TRW
DEFENSE AND SPACE SYSTEMS GROUP
ONE SPACE PARK • REDONDO BEACH, CALIFORNIA

FSCM NO. 11982

TITLE

RADIOFREQUENCY FIELD ENHANCEMENT NEAR ION GYRORESONANCE

Francis F. Chen

DATE 21 January 1981 NO. Task II-3552

SUPERSEDING: ______

PREPARED BY: Francis F. Chen
Francis F. Chen

APPROVAL SIGNATURES:

UNCLASSIFIED: T. E. Romm...
Authorized Derivative Classifier

______ DATE ______ DATE

______ DATE ______ DATE

______ DATE ______ DATE

Radiofrequency field enhancement near ion gyroresonance

Francis F. Chen[a)]

TRW Defense and Space Systems Group, One Space Park, Redondo Beach, California 90278

(Received

In the near field of an antenna which induces an electric field component E_z along $\underline{B}_o$, the rf field inside the plasma is enhanced through the buildup of electrostatic charge. Analytic expressions for the enhancement factor are given for slab geometry. Results for an infinite plasma with internal excitation and for a finite plasma with external coils are qualitatively different. In the latter case, it is found that the field enhancement is not diminished at exact ion cyclotron resonance. Application is made to ion acceleration and to rf plugging of magnetic mirrors and cusps.

PACS numbers 52.40.Db, 52.40.Fd, 52.50.Gj, 52.55.Ke

I. INTRODUCTION

The extent to which rf fields near the ion cyclotron frequency can be made to penetrate to the interior of a dense plasma from an external coil is of importance both to cyclotron acceleration of different ion species[1] and to rf plugging of mirrors and cusps[2]. In numerical computations[3] of plasma response to excitation by helical coils, it was noticed that the rf field not only penetrated but was actually enhanced. This effect did not appear in calculations[4] where the induced electric field did not contain a component E_z along the dc magnetic field $\underline{B}_o = B_o\hat{\underline{z}}$ and did not vary along $\hat{\underline{z}}$ with a wave-number k_z, nor was the effect seen in experiments[5] where $\underline{E}$ was applied electrostatically.

The origin of the enhancement effect was probably first pointed out by J. M. Dawson[6] and is illustrated in Fig. 1. The excitation current $\underline{J}$ flows in a coil which contains segments periodic in z and lying in the $\hat{\underline{z}}$ direction. The return current flows in segments which go around the plasma. As $\underline{J}$ oscillates, it generates an oscillating field $\underline{B}_1$, as shown. This field, in turn, induces an electric field $\underline{E}$ in the plasma, such that

$$\oint \underline{E} \cdot d\underline{L} = -\int (\partial/\partial t)(\underline{B} \cdot dS), \qquad (1)$$

The line integral being taken along any path in the plasma such as the one indicated by the dashed line. If the plasma is a good conductor, E_z must vanish, so that $\underline{E}$ exists only on the perpendicular legs of the path. This $\underline{E}_\perp$ is seen to be in the same direction as that induced by the current in the perpendicular segments of the coil, and in this sense the rf field has been enhanced. Electrons responding to the induced E_z move along $\underline{B}_o$ and, because E_z is periodic in z, cause a periodic space charge ρ to build up on each line of force. The magnitude of ρ is such that its electrostatic field E_z exactly cancels the induced E_z due to the coil current J_z, making the total E_z zero. It is this space charge that causes the enhanced $\underline{E}_\perp$.

From this simple picture it is evident that ρ will be large if the coil is long, and that the $E_\perp$ caused by ρ will be large if the coil diameter is small. Indeed, we show in this paper that the enhancement factor depends only on the aspect ratio of the coil over a wide range of parameters. It is also clear that the plasma cannot easily screen the E-field from its interior; for to do so would require plasma currents large enough to cancel $\underline{B}_1$. Large, low-frequency currents cannot flow along $\underline{B}_o$ because of the accumulation of electron space charge. Large ion currents perpendicular to $\underline{B}_o$ might be possible at cyclotron resonance, but these would be in the wrong direction to oppose $\underline{B}_1$.

The field enhancement effect has been observed in rf plugging experiments at Nagoya[2] using the so-called Type III coil, which induces an E_z. A theory[2,7] was given for this effect, which was called plasma paramagnetism[7]. However, only the internal electromagnetic field $\underline{B}_1$ was calculated, and only the electromagnetic field was measured using magnetic probes. Since the main enhancement is due to the electrostatic field, it seems that previous workers have missed the point. Comparison with the Nagoya results will be made in Sec. V.

In actual experiments the antenna structures used--that is, the Type III coil at Nagoya and helical coils at TRW--are not the same as that depicted in Fig. 1. Rather than treat a realistic coil, in this paper we consider an idealized slab model, so as to simplify the algebra and obtain parametric dependences. If one were to use rf to plug the thin sheets of plasma escaping from the line cusps of a cusp reactor, the excitation method considered here may have direct applicability.

II. INFINITE PLASMA PROBLEM

Consider an infinite, uniform plasma in a uniform magnetic field $B_o\hat{\underline{z}}$. We wish to compute the plasma response to a periodic driving field

$$\underline{E}_o = E_o[\hat{\underline{y}} - (k_y/k_z)\hat{\underline{z}}] \exp[i(k_x x + k_y y + k_z z - \omega t)]. \quad (2)$$

This field is presumed to be induced by a similarly periodic current $\underline{j}_o$, which is distributed throughout the plasma but does not interact with it. The direction of $\underline{E}_o$ has been chosen to satisfy $\underline{\nabla} \cdot \underline{E}_o = 0$. Although this is a common formulation of the problem, the excitation mechanism is unrealistic; and we shall show that the results are misleading. In gaussian units, the vacuum field satisfies the equation

$$\underline{\nabla} \times \underline{\nabla} \times \underline{E}_o = \frac{4\pi i\omega}{c^2} \underline{j}_o + k_o^2 \underline{E}_o \quad (3)$$

where

$$k_o \equiv \omega/c. \quad (4)$$

In the presence of plasma, the total field $\underline{E}$ satisfies

$$\underline{\nabla} \times \underline{\nabla} \times \underline{E} = \frac{i\omega}{c^2} (4\pi \underline{j}_o + \underline{\underline{\varepsilon}} \cdot \dot{\underline{E}}), \quad (5)$$

where

$$\underline{E} = \underline{E}_o + \underline{E}_p, \quad (6)$$

and the plasma response $\underline{E}_p$ is of the form

$$\underline{E}_p = (E_x\hat{\underline{x}} + E_y\hat{\underline{y}} + E_z\hat{\underline{z}}) \exp[i(k_x x + k_y y + k_z z - \omega t)]. \quad (7)$$

Subtracting Eq. (5) from Eq. (3), we obtain

$$-\underline{\nabla} \times \underline{\nabla} \times \underline{E}_p + k_o^2 \underline{\underline{\varepsilon}} \cdot \underline{E}_p = k_o^2 (\underline{\underline{I}} - \underline{\underline{\varepsilon}}) \cdot \underline{E}_o \quad (8)$$

For the dielectric tensor we take the cold-plasma elements in the notation of Stix[8]:

$$\underline{\underline{\varepsilon}} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix} , \qquad (9)$$

where

$$\left. \begin{aligned} S &= \frac{1}{2}(R+L), \qquad D = \frac{1}{2}(R-L) \\ R &= 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega \pm \omega_{cs})} \\ L &= 1 - \sum_s \frac{\omega_{ps}^2}{\omega(\omega \mp \omega_{cs})} \\ P &= 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} \end{aligned} \right\} \qquad (10)$$

The sum is over species, the ± stands for ions (top sign) or electrons (bottom sign), and other symbols are standard.

We now make the basic approximation of infinite conductivity along $\underline{B}_o$. This allows us to reduce Eq. (8) to two simultaneous equations for E_x and E_y, replacing the z equation with

$$E_z^{tot} = E_{oz} + E_z = 0, \qquad E_z = - E_{oz} = (k_y/k_z)\, E_o . \qquad (11)$$

This approximation is almost always a good one physically, unless the coil is exceedingly long. Setting E_z^{tot} to zero means that we cannot expect the solution to converge to the vacuum solution as the density is reduced to zero, because it takes some electrons to provide the conductivity. Though this quirk may cause havoc with computer programs, we shall turn it to an advantage in Sec. III when an analytic expression is obtained. If finite conductivity or electron inertia is important, the element $\varepsilon_{zz} = P$ can be replaced with an expression containing collisions, a fluid thermal term, or even a Landau term.

Eq. (8) must then be solved for three unknowns. The effect of finite temperature on the other components of $\underline{\underline{\varepsilon}}$ would greatly complicate the problem, but these finite Larmor radius effects are generally negligible in a uniform plasma.

Next we divide $\underline{E}_p$ into its transverse and longitudinal parts:

$$\underline{E}_p = \underline{E}^t - \underline{\nabla}\phi \tag{12}$$

(Omission of this step leads to the trivial solution $\underline{E}^{tot} = 0$.) Defining the index of refraction

$$\underline{n} = c\,\underline{k}/\omega = \underline{k}/k_o, \tag{13}$$

we can write Eq. (8) as

$$-n^2\,\underline{E}^t + \underline{\underline{\varepsilon}}\cdot(\underline{E}^t - \underline{\nabla}\phi) = (\underline{\underline{I}} - \underline{\underline{\varepsilon}})\cdot\underline{E}_o \tag{14}$$

This contains four scalar unknowns, of which two can be eliminated by use of Eq. (11) and the condition

$$\underline{\nabla}\cdot\underline{E}^t = 0 \tag{15}$$

The resulting 2 × 2 matrix equation is

$$\begin{bmatrix} (S-n^2)n_z^2 + n_x(Sn_x - iDn_y) & -iDn_z^2 + n_y(Sn_x - iDn_y) \\ iDn_z^2 + n_x(Sn_y + iDn_x) & (S-n^2)n_z^2 + n_y(Sn_y + iDn_x) \end{bmatrix} \begin{bmatrix} E_x{}^t \\ E_y{}^t \end{bmatrix} = - E_o \begin{bmatrix} -iDn_z^2 + n_y(Sn_x - iDn_y) \\ (S-1)n_z^2 + n_y(Sn_y + iDn_x) \end{bmatrix} . \tag{16}$$

The second column in the matrix on the left is identical to the vector on the right except for the factor $(S-n^2)$, which occurs instead of $(S-1)$. If $n^2 = 1$, the solution would be the trivial one: $E_x^t = 0$, $E_y^t = -E_o$. Since $n^2 = c^2k^2/\omega^2$ is normally much larger than 1, the solution of Eq. (16) is non-trivial and is easily found by considering the difference between the $n^2 = 1$ and $n^2 > 1$ cases. Knowing E_x^t and E_y^t, we can then calculate E_z^t from Eq. (15) and ϕ from Eq. (11). Adding $\underline{E}^t$ and $-\underline{\nabla}\phi$ to $\underline{E}_o$, we obtain for the total field

$$\begin{aligned} E_x^{tot} &= \mathbb{D}^{-1} E_o(n^2-1)(iD-n_xn_y) \\ E_y^{tot} &= \mathbb{D}^{-1} E_o(n^2-1)(S-n_y^2-n_z^2) \\ E_z^{tot} &= 0 \quad \text{(by assumption)}, \end{aligned} \tag{17}$$

where

$$\mathbb{D} \equiv n^2(S-n_z^2) + Sn_z^2 - RL \tag{18}$$

and the identity $S^2 - D^2 = RL$ has been used. The circularly polarized components

$$E^{L,R} = 2^{-1/2}(E_x \pm iE_y) \tag{19}$$

then can be written

$$\begin{aligned} E^R/E_o^R &= \mathbb{D}^{-1}(n^2-1)(L-n_y^2-n_z^2-in_xn_y) \\ E^L/E_o^L &= \mathbb{D}^{-1}(n^2-1)(R-n_y^2-n_z^2+in_xn_y). \end{aligned} \tag{20}$$

This solution has the same behavior as that found by previous workers[2,7]. The field in the plasma is large when the determinant $\mathbb{D}$ vanishes and the dispersion relation for waves in the plasma is satisfied; namely, when

$$n_x^2 = \frac{RL-Sn_z^2}{S-n_z^2} - n_y^2 - n_z^2 . \tag{21}$$

For low frequencies $\omega \ll \omega_{ce}$, Eq. (10) reduces to

$$S \simeq 1 - \frac{\Omega_p^2}{\omega^2 - \Omega_c^2}, \quad RL \simeq \left(1 + \frac{\Omega_p^2/\Omega_c}{\Omega_c + \omega}\right)\left(1 + \frac{\Omega_p^2/\Omega_c}{\Omega_c - \omega}\right) \simeq S^2, \tag{22}$$

where Ω_p, Ω_c are the plasma and cyclotron frequencies of a single ion species. Eq. (21) then becomes

$$n_x^2 = S - n_y^2 - n_z^2 \simeq 1 - \frac{\Omega_p^2}{\omega^2 - \Omega_c^2} - n_y^2 - n_z^2. \tag{23}$$

To recover the Nagoya results, we set $n_x^2 = n_y^2 = 0$ and neglect the "1", obtaining

$$\omega^2 = \frac{\Omega_c^2}{1 + (\Omega_p^2/c^2 k_z^2)} \quad . \tag{24}$$

This is identical with Eq. (1) of Ref. 7 except for our neglect of a small thermal correction $k_x^2 c_s^2$ in the numerator, which represents an electrostatic ion cyclotron wave. It is clear that real n_x requires ω^2 less than the critical value given in Eq. (24); this feature is in general agreement with experimental results[7].

Here the resemblance ends. Consider an excitation field with $\lambda_x \simeq \lambda_y \simeq \lambda_z/5 \simeq 10$ cm. Then for average plasma parameters the condition $n^2 \gg |S| \gg 1$ is satisfied away from exact cyclotron resonance. Eq. (20) then gives an enhancement factor

$$\left|\frac{E^{R,L}}{E_o^{R,L}}\right| \simeq \left|\frac{n_y^2 + n_z^2 \pm i n_x n_y}{n_z^2}\right| \simeq 2^{\frac{1}{2}} \frac{n_y^2}{n_z^2} \simeq 35. \tag{23}$$

This field enhancement is a result of the electrostatic plasma field and does not occur if k_y is zero. At exact resonance $\omega = \Omega_c$, L goes to ∞ and S to L/2, while R remains finite. Eq. (20) then shows that E^L goes to zero while E^R

remains finite. The fact that the plasma shields itself against cyclotron fields rotating in the same direction as the ion Larmor gyration seems reasonable at first, but it is a spurious result stemming from the unphysical assumption that the driving field is distributed throughout the plasma by a system of non-interacting currents.

III. FINITE PLASMA SLAB PROBLEM

A. Description of Problem

The configuration treated is shown in Fig. 2. A uniform plasma is infinite in the y and z directions and has width 2a in the x direction. The uniform magnetic field $\underline{B}_o$ is in the z direction. At $x = \pm b$ there are sheet currents $\underline{K}_o$ which are periodic in y and z and are antisymmetric about $x = 0$. Only the lowest Fourier modes in y and z are treated. The currents flow in space and are not attached to conductors. Perfectly conducting boundaries are assumed at $x = \pm d$, outside the current sheets. Let $\underline{K}_o$ at $x = \pm b$ have the form

$$\underline{K}_o = \pm K_o' \, e^{-i\omega t} (\hat{y} |n_y|^{-1} \sin |k_y| y \cos |k_z| z - \hat{z} \, |n_z|^{-1} \cos |k_y| y \ \sin |k_z| z). \tag{24}$$

This is the sum over four waves of the form

$$\underline{K}_o = \pm K_o \left(\frac{\hat{y}}{n_y} - \frac{\hat{z}}{n_z} \right) e^{i(n_y y + n_z z - \omega t)} \tag{25}$$

with n_y taking on the values $\pm|n_y|$ and n_z and values $\pm|n_z|$. Here y and z are in units of $k_o^{-1} = c/\omega$, and $K_o = -iK_o'/4$.

The vacuum field induced by the latter $\underline{K}_o$ has the form

$$\underline{E}_o = E_o \left(\frac{\hat{y}}{n_y} - \frac{\hat{z}}{n_z} \right) e^{i(n_y y + n_z z - \omega t)} \sinh n_x x, \tag{26}$$

where

$$n_x = (n_y^{\,2} + n_z^{\,2} - 1)^{\frac{1}{2}}. \tag{27}$$

It is tempting to substitute this $\underline{E}_o$ into Eq. (14), expand the solution in $\sinh n_x x$ and $\cosh n_x x$ terms, and obtain a matrix equation of the form of Eq. (16). However, when this is done carefully with displacement current retained, there is an exact cancellation: the second column on the left-hand side of Eq. (16) becomes identical with the vector on the right-hand side. This leads to the trivial solution $\underline{E}_p = -\underline{E}_o$, $\underline{E}^{tot} = 0$. There is also a non-trivial solution in the sense that the equation $f'' = -a^2 f$ has both a trivial solution $f = 0$ and a non-trivial solution $f = \exp(\pm iax)$. That the non-trivial solution cannot be found in this manner is not surprising, since we are dealing now with a boundary-value problem rather than an infinite plasma. However, a nearly correct answer for the enhancement factor can be obtained by this shortcut method if the transverse plasma field $\underline{E}^t$ is neglected and only the longitudinal field $-\underline{\nabla}\phi$ is retained. This fact confirms our intuition that the vacuum field $\underline{E}_o$ cannot easily be shielded out by plasma currents.

B. General Solution

To solve the boundary-value problem requires matching the solutions in the three regions at the interfaces $x = \pm a$ and $x = \pm b$. Since $E_y = E_z = 0$ at $x = d$ and $\underline{\nabla} \cdot \underline{E} = 0$, the exterior field $\underline{E}^e$ between $x = b$ and $x = d$ has the form

$$E_y^e = A_y^e \, e^{n_x x} \{1 - \exp[2n_x(d-x)]\}$$

$$E_z^e = A_z^e \, e^{n_x x} \{1 - \exp[2n_x(d-x)]\} \tag{28}$$

$$E_x^e = -in_x^{-1}(n_y A_y^e + n_z A_z^e) \, e^{n_x x} \{1 + \exp[2n_x(d-x)]\},$$

where n_x satisfies Eq. (27).

We again make the basic approximation that E_z vanishes in the plasma. The solution in the middle region, $\underline{E}^m$, must have $E_z^m = 0$ at $x = a$. We therefore take $\underline{E}^m$ to have the form

$$E_x^m = A \sinh \xi + B \cosh \xi$$

$$E_y^m = C \sinh \xi + D \cosh \xi \tag{29}$$

$$E_z^m = E \sinh \xi , \qquad \xi \equiv n_x(x-a).$$

Two of the coefficients can be determined from $\underline{\nabla} \cdot \underline{E}^m = 0$. $\underline{E}^m$ is matched to $\underline{E}^e$ by the continuity of all three components of $\underline{E}$ at $x = b$ and by the jump in B_y and B_z caused by $\underline{K}_o$.

The interior solution must have E_y^i antisymmetric in x the way E_o is; but E_x^i, which is due entirely to space charge, can be seen from Figs. 1 and 2 to be symmetric in x. Thus we take the interior solution for $x \leq a$ to be of the form

$$E_x^i = A_x^i \cosh g x$$

$$E_y^i = A_y^i \sinh g x \tag{30}$$

$$E_z^i = 0.$$

Furthermore, $\underline{E}^i$ satisfies the homogeneous equation

$$\nabla^2 \underline{E}^i - \underline{\nabla}(\underline{\nabla} \cdot \underline{E}^i) + \underline{\underline{\epsilon}} \cdot \underline{E}^i = 0, \tag{31}$$

leading to the dispersion relation [cf. Eq. (21)]

$$g^2 = n_y^2 + n_z^2 + \frac{S n_z^2 - RL}{S - n_z^2} . \tag{32}$$

The tangential components of $\underline{E}$ and $\underline{B}$ are continuous across x = a, but E_x is not necessarily so because of the jump in $\underline{\underline{\varepsilon}}$.

The matching procedure yields the following general solution:

$$E_y^i = \frac{4\pi i}{c}\frac{K_o}{n_y}\left(1+\frac{n_y^2}{n_z^2}\right)\frac{\sinh gx}{Nn_x \sinh ga + Mg\cosh ga}$$

$$E_x^i = -\frac{4\pi}{c}\frac{K_o}{n_z^2 M}\left(\frac{1-gn_x(M/N)\text{ctnh } ga}{n_x^2 + gn_x(M/N)\text{ctnh } ga}\right)\frac{\cosh gx}{\cosh ga}, \quad (33)$$

where

$$M \equiv \cosh n_x(b-a) + \sinh n_x(b-a)\ \text{ctnh } n_x(d-b)$$

$$N \equiv \sinh n_x(b-a) + \cosh n_x(b-a)\ \text{ctnh } n_x(d-b), \quad (34)$$

and g and n_x follow Eqs. (32) and (27), respectively. Here K_o is defined by Eq. (25), and the oscillating factor has been omitted.

The vacuum field $\underline{E}_o^i$ <u>cannot</u> be found by taking the limit $\omega_p^2 \to 0$ in Eq. (32), because the $E_z^i = 0$ approximation breaks down in this limit. A boundary matching procedure yields

$$E_{ox}^i = 0$$

$$E_{oy}^i = \frac{4\pi i}{c}\frac{K_o}{n_x n_y} e^{-n_x b} \sinh n_x x \quad (35)$$

$$E_{oz}^i = -\frac{4\pi i}{c}\frac{K_o}{n_x n_z} e^{-n_x b} \sinh n_x x$$

C. The Enhancement Factor

To simplify the algebra, let us discuss the case d → ∞, a = b, where the excitation currents are right next to the plasma. Then M = N = 1, and

Eq. (33) reduces to

$$E_y^i = \frac{4\pi i}{c}\frac{K_o}{n_y}\left(1 + \frac{n_y^2}{n_z^2}\right)\frac{\sinh g x}{n_x \sinh g a + g \cosh g a}$$

$$E_x^i = -\frac{4\pi}{c}\frac{K_o}{n_z^2}\frac{1 - g n_x \operatorname{ctnh} g a}{n_x^2 + g n_x \operatorname{ctnh} g a}\frac{\cosh g x}{\cosh g a}, \qquad (36)$$

with $E_z^i = 0$. Here g follows the dispersion relation, Eq. (32), with S and RL given by Eq. (22). At sufficiently low density, the condition

$$n_z^2 >> |S| , |R|, |L| \qquad (37)$$

is obeyed except in the vicinity of $\omega = \Omega_c$. In that case, Eq. (33) becomes, with Eq. (27),

$$g^2 \approx n_y^2 + n_z^2 \approx n_x^2 . \qquad (38)$$

A great simplification results from the low-density approximation $g = n_x$; that is, the wavelength in the plasma is approximately the vacuum wavelength. As will be shown in Sec. IV, this approximation is valid over a wide parameter range. In the low-density limit, Eq. (36) becomes

$$E_y^i = \frac{4\pi i}{c}\frac{K_o}{n_x n_y}\left(1 + \frac{n_y^2}{n_z^2}\right) e^{-n_x a} \sinh n_x x$$

$$E_x^i = \frac{4\pi}{c}\frac{K_o}{n_x^2 n_z^2}(n_x^2 - \tanh n_x a)\, e^{-n_x a} \cosh n_x x \qquad (39)$$

Since a = b has been assumed, division by the vacuum field of Eq. (35) gives the enhancement factor

$$Q = \frac{E^i_y}{E^i_{oy}} = 1 + \frac{n_y^2}{n_z^2} . \qquad (40)$$

Note that this is not unity, because the assumption $E^i_z = 0$ does not allow Eq. (36) to converge to the vacuum solution as the density is reduced to zero. Eq. (40) shows that the enhancement factor in the low-density limit depends only on the geometry of the coil. Furthermore, the factor Q does not vanish at $\omega = \Omega_c$. At resonance, L approaches ∞, and S approaches L/2. Eq. (32) then becomes

$$g^2 \simeq n_y^2 + n_z^2 + \frac{Ln_z^2 - 2RL}{L-2n_z^2} \simeq n_y^2 + 2n_z^2 \simeq n_x^2 + n_z^2 . \qquad (41)$$

Since $n_x^2 >> n_z^2$, the approximation $g = \mu_x$ is still valid, and Q is still approximated correctly by Eq. (40). This result confirms the intuitive argument that the divergenceless induced field cannot be shielded out by electrostatic fields and that transverse plasma currents large enough to buck out the coil currents cannot flow in a strong magnetic field.

Besides the y component, Eq. (39) gives an x component of $\underline{E}$ which did not exist in vacuum. E_x and E_y can be combined to give the circularly polarized components E^L and E^R. It is found that cancellation between E_x and E_y causes E^L and E^R to peak on opposite sides of the plasma. This circumstance is caused by the use of Eq. (25) for $\underline{K}_o$ instead of Eq. (24). We must add four solutions with $n_y = \pm|n_y|$, $n_z = \pm|n_z|$ to obtain a result pertinent to a physical coil. The interior field, which is mostly electrostatic, is then found to be linearly polarized, so that computation of E^L and E^R is not necessary; the left- and right-hand components have identical enhancement factors. The result is

$$E_y^i = \frac{4\pi i}{c} \frac{K_o'}{|n_x n_y|} e^{-n_x a} \left(1 + \frac{n_y^2}{n_z^2}\right) \sinh n_x x \sin|n_y|y \cos|n_z|z$$

(42)

$$E_x^i = -\frac{4\pi i}{c} \frac{K_o'}{|n_x n_y|^2} e^{-n_x a} (n_x^2 - \tanh n_x a) \cosh n_x x \cos|n_y|y \cos|n_z|z.$$

When K_o' is in A/cm and E is in V/cm, the coefficient $4\pi i/c$ is to be replaced by $120\pi i$. The enhancement factor for the y component is again given by Eq. (40). The additional enhancement due to the x component is

$$\frac{E_x^i}{E_{oy}^i} = -\frac{|n_y|}{n_x n_z^2} (n_x^2 - \tanh n_x a) \text{ ctnh } n_x x \text{ ctn } |n_y|y. \qquad (43)$$

This becomes infinite at $x = 0$ because E_{oy} vanishes while E_x^i is finite there. At the plasma edge, ctnh $n_x a \simeq 1$. The y dependence is due to 90° shift in phase. The coefficient in front is, from Eq. (27),

$$\left|\frac{n_x n_y}{n_z^2}\right| \simeq \left|\frac{n_y}{n_z}\right| \left(1 + \frac{n_y^2}{n_z^2}\right)^{1/2} \simeq \frac{1}{2} + \frac{n_y^2}{n_z^2}. \qquad (44)$$

Thus, for $n_y^2 \gg n_z^2$, E_x^i contributes almost as much to Q as does E_y^i.

IV. APPLICATION TO ION ACCELERATION

In the last section we showed that the low-density approximation $g = n_x$ permits the rf field enhancement to be reduced to very simple form. We now illustrate the range of validity of this approximation by specifying the set of experimental parameters appropriate to ion acceleration at cyclotron resonance[1]. Consider a plasma slab with half-thickness a = 10 cm, confined by a 20 kG magnetic field. The Xe ions have $M = 129\ M_H$ and $Z = 1$. The excitation currents have $\lambda_y = 20\pi$ cm and $\lambda_z = 60\pi$ cm, so that $k_y/k_z = 3$. For $\omega \simeq \Omega_c$, k_o is 5.44×10^{-5} cm^{-1}, so that $n_y = 1.84 \times 10^3$, $n_z = 6.13 \times 10^2$, $n_x = 1.94 \times 10^3$, and $k_x a = 1.055$. To study the behavior of $g(\omega)$ near $\omega = \Omega_c$, we define

$$\begin{aligned} \Delta &\equiv (\omega-\Omega_c)/\Omega_c \\ A &\equiv 1 + (k_y^2/k_z^2) = 10 \\ \gamma &\equiv \Omega_p^2/\Omega_c^2 . \end{aligned} \tag{45}$$

Substituting the low-frequency dielectric elements of Eq. (22) into Eq. (32), we obtain

$$g^2 \simeq An_z^2 + \frac{n_z^2 - \gamma}{1-(n_z^2/\Omega_p^2)(\Omega_c^2-\omega^2)} \simeq \frac{n_z^2(A+1+2An_z^2\Delta/\gamma)-\gamma}{1+2n_z^2\Delta/\gamma} \tag{46}$$

Here we have neglected the "1" in S, R, and L and have set $\omega + \Omega_c \simeq 2\Omega_c$. As we have already seen, g^2 is well behaved at $\Delta = 0$; however, it becomes infinite at $\Delta = -\frac{1}{2}\gamma/n_z^2$ and zero at approximately $(A+1)/A = 1.1$ times this value. The last γ in the numerator is negligible at all densities of interest, so that g^2/n_z^2 is $\simeq A+1$ at $\Delta = 0$ and is $\simeq A$ far from resonance, where the Δ terms dominate. the behavior of g^2/n_z^2 with Δ is shown in Fig. 3.

Since $An_z^2 = n_y^2 + n_z^2 = n_x^2 + 1 \simeq n_x^2$, we see that the approximation $g = n_x$ is valid except in a very narrow region. The width of this region is of the order of the distance between the zero and the infinity of g, which is $\delta\Delta = \gamma/2An_z^2$. In this example, this width is only 0.08% of Ω_c. Since magnetic fields are not uniform to this degree of accuracy in practice, one would not expect to observe any effect of large g^2 (rapid evanescence) or of negative g^2 (wave propagation). In multi-species plasmas, let there be a minority species with a different mass, say $M_2 = 128\, M_H$. The g^2/n_z^2 curve will have a second resonance near Ω_c of the second species; this resonance will be narrower because γ is smaller. Since the Ω_c's differ by 0.8% while the resonance zone is only 0.16% wide, the resonances do not overlap, and the low density approximation is valid also in this two-ion plasma. Fig. 4 shows the radial variation of E_x, E_y, and E_{oy} for this example. These fields vary sinusoidally in y and z; only the peak field is shown.

We next show the density dependence of the enhancement factor Q, defined as $(E_x^2 + E_y^2)^{\frac{1}{2}}/E_{oy}$. This ratio varies with x and y, so we have taken x = a and tan $|n_y|y = 1$. A two-ion plasma is assumed, with $M_2/M_1 = 0.992$, $n_2/n_1 = 0.1$, and $\omega = \Omega_2$ exactly. The low-density approximation is not made. An equation analogous to Eq. (46) with $\Delta = 0$ is used to compute g for the two-ion case; namely,

$$g^2 = n_z^2(A+1) - (1+\frac{n_2}{n_1})\,\frac{\Omega_{p2}^2}{\Omega_{c2}^2}\,. \qquad (47)$$

E_x and E_y are then computed from Eq. (36). The result is shown in Fig. 5, where Q is plotted against the density n_1 of the major species. In this case the low density approximation is seen to be good up to $n_1 = 3 \times 10^{14}$ cm^{-3}, where g^2 becomes negative.

In experiments on cylindrical plasmas, it is convenient to use a helical excitation coil, like the m = 1 bifilar winding shown in Fig. 6. Though our slab geometry results are applicable to such a coil as far as field enhancement and penetration are concerned, some modifications are needed to account for differences in the relative phase of E_x and E_y. In slab geometry, Eq. (42) shows that both E_x^i and E_y^i vary as cos $k_z z$, so that $\underline{E}^i$ vanishes althogether for some positions along z. In the cylindrical case, the electrostatic charges maximize on a helical line between the windings, giving rise to a linearly polarized E-field whose magnitude on axis is constant with z but whose plane of polarization rotates with z. The helical coil fills the plasma volume with rf field more effectively than the purely antisymmetric planar coils. To make ion heating more efficient, a second bifilar winding can be added in the spaces between the wires of the first coil to give an electrostatic field perpendicular to that of the first coil. When the two windings are driven 90° out of phase, a left-hand rotating E-field can be produced at each z. However, the phase of this field varies with z, so that parallel motions of ions will Doppler shift their cyclotron resonances.

V. APPLICATION TO RF PLUGGING

Although the internal rf field has been measured[5] in the case where it is applied electrostatically at the ends of the lines of force, there has not yet been a direct confirmation of the field enhancement factor for induced cyclotron fields. An exception to this is the work on rf plugging of cusps done at Nagoya[2,7]. Unfortunately, their experiments and calculations have sufficiently different geometry from ours that a direct comparison is not enlightening.

In Ref. 2, the plasma slab is finite in both the x and z directions, and the excitation current, in the z direction only, does not vary in the y direction; since $k_y = 0$, our theory would predict no enhancement in that case unless a propagating wave is excited. In Ref. 7, periodic coils such as ours are assumed, but they lie in the x-z plane rather than the y-z plane. Since the plasma is infinite in the y direction, the coils must pass through the plasma (at least in the simulation). In both papers the enhancement in oscillating <u>magnetic</u> field is calculated and measured with magnetic probes. Since the largest enhancement is due to the <u>electrostatic</u> field, the main physical effect seems to have been overlooked.

The experimental parameters of the Nagoya work with the Type-3 coil are approximately as follows[7]: $B = 5$ kG, $M = 1.67 \times 10^{-24}$ g, $T_e \simeq T_i \simeq 4$ eV, $n = 10^{13} - 10^{15}$ cm^{-3}, $a = 2.5$ cm, $\lambda_z = 16$ cm, $\omega/\Omega_c = 0.8$. These yield $\Omega_c = 4.79 \times 10^7$ sec^{-1}, $c_s = 3.92 \times 10^6$ cm/sec, $k_z = 0.39$ cm^{-1}, and $\Omega_p{}^2 = 1.73 \times 10^{19}\, n_{13}$, where n_{13} is density in units of 10^{13} cm^{-3}. The one point of agreement is that something happens at a critical frequency given by

$$\omega_m{}^2 = (\Omega_c{}^2 + c_s{}^2/a^2)(1 + \Omega_p{}^2/c^2k_z{}^2)^{-1}. \qquad (48)$$

This is the same as our Eq. (24) except for the addition of the thermal term $c_s{}^2/a^2$, which amounts to $10^{-3}\Omega_c{}^2$ in this case. At this frequency, waves change from evanescence ($\omega > \omega_m$) to propagation ($\omega \lesssim \omega_m$). The experiments showed that the electromagnetic enhancement factor Q_m has a peak value $\simeq 1.25$ at approximately the frequency

$$\omega = \omega_m \simeq \Omega_c(1 + 0.124\, n_{13})^{-\frac{1}{2}} \qquad (49)$$

predicted by Eq. (48) and fell below unity for $\omega > \omega_m$. Analytic theory[2] predicted

$$Q_m = \frac{B_y}{B_{yo}} = \frac{1 - \omega^2/(\Omega_c{}^2 + c_s{}^2/a^2)}{1 - \omega^2/\omega_m{}^2}\,. \qquad (50)$$

which becomes infinite at $\omega = \omega_m$. This behavior, which does not agree well with experiment, corresponds to g^2 going negative in our Fig. 3, causing Q to become infinite. The observed behavior has been reproduced in computer simulation[7], but only in a slab two ion gyro-diameters thick, with a mass ratio of 20 and an electron temperature scaled to 20 keV.

In an attempt to fit the Nagoya measurements with our calculations, we have computed the electromagnetic part of the internal field by taking the curl of $\underline{E}^i$, as given in Eq. (36). The low-density approximation is not necessarily valid here, and it has not been employed. The result is

$$\frac{B_x^i}{B_{ox}} = \frac{e^{n_x a}}{\sinh g a + (g/n_x)\cosh g a} \; \frac{\sinh g x}{\sinh n_x x}$$

$$Q_m \equiv \frac{B_y^i}{B_{oy}} = \frac{e^{n_x a}}{g n_x} \; \frac{g n_x - \tanh g a}{(n_x/g)\sinh g a + \cosh g a} \; \frac{\cosh g x}{\cosh n_x x} \qquad (51)$$

$$\frac{B_y^i}{B_{oz}} = e^{n_x a} \; \frac{g n_x + (n_y^2/n_z^2)\tanh g a}{\sinh g a + (g/n_x)\cosh g a} \; \frac{\cosh g x}{\cosh n_x x}$$

The x and y components show enhancement factors of order unity when either $g = n_x$ (low-density approximation) or $g = 0$ and $x = 0$. The z component has $Q_m \simeq n_x^2 >> 1$ in these limits, but only because B_{oz} is small compared with B_{oy} and B_{ox}. As expected, elimination of the electrostatic field has reduced Q_m to the order of unity.

To be more specific, we have computed Q_m for $n = 10^{13}$ cm^{-3}, B = 5 kG, a = 2.5 cm, $k_z = 0.196$ cm^{-1}, $k_y = 0$, and x = 0, using Eq. (46) (with A = 1) to find g and Eq. (51) to find Q_m. Since ω was varied over two orders of magnitude, the variation of k_o, and hence n_x, with ω was taken into account. [In Eq. (50), a is dimensionless, so that $n_x a$ does not depend on k_o.] The result is shown in Fig. 7. The value of Q_m below $\omega = \omega_m$ is 1.3, in good agreement with the observed value of 1.25. The sharp behavior of Q_m near ω_m, due to g becoming sucessively 0 and ∞, would hardly have been noticeable in experiment, though thermal effects and inhomogeneities there could broaden the resonance. The fact that our curve does not drop to unity at low frequencies (it approaches $Q_m = 1.35$) is probably due to the neglect of collisions.

We conclude that the main features of the Nagoya measurements can be obtained from this simplified calculation. Good agreement cannot be expected because of differences in coil geometry and the neglect of density gradients, finite Larmor radius effects, and collisions. The $T_e = 0$ approximation, though valid in regard to the neglect of $k_\perp^2 c_s^2$ relative to Ω_c, was only marginal here in regard to electron Landau damping, since $\omega/k_z v_{the} \simeq O(1)$. The main field enhancement due to the electrostatic field would have been much larger than Q_m but was missed in the Nagoya work. With a coil designed to give large k_y/k_z, we believe that rf plugging can be much more effective than presently thought.

VI. SUMMARY

The conclusions of this work are two-fold: 1) enhancement of inductively applied rf fields can be made large by properly designing the excitation coil to favor the buildup of electrostatic charges inside the plasma; and 2) solving the boundary value problem gives a qualitatively different result from solving for the plasma response to a given driving field in an infinite plasma. In the latter case, the left-hand circularly polarized field $\underline{E}^L$ vanishes at $\omega = \Omega_c$, presumably because the ion current is infinite at $\omega = \Omega_c$ and can easily cancel the internal driving current that produces $\underline{E}_o$. When $\underline{E}_o$ is produced by an external coil, however, it can be cancelled only by a surface current which is as large as the current in the coil. Such a plasma current across $\underline{B}_o$ cannot arise in practice unless the ion Larmor radius is extremely large, and we have legislated it out of the model by requiring that the tagential component of $\underline{B}$ be continuous across the plasma surface. The result is that $\underline{E}^L$ not only penetrates into the plasma but is even enhanced over its vacuum value.

Our simple model was designed for the problem of ion acceleration and fits well with the parameters of that problem. It fits less well with the parameters of the rf plugging experiments, but the qualitative results should still be applicable. The question arises as to whether the near-field effects treated here have any relevance to ICRF (ion-cyclotron-range-of-frequencies) heating of tokamaks, where the emphasis has been on the propagation of waves in the far-field and their subsequent mode conversion and damping. To clarify this point, we take the case $A = 1 + (k_y^2/k_z^2) >> 1$ and rewrite Eq. (46) in the form

$$g^2 = n_x^2 \frac{1 + (2\Delta/\alpha A) - \alpha}{1 + (2\Delta/\alpha A)} . \tag{52}$$

Here $\alpha \equiv \gamma/n_x^2$, and we have assumed $n_x^2 = n_y^2 + n_z^2 - 1 = An_z^2 - 1 \simeq An_z^2$. The parameter α is essentially the number of ion collisionless skin depths c/Ω_p in a plasma radius:

$$\alpha = \gamma/n_x^2 = (\Omega_p^2/\Omega_c^2)(k_o^2/k_x^2) \simeq \Omega_p^2/c^2k_x^2 , \tag{53}$$

for $\omega \simeq \Omega_c$. Since $A >> 1$, $k_x^2 \simeq k_y^2 \simeq m^2/a^2$, where m/a is the cylindrical equivalent of k_y at r = a. Thus

$$\alpha \simeq (\Omega_p/c)^2(a/m)^2 . \tag{54}$$

When $\alpha << 1$, Eq. (52) gives $g^2 \simeq n_x^2$, which is the low-density approximation made earlier. When α is not small, g^2 is negative (and wave propagation occurs) between the values $\Delta = \frac{1}{2} A\, \alpha(\alpha-1)$, where the numerator of Eq. (52) vanishes and the value $\Delta = -\frac{1}{2} A\alpha$, where the denominator vanishes. Thus the narrow propagation region ($g^2 < 0$) in Fig. 3 becomes wide, with $\Delta = (\omega-\Omega_c)/\Omega_c = 0(1)$, for $\alpha > 2/A$. This region is shown in Fig. 8, where density n is plotted against a/m for $\alpha = 1$ and 0.1. For heavy atoms it is seen that

$\alpha < .01$ up to very high densities for laboratory-size plasmas. For hydorgen or deuterium plasmas in rf plugging experiments, $\alpha = 0.1$ can be achieved if the density is kept below 10^{13} cm^{-3} and a/m below 2.5 cm. This may be possible in reactor-size plasmas if the cusp region is kept thin. For tokamak reactors with $n > 10^{14}$ cm^{-3} and $a/m > 20$ cm, the near-field effects should be negligible. However, near-term rf heating experiments with $n \leq 10^{13}$ cm^{-3} and $a \leq 20$ cm can benefit from the field enhancement described here if $m \gtrsim 4$. Of course, if m is too large the vacuum field decays too rapidly away from the coil.

We wish to thank A. Bers, J. Dawson, B. McVey, and T. Romesser for enlightening discussions. This work was supported by the U.S. Department of Energy, Contract EN-77-C-03-1461.

REFERENCES

a) Permanent address: Electrical Engineering Department, University of California, Los Angeles, California 90024.

1. J. M. Dawson, H. C. Kim, D. Arnush, B. D. Fried, R. W. Gould, L. O. Heflinger, C. F. Kennel, T. E. Romesser, R. L. Stenzel, A. Y. Wong, and R. F. Wuerker, Phys. Rev. Lett. 37, 1547 (1976). J.P.M. Schmitt, Phys. Fluids 21, 1461 (1978); E.S. Weibel, Phys. Rev. Letters 44, 377 (1980).

2. T. Watari, T. Hatori, R. Kumazawa, S. Hidekuma, T. Aoki, T. Kawamoto, M. Inutake, S. Hiroe, A. Nichizawa, K. Adati, T. Sato, T. Watanabe, H. Obayashi, and K. Takayama, Phys. Fluids 21, 2076 (1978).

3. B. D. McVey, TRW Technical Reports Task II-2117 and 2584 (1979, unpublished); J. M. Kindel, A. T. Lin, J. M. Dawson, and R. M. Martinez, Phys. Fluids 24, 498 (1981).

4. B. D. Fried, TRW Technical Report Task II-THA-01 (1977, unpublished); M. Z. Caponi, TRW Technical Report Task II-2231 (1978, unpublished).

5. J. T. Tang, R. L. Stenzel, and H. C. Kim, Phys. Fluids 22, 1907 (1979).

6. J. M. Dawson, private communication.

7. Y. Ohsawa, M. Inutake, T. Tajima, T. Hatori, and T. Kamimura, Phys. Rev. Lett. 43, 1246 (1979).

8. T. H. Stix, Theory of Plasma Waves (McGraw-Hill, New York, 1962).

FIGURE CAPTIONS

Fig. 1 Physical mechanism of rf field enhancement.

Fig. 2 Geometry of the slab problem.

Fig. 3 Variation of "radial" attenuation coefficient g with frequency in the neighborhood of $\omega = \Omega_c$. Here Δ is $(\omega - \Omega_c)/\Omega_c$ and A is the off-resonance amplification factor.

Fig. 4 Radial variation of the vacuum field E_{oy} and the enhanced internal fields E_x and E_y (in arbitrary units).

Fig. 5 Dependence of the enhancement factor Q on density at cyclotron resonance.

Fig. 6 Space charge distribution under a helical coil.

Fig. 7 The electromagnetic enhancement factor Q_m evaluated for the parameters of the Nagoya Type-3 coil experiments.

Fig. 8 Regime in which direct cyclotron acceleration in the near-field of an antenna is feasible: the region $\alpha \ll 1$. Here $\alpha^{\frac{1}{2}} = \Omega_p/ck_x$ is essentially the number of ion collisionless skin depths in a plasma radius a, n is the density, m is the azimuthal mode number of the antenna, and Xe and D are xenon and deuterium.

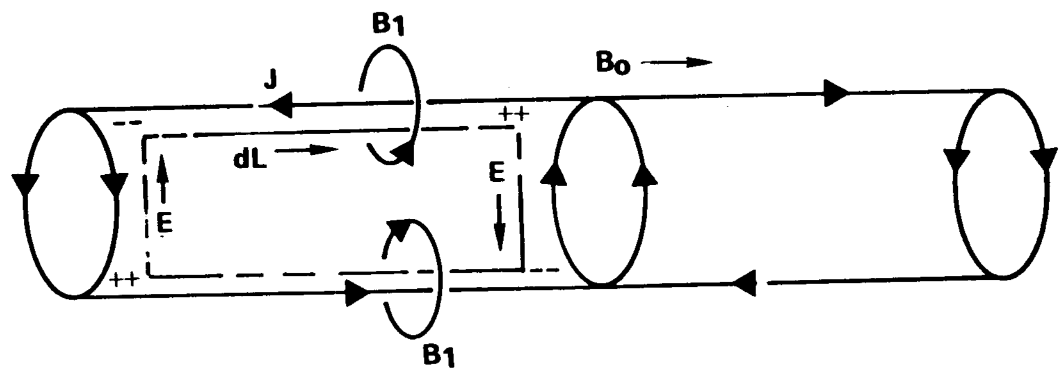

Figure 1

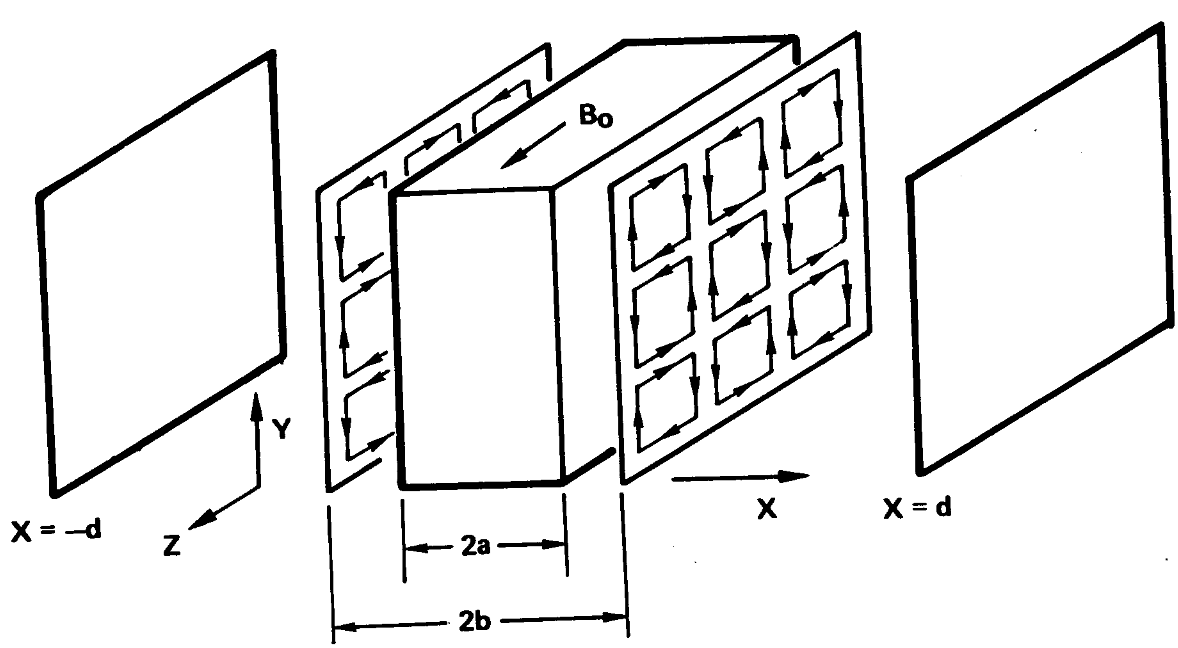

Figure 2

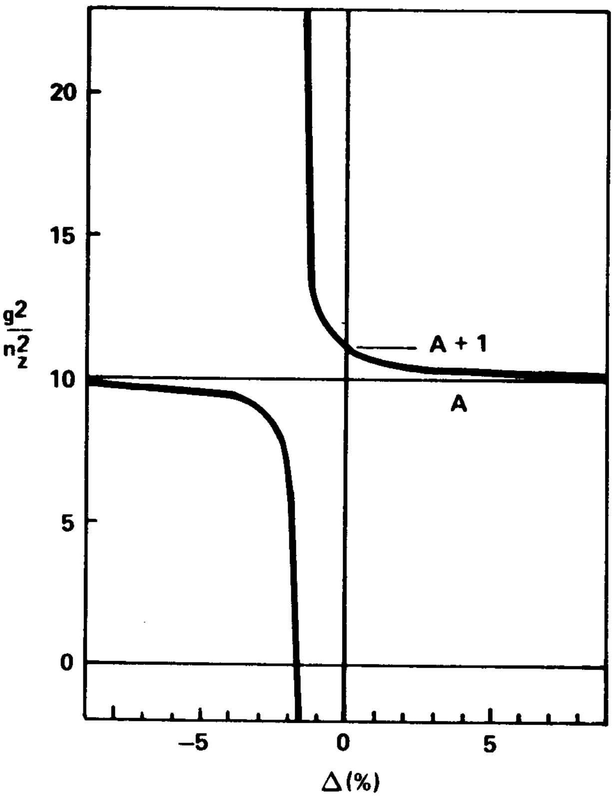

Figure 3

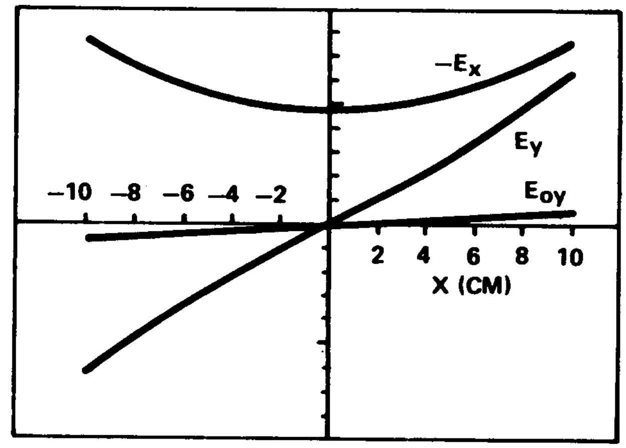

Figure 4

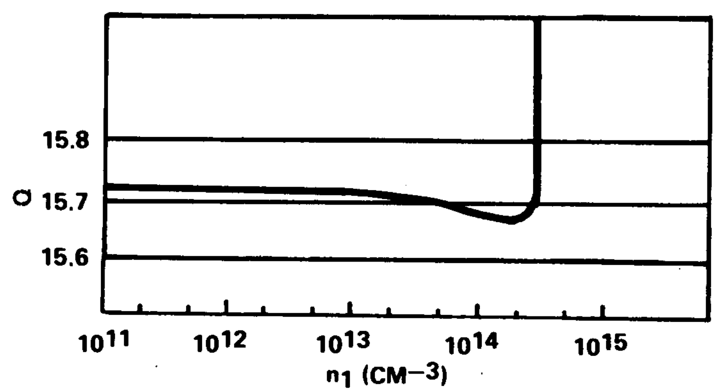

Figure 5

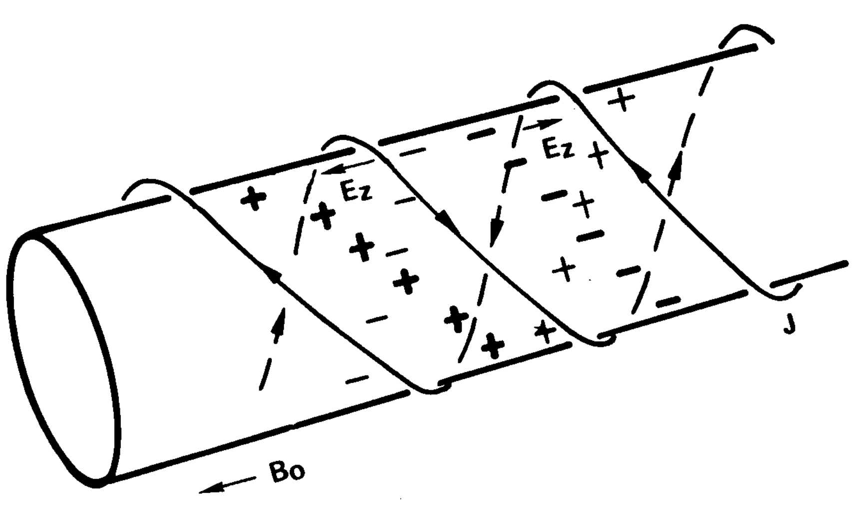

Figure 6

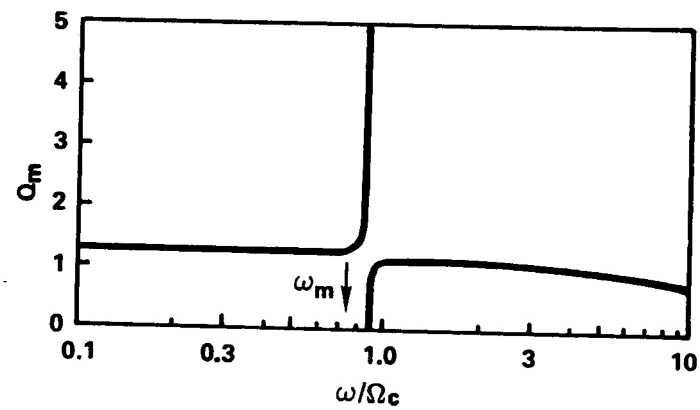

Figure 7

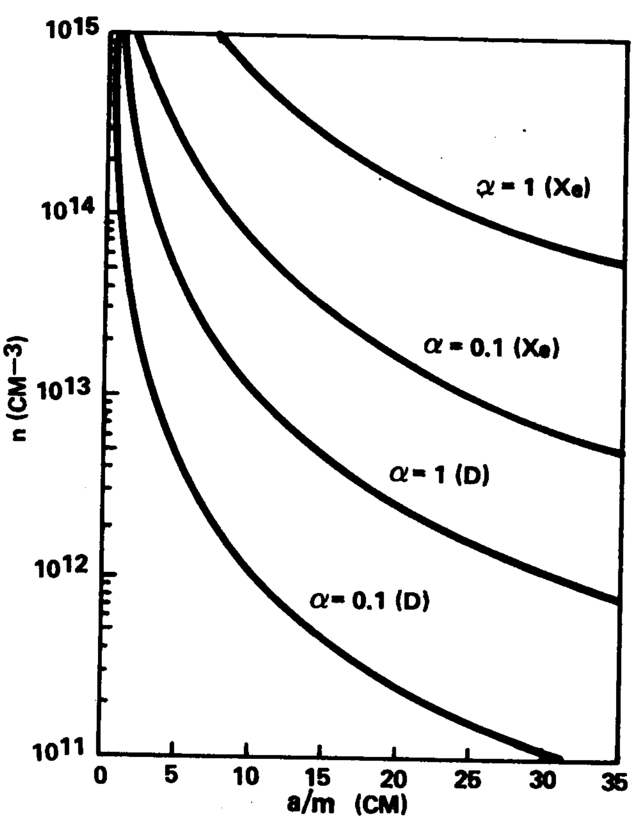

Figure 8