq. (13). We then have

$$\frac{3}{2} n\kappa \frac{\partial T}{\partial t} - \frac{\partial}{\partial z}\left(K_{\shortparallel} \frac{\partial T}{\partial z} \right) = Q \quad , \qquad \text{ergs/cm}^3\text{/sec} \tag{36}$$

with $K_{\shortparallel}$ and Q given by Eqs. (9) and (18), respectively. Again, we may look for a steady-state solution by neglecting the first term. Since $K_{\shortparallel} T_{eV}^{-5/2}$ is a constant, according to Eq. (9), Eq. (36) becomes

$$\frac{\partial}{\partial z}\left(T_{eV}^{+5/2} \frac{\partial T}{\partial z} \right) = - \frac{Q}{K_{\shortparallel}} T_{eV}^{5/2} \quad , \tag{3 }$$

or

$$\frac{\partial^2}{\partial z^2} T_{eV}^{7/2} = - \frac{7}{2} (11{,}600)^{-1} \frac{Q T_{eV}^{5/2}}{K_{\shortparallel}} \tag{38}$$

$$= -11.3 \; Z n_{16}^2 I_{MW} \ln \Lambda(\nu) \ln \Lambda \; T_{eV}^{-3/2} \tag{39}$$

$$\simeq -300 \; n_{16}^2 I_{MW} T_{eV}^{-3/2} \quad . \tag{40}$$

In the last step, we have taken Z = 1 and $\ln \Lambda(\nu) \ln \Lambda = 25$. If we consider the region inside the beam and let $T_{eV} = T_0 + T_1$, where T_0 is the uniform initial temperature, Eq. (40) can be written

$$T_{eV}^{3/2} \left(T_{eV}^{5/2} T_1'' + \frac{5}{2} T_{eV}^{3/2} T_1'^2 \right) = -88 \; n_{16}^2 I_{MW} \quad . \tag{41}$$

Let $I_{MW} = P_{MW}/\pi b^2$ and let ℓ be the scale length of T_1. Eq. (41) then becomes

$$T_{eV}^4 (T_{eV} - T_0) + \frac{5}{2} T_{eV}^3 (T_{eV} - T_0)^2 = 28 \; n_{16}^2 P_{MW} (\ell^2/b^2) \quad . \tag{42}$$

The result previously obtained by Jassby and Marhic[5] corresponds to neglecting the second term in Eq. (42) and replacing the factor 28 by 42.

As explained in Sec. IV, however, the temperature gradient outside the beam is more important than that inside. We should therefore examine the region $|z| > b$, where $I_{MW} = 0$. Eq. (40) then yields

$$T_{eV}^{7/2} = Az + B \quad , \tag{43}$$

or, for $T_{eV} = T_0$ at $z = a$,

$$T_b^{\ 7/2} - T_0^{\ 7/2} = -A(a-b) \quad . \tag{44}$$

The heat flux across the boundary $z = b$ is

$$q = -K_{\parallel} \frac{\partial T}{\partial z} = -K_{\parallel}(11{,}600) \frac{\partial T_{eV}}{\partial z} \quad . \tag{45}$$

From Eq. (43), we have

$$\frac{7}{2} T_{eV}^{5/2} T_{eV}' = A \quad , \tag{46}$$

so that

$$-A = \frac{7}{2} T_b^{\ 5/2} q/11{,}600\ K_{\parallel} \quad . \tag{47}$$

The total heat input into the beam of focal depth L is

$$\pi b^2 LQ = 2bLq \quad . \tag{48}$$

Eqs. (18) and (47) then give

$$-A = \frac{7}{2} T_b^{5/2} \frac{\pi b^2 (9.7 \times 10^{-4}) Z n_{16}^2 \ln \Lambda(\nu) \overline{I}\, T_{eV}^{-3/2}}{2b(11{,}600) 2.6 \times 10^5\ T_b^{5/2}/\ln \Lambda}$$

$$= 18b\ Z n_{16}^2 \overline{I}_{MW} \ln \Lambda(\nu) \ln \Lambda / T_{eV}^{3/2} \quad . \qquad (49)$$

Here T_{eV} is the average temperature inside the beam and may be approximated by the edge temperature T_b. Again taking $Z = 1$, $\ln \Lambda(\nu) \ln \Lambda = 25$, we have

$$-A \simeq 440\ b n_{16}^2 \overline{I}_{MW} / T_b^{3/2} \qquad (50)$$

and

$$T_b^{3/2}(T_b^{7/2} - T_0^{7/2}) = 440 b(a-b) n_{16}^2 \overline{I}_{MW} \quad . \qquad (51)$$

$$\simeq 440 a b n_{16}^2 \overline{I}_{MW} = 140 (a/b) n_{16}^2 P_{MW} . \qquad (52)$$

Comparing this with Eq. (30) for transverse conduction, we see that the expression for T_b depends on n^2 rather than on B^2. Specifically, for $T_b >> T_0$, we have

$$T_b \simeq 2.7 (a/b)^{1/5} n_{16}^{2/5} P_{MW}^{1/5} \qquad \text{(transverse beam)} \qquad (53)$$

$$T_b \simeq 0.15 [\ln(a/b)]^{1/2} B_{kG} P_{MW}^{1/2} \qquad \text{(axial beam)} \ . \qquad (54)$$

The dimension a is difficult to estimate, but it enters only weakly; it is related to the length of the plasma column in Eq. (53) and to the radius in Eq. (54). For example, if B = 4kG, P = 1 GW, $n = 10^{16}$ cm^{-3}, b = .05 cm, and a = 2 cm and 0.5 cm, respectively, Eqs. (53) and (54) give

T_b = 22 eV (transverse beam)

T_b = 29 eV (axial beam) .

VI. Heating in Absence of Magnetic Field

If $\omega_c/\nu_{ei} \ll 1$, we must use $K_{||}$ in the radial heat equation. Eq. (14) becomes

$$\frac{3}{2} nK \frac{\partial T}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}\left(rK_{||}\frac{\partial T}{\partial r}\right) = Q \ , \qquad (55)$$

where, from Eq. (18),

$$Q = 9.7 \times 10^3 (\lambda_\mu/10.6)^2 Z n_{16}^2 I_W \ell n\Lambda(\nu) T_{eV}^{-3/2} \quad \text{ergs/cm}^3\text{/sec} \quad . \qquad (56)$$

Here I_W is I_o in W/cm^2. Using Eq. (9) for $K_{||}$ and measuring all T's in eV, we obtain

$$T^{3/2} n_{16} \frac{\partial T}{\partial t} - \frac{1.26 \times 10^5}{\ell n \ \Lambda} \frac{T^{3/2}}{r} \frac{\partial}{\partial r}\left(rT^{5/2}\frac{\partial T}{\partial r}\right) = 0.4 \ Z \left(\frac{\lambda_\mu}{10.6}\right)^2 n_{16}^2 I_W \ell n\Lambda(\nu) \ . \qquad (57)$$

The steady state solution is obtained by solving

$$\frac{T^{3/2}}{r} \frac{\partial}{\partial r}\left(rT^{5/2}\frac{\partial T}{\partial r}\right) = -3.2 \times 10^{-6} Z \left(\frac{\lambda_\mu}{10.6}\right)^2 n_{16}^2 I_W \ell n\Lambda \ell n\Lambda(\nu) \quad . \qquad (58)$$

In the interior region $r < b$, we have

$$T^{3/2} \frac{\partial}{\partial r}\left(rT^{5/2}\frac{\partial T}{\partial r}\right) = -Ar \ , \qquad (59)$$

where A is a constant if n and I_W are constant, and this has no simple solution.

In the exterior region $r > b$, we have $I_W = 0$ and hence

$$[r(T^{7/2})']' = 0 ,$$

$$T^{7/2} - T_o^{7/2} = C \ln(b/r) , \tag{60}$$

where $T_o = T(a)$. The boundary condition is determined by the heat flux condition

$$\pi b^2 Q = 2\pi b q = -2\pi b K_{||} T' . \tag{61}$$

From Eqs. (9) and (56) we obtain

$$T'(b) = -\frac{b}{2}\frac{Q}{K_{||}} = -\frac{b}{2}\frac{9.7 \times 10^3 Z(\lambda_\mu/10.6)^2 n_{16}^2 I_W \ln\Lambda(\nu) T_b^{-3/2}}{(2.6 \times 10^5 T_b^{5/2}/\ln\Lambda)\ (11,600)} , \tag{62}$$

while from Eq. (60) we obtain

$$T'(b) = -\frac{2}{7}\frac{C}{b} T_b^{-5/2} . \tag{63}$$

Equating these, we obtain

$$C = \frac{7}{4} b^2 \ (3.2 \times 10^{-6}) Z \left(\frac{\lambda_\mu}{10.6}\right)^2 n_{16}^2 \ I_W \ \ln\Lambda \ \ln\Lambda(\nu) \ T_b^{-3/2} ; \tag{64}$$

or since $\pi b^2 I_W = P_W$, we can write C in terms of P_{MW}, whereupon Eq. (60) becomes

$$T_b^{3/2}(T_b^{7/2} - T_o^{7/2}) = 1.79 \left(\frac{\lambda_\mu}{10.6}\right)^2 Zn_{16}^2 P_{MW} \, \ell n\Lambda \, \ell n\Lambda(\nu) \, \ell n(a/b) \ . \quad (65)$$

If $T_o^{7/2}$ is negligible, we have, for $\lambda = 10.6$ μm,

$$T^5(b) \simeq 1.44 \, Zn_{16}^2 P_{MW} \, \ell n\Lambda \, \ell n\Lambda(\nu) \, \ell n(a/b) \ . \quad (66)$$

The risetime can be estimated by neglecting the $K_{||}$ term in Eq. (55):

$$\frac{3}{2} n \frac{\partial T}{\partial t} = \frac{Q}{1.6 \times 10^{-12}} \ . \quad (67)$$

From Eq. (56) we then obtain

$$\frac{2}{5} \frac{\partial}{\partial T} (T^{5/2}) = 0.4 \, Z \left(\frac{\lambda_\mu}{10.6}\right)^2 n_{16} I_W \, \ell n\Lambda(\nu) \ . \quad (68)$$

The solution is

$$T_b(t) = [1.0 \, Z(\lambda_\mu/10.6)^2 n_{16} I_{MW} \, \ell n\Lambda(\nu) t_{\mu sec}]^{2/5} \ , \quad (69)$$

where I_{MW} is I_o in MW/cm^2 and $t_{\mu sec}$ is t in μsec.

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