CAPACITOR TUNING CIRCUITS FOR INDUCTIVE LOADS

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This report was written for the use of our students, and is not intended for publication in its present form.

ABSTRACT and SUMMARY

The following simple formulas have been derived for the loading and tuning capacitors, C_1 and C_2, in networks for matching to a load $R + jX$. All impedances are normalized to R_0, and ω is absorbed in the definition of C_1, C_2.

Standard circuit:

$$\begin{aligned} C_1 &= \left[1-(1-2R)^2\right]^{1/2}/2R \\ C_2 &= [X-(1-R)/C_1]^{-1} \end{aligned}$$

Alternate circuit:

$$\begin{aligned} C_1 &= R/B\,, \qquad C_2 = (X-B)/T^2 \\ T^2 &\equiv R^2+X^2 \\ B^2 &\equiv R(T^2-R) \end{aligned}$$

No direct match is possible for a capacitive load $(X < 0)$. However, if impedances are transformed by a line of length kz, R and X above should be replaced by

$$\begin{aligned} R &= R_L/D\,, \qquad D \equiv (\cos kz - X_L \sin kz)^2 + R_L^2 \sin^2 kz \\ X &= \left\{\left[1-\left(R_L^2+X_L^2\right)\right]\sin kz \cos kz + X_L(cos^2 kz - \sin^2 kz)\right\}/D\quad , \end{aligned}$$

where the actual load is $R_L + jX_L$.

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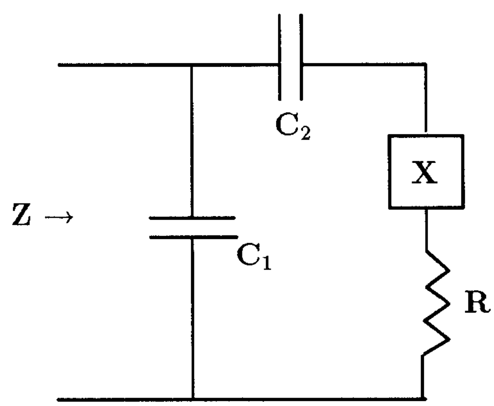
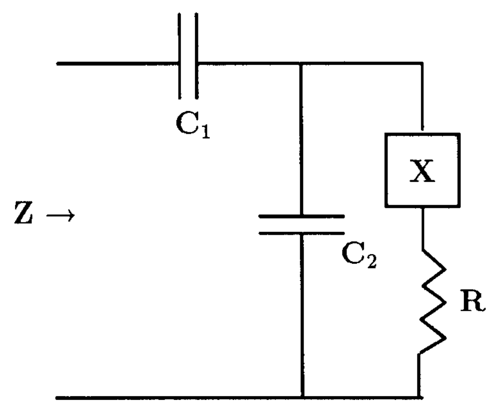
I. Introduction

The advent of fast personal computers makes it unnecessary to use Smith charts and such devices for designing rf transmission lines and matching circuits. By reducing the equations to the simplest form, it is not only possible to generate the results easily on a PC, but the reasons for the behavior of the curves are also more transparent.

RF power supplies for generating plasmas for materials processing are usually fixed in frequency at 13.56 MHz and come with a tuning circuit for matching to a capacitive load, such as that of a parallel-plate RIE (Reactive Ion Etching) reactor. Newer plasma sources use frequencies from 7 to 30 MHz and are coupled to the plasma by an antenna, which forms a primarily inductive load. The two most common circuits, which we shall call the Standard Circuit and the Alternate Circuit, are shown below.



Standard Circuit



Alternate Circuit

Here, **R** and **X** are the real and imaginary parts of the load reactance, respectively, and $\mathbf{C}_1$ is the "load" capacitor, while $\mathbf{C}_2$ is the "tuning" capacitor. The circuit is to have a net impedance $\mathbf{Z}_0$ (usually $50 + j0$ ohms) to match the generator and transmission line. In this paper, we derive explicit formulas for the values of C_1 and C_2 and give computed curves for designing matching circuits. We point out the differences between inductive and capacitive loads, paying special attention to how a circuit designed for capacitive loads can be adapted to inductive loads.

II. The Standard Circuit

Terminology. For convenience, all impedances are normalized to R_o. The factor ω is absorbed into the definitions of L and C. Thus, for inductive loads, $X = X_L = L$; and for capacitive loads, $X = X_C = -1/C$. The impedance Z is given by

$$Z = \left[Z_1^{-1} + Z_2^{-1} \right]^{-1} \quad , \tag{1}$$

where

$$Z_1 = -j/C_1 \ , \qquad Z_2 = R + jX - j/C_2 \quad . \tag{2}$$

One might think that it would be a simple matter to use a program that can handle complex numbers, and simply solve for the real and imaginary parts of Z, set the real part to 1 and the imaginary part to 0, and then iterate to get C_1 and C_2 for different values of R and X. However, such a search usually gets stuck around the wrong root.

Solution. Therefore, we proceed analytically as follows.

$$Z = \frac{R + j(X - 1/C_2)}{1 - C_1(X - 1/C_2) + jRC_1} = \frac{(R + jQ)(1 - C_1 Q - jRC_1)}{(1 - C_1 Q)^2 + R^2 C_1^2} \quad , \tag{3}$$

where

$$Q \equiv X - 1/C_2 \quad . \tag{4}$$

Defining the denominator

$$D \equiv (1 - C_1 Q)^2 + R^2 C_1^2 \quad , \tag{5}$$

we can write

$$D\, Re(Z) = R(1 - C_1 Q) + RC_1 Q = R \quad , \tag{6}$$

$$D\, Im(Z) = Q(1 - C_1 Q) - R^2 C_1 = Q - C_1(R^2 + Q^2) \quad . \tag{7}$$

For matching to R_o, we set $Im(Z) = 0$, obtaining

$$Q = C_1(R^2 + Q^2) \ , \qquad C_1 Q = C_1^2 R^2 + C_1^2 Q^2 \quad . \tag{8}$$

Substituting this into Eq. (5), we have

$$D = 1 - 2C_1 Q + C_1^2 Q^2 + R^2 C_1^2 = 1 - C_1 Q \quad . \tag{9}$$

Since Z is normalized to R_o, we next set $Re(Z) = 1$, so that

$$Re(Z) = R/(1 - C_1 Q) = 1 \ , \qquad C_1 = (1 - R)/Q \quad . \tag{10}$$

Eq. (8) shows that Q satisfies the quadratic

$$C_1 Q^2 - Q + C_1 R^2 = 0 \quad , \tag{11}$$

whose solution is

$$2C_1Q = 1 \pm (1 - 4C_1^2R^2)^{1/2} \quad . \tag{12}$$

The capacitor C_2, which is in Q, is eliminated by substituting this value of C_1Q into Eq. (10):

$$\begin{aligned} 2(1-R) &= 1 \pm (1 - 4C_1^2R^2)^{1/2} \\ 1 - 2R &= \pm(1 - 4C_1^2R^2)^{1/2} \quad . \end{aligned} \tag{13}$$

Squaring this removes the ambiguity of the two roots, and we obtain

$$(1-2R)^2 = 1 - 4C_1^2R^2 \tag{14}$$

$$C_1 = [1 - (1-2R)^2]^{1/2}/2R \quad . \tag{15}$$

To obtain C_2, we equate the expressions for Q from Eqs. (10) and (4):

$$Q = (1-R)/C_1 = X - 1/C_2 \quad , \tag{16}$$

obtaining

$$C_2 = \left(X - \frac{1-R}{C_1}\right)^{-1} \quad . \tag{17}$$

In conventional units, the result is, finally,

$$C_1 = \frac{1}{2\omega R}\left[1 - \left(1 - \frac{2R}{R_o}\right)^2\right]^{1/2} \quad ; \tag{18}$$

$$C_2 = \left[\omega X - \frac{1 - R/R_o}{C_1}\right]^{-1} \quad . \tag{19}$$

Discussion. Note that the loading capacitor C_1 depends only on R and is independent of the load reactance. For an inductive load (R, L), Eq. (19) gives

$$C_2^{-1} = \omega^2 L - (1 - R/R_o)/C_1 \quad . \tag{20}$$

There is therefore a critical inductance $L_C = (1 - R/R_o)/\omega^2 C_1$ below which there is no positive solution for C_2, and for large L's, C_2 depends only weakly on L. For a capacitive load (R, C), Eq. (19) gives

$$C_2^{-1} = -\frac{1}{C} - \frac{1}{C_1}\left(1 - \frac{R}{R_o}\right) \quad . \tag{21}$$

We see that C_2 is negative in the usual case $R/R_o \ll 1$, so that this circuit cannot be used to match to a purely capacitive load. For $R/R_o \ll 1$, we can expand Eqs. (18) and (19) to obtain the approximate formulas

$$\omega C_1 \simeq (RR_o)^{-1/2} \, , \qquad \omega C_2 \simeq \left[X - (RR_o)^{1/2}\right]^{-1} \quad . \tag{22}$$

There are three mitigating factors which allow the Standard Circuit to be used with capacitive discharges, however. First, Eq. (21) is valid only if the matching circuit is connected directly to the load. If the connection is made through even one foot of cable, there will normally be sufficient inductance to exceed L_C. Second, a small inductor consisting of, say, two turns of tube or wire can be inserted between C_1 and C_2 to provide the inductance. Third, a parallel-plate discharge is not a pure capacitance when there is plasma present, because the sloshing of the electrons occurs with a time lag which is effectively an inductance.

An inductive antenna would not be sensitive to these delicate effects, and would give rise to more stable tuning with this circuit. Tuning curves based on Eqs. (18) and (20) are given in Fig. 1 to 4, in both dimensionless and dimensional form. The characteristic impedance is in all cases assumed to be 50 ohms, and curves are given for 13.56 and 27.12 MHz. For other frequencies, one simply scales L or C by the frequency ratio, since ωL and ωC are invariant. One notes that the curves are much less sensitive to R than to L, because the values of R chosen are much less than the values of ωL. If the reactance were not dominant, there would not be a need for a matching circuit in the first place.

III. The Alternate Circuit

Using the same terminology as before, we have for the second circuit

$$\begin{aligned} Z &= Z_1 + Z_2 , \qquad Z_1 = -j/C_1 \\ Z_2 &= \left(jC_2 + \frac{1}{R + jX} \right)^{-1} = \frac{R + jX}{1 + jC_2(R + jX)} \end{aligned} \tag{23}$$

The impedance looking into the circuit and load (R, X) is then

$$Z = \frac{R + jX}{1 + jC_2(R + jX)} - \frac{j}{C_1} = \frac{(C_1 + C_2)(R + jX) - j}{C_1(1 - XC_2) + jRC_1C_2} . \tag{24}$$

Defining the abbreviations

$$Y \equiv 1 - XC_2 , \qquad C \equiv C_1 + C_2 , \tag{25}$$

we have

$$Z = \frac{RC - j(1 - XC)}{C_1 Y + jRC_1C_2} = \frac{1}{C_1} \frac{RCY - RC_2(1 - XC) - j[Y(1 - XC) + R^2CC_2]}{Y^2 + R^2C_2^2} . \tag{26}$$

In terms of the denominator

$$D \equiv C_1 \left(Y^2 + R^2 C_2^2 \right) \tag{27}$$

the real and imaginary parts of Z are given by

$$DRe(Z) = R[C - XCC_2 - C_2 + XCC_2] = RC_1 , \tag{28}$$

$$DIm(Z) = -Y(1 - XC) - R^2CC_2 . \tag{29}$$

Setting $Re(Z) = 1$, we obtain

$$D = RC_1 \quad . \tag{30}$$

Using this in Eq. (27) we obtain

$$Y^2 + R^2C_2^2 - R = 0 \quad . \tag{31}$$

Setting $Im(Z) = 0$ and splitting C into $C_1 + C_2$, we obtain

$$Y(Y - XC_1) + R^2C_1C_2 + R^2C_2^2 = 0 \quad . \tag{32}$$

Two of these terms can be replaced by R, according to Eq. (31), so that

$$-XC_1(1 - XC_2) + R^2C_1C_2 + R = 0 \quad . \tag{33}$$

Defining

$$T^2 \equiv R^2 + X^2 \quad , \tag{34}$$

we find a relation between C_1 and C_2:

$$C_1\left(T^2C_2 - X\right) + R = 0 \ , \qquad C_1 = R/(X - T^2C_2) \quad . \tag{35}$$

C_2 can be found from Eq. (31), which yields the quadratic

$$1 - 2XC_2 + T^2C_2^2 - R = 0 \quad . \tag{36}$$

The solution is

$$T^2C_2 = X \pm B \quad , \tag{37}$$

where

$$B^2 = X^2 - T^2(1 - R) = R(T^2 - R) \quad . \tag{38}$$

Substituting T^2C_2 into Eq. (35), we find

$$C_1 = \frac{R}{X - (X \pm B)} = \frac{R}{B} \quad . \tag{39}$$

Note that only the negative sign in Eq. (37) will give a positive value for C_1; thus, Eqs. (35) and (39) give

$$C_2 = (X - B)/T^2 \quad . \tag{40}$$

Here again, a capacitive load with $X < 0$ cannot be matched directly by this circuit.

Converting back to normal units, we can write the solution as

$$\omega C_1 = (R/R_o)/R_oB \tag{41}$$

$$\omega C_2 = (X - R_oB)/(X^2 + R^2) \tag{42}$$

where

$$R_oB = (R/R_o)^{1/2}\left[X^2 + R(R - R_o)\right]^{1/2} \quad . \tag{43}$$

For $(X/R_o)^2 \gg R/R_o$, approximate solutions are given by

$$\omega C_1 \simeq (R/R_o)^{1/2}/X \quad (44)$$

$$\omega C_2 \simeq \left[1-(R/R_o)^{1/2}\right]/X \quad . \quad (45)$$

Note that C_1 contains the small factor $(R/R_o)^{1/2}$; the advantage of the alternate circuit is the smallness of C_1. In these formulas, X is to be replaced by ωL; there is no solution for capacitive loads. The reactance can be changed by addition of a transmission line, as in the next section. Figs. 5 to 10 are tuning curves for the Alternate Circuit in both dimensionless and dimensional form. Frequiencies of 13.56 and 27.12 MHz have been chosen, but these curves can be scaled easily to other frequencies.

IV. Impedance Transformation with Transmission Line

Capacitive loads can be transformed into inductive loads, and vice versa, by adding a length of transmission line between the load and the matching circuit. In doing so, the value of R seen by the matching circuit may become much higher than the real resistance of the load. This has the effect of decreasing the size of the tuning capacitors and increasing the impedance of the whole circuit. Since we neglect the losses in both the line and the match box, the power delivered by the rf generator will still end up in the plasma, but the generator will have to produce higher voltage at lower current in order to do it.

In a transmission line of impedance $R_o = (L/C)^{1/2}$, where L and C are the inductance and capacitance per unit length, the phase velocity is $v_p = 1/(LC)^{1/2} = 1/R_oC$, and the propagation constant is* $k = \omega R_o C = 2\pi/\lambda = 2\pi f R_o C$. If the antenna impedance is $Z_L = R_L + jX_L$, the impedance seen by the tuning circuit after a line of length z is inserted is

$$Z = \frac{Z_L \cos kz + j \sin kz}{\cos kz + jZ_L \sin kz} \quad . \quad (46)$$

This can be split into real and imaginary parts with the result

$$DRe(Z) = R_L$$

$$DIm(Z) = \left(1 - |Z_L|^2\right) \sin kz \cos kz + X_L(1 - 2\sin^2 kz) \quad , \quad (47)$$

where

$$D = (\cos kz - X_L \sin kz)^2 + R_L^2 \sin^2 kz \quad . \quad (48)$$

Here all the impedances have been normalized to R_o. For inductive and capacitive loads, respectively, X_L really stands for

$$X_L = (\omega/R_o)10^{-6}L_{\mu H} \ , \qquad X_C = 10^{12}/R_o\omega C_{pF} \quad . \quad (49)$$

Fig. 11 shows R and X as a function of kz for $R_L = 2/50$ and $X_L = 2 \times 10^{-7}\omega/50$. It is seen that these show a resonance behavior in the region where D has a minimum. Since,

*If ε_R is known instead of R_0 and C, k is simply $k = k_0\sqrt{\varepsilon_R} = (\omega/c)\sqrt{\varepsilon_R}$.

as noted before, the values of R_L are likely to be much less than those of X_L, the resonance peak for R depends on X_L much more than on R_L, as seen from Eq. (48). The sign reversal posiiton for X_L also depends more on X_L than on R_L, as seen from Eq. (47). Various curves for R and X as functions of R_L and X_L, in both dimensionless and dimensional forms, are given in Figs. 12-17. The line length needed to transform a capacitive load to an inductive load is apparent.

V. Combination of Tuning Circuit with Transmission Line

The results for the last three sections can be combined to give the values of C_1 and C_2 for the standard and alternate circuits to match to a load R_L, X_L through a transmission line of length z. There are too many combinations to plot fully, so we have chosen two cases to give in dimensionless units. Fig. 18 gives the capacitor values vs. kz for a normalized inductive load which corresponds to $R_L = 2$ ohms, $L = 0.2$ uH, at 27.12 MHz. Fig. 19 gives the capacitor values for a normalized capacitive load corresponding to $R_L = 2$ ohms, $C = 200$ pF, at 13.56 MHz. Note that the capacitive load requires a minimum line length for C_2 to be positive, while the inductive load cannot be matched if the line is so long that the effective load becomes capacitive. The advantage of the alternate circuit in reducing the size of the capacitors is clear from these two figures.

VI. Conclusion

We have shown how inductive loads are much more easily matched by capacitor circuits than capacitive loads are, and we have reduced the equations to trivially simple form. Matching circuits designed for capacitive loads have to have added inductances, and these can be removed when using inductive loads. We have also shown how the alternate circuit can reduce the required capacitances, but of course the voltage ratings of the capacitors may have to be raised. These computations, which neglect losses in the circuitry, are useful only for preliminary design of matching circuits. In practice, there are resistive losses in the inductive elements of the circuit, especially in eddy currents induced on nearby, grounded conductors. The final design is best done by trial and error (with insight, of course).

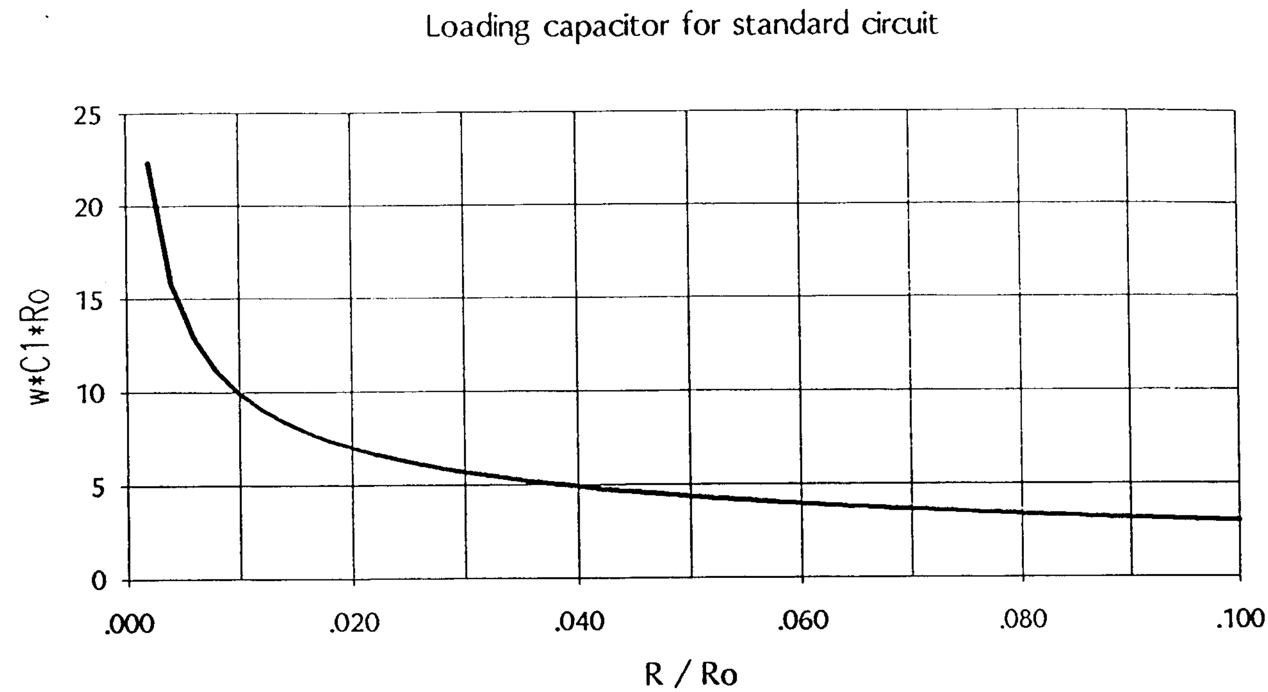


Figure 1: Standard circuit: normalized loading capacitance vs. normalized load resistance.

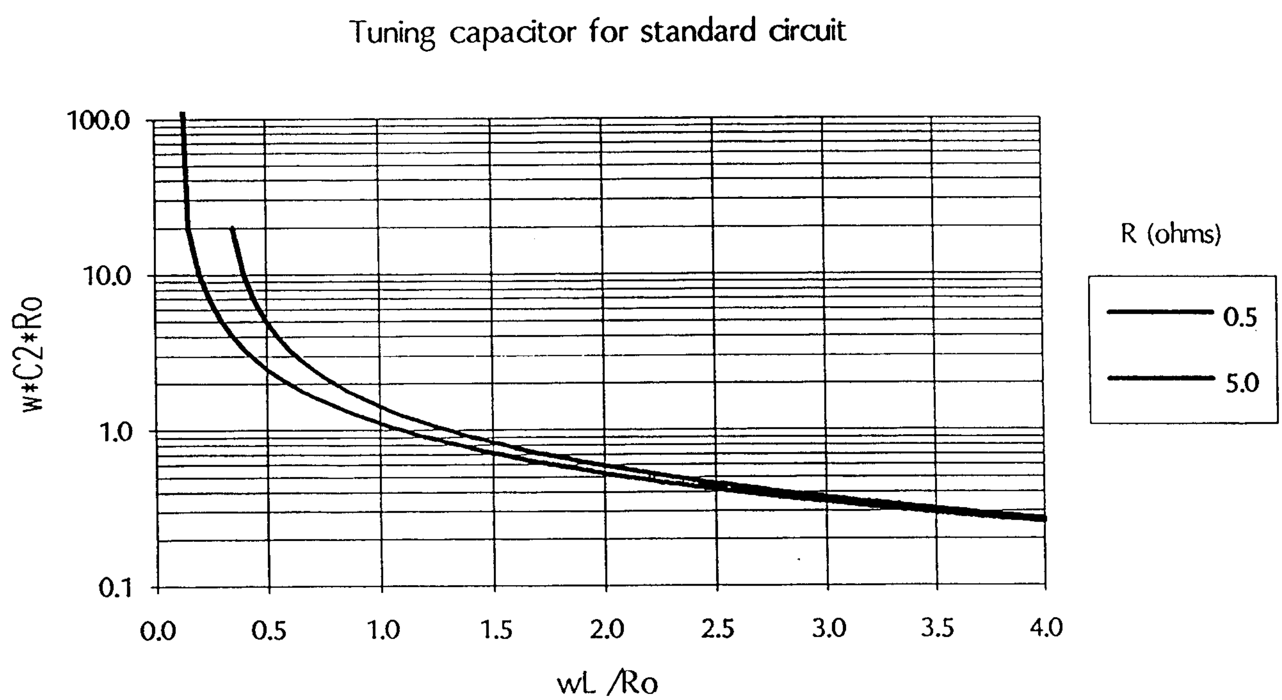


Figure 2: Standard circuit: normalized tuning capacitance vs. normalized load inductance, for two values of load resistance.

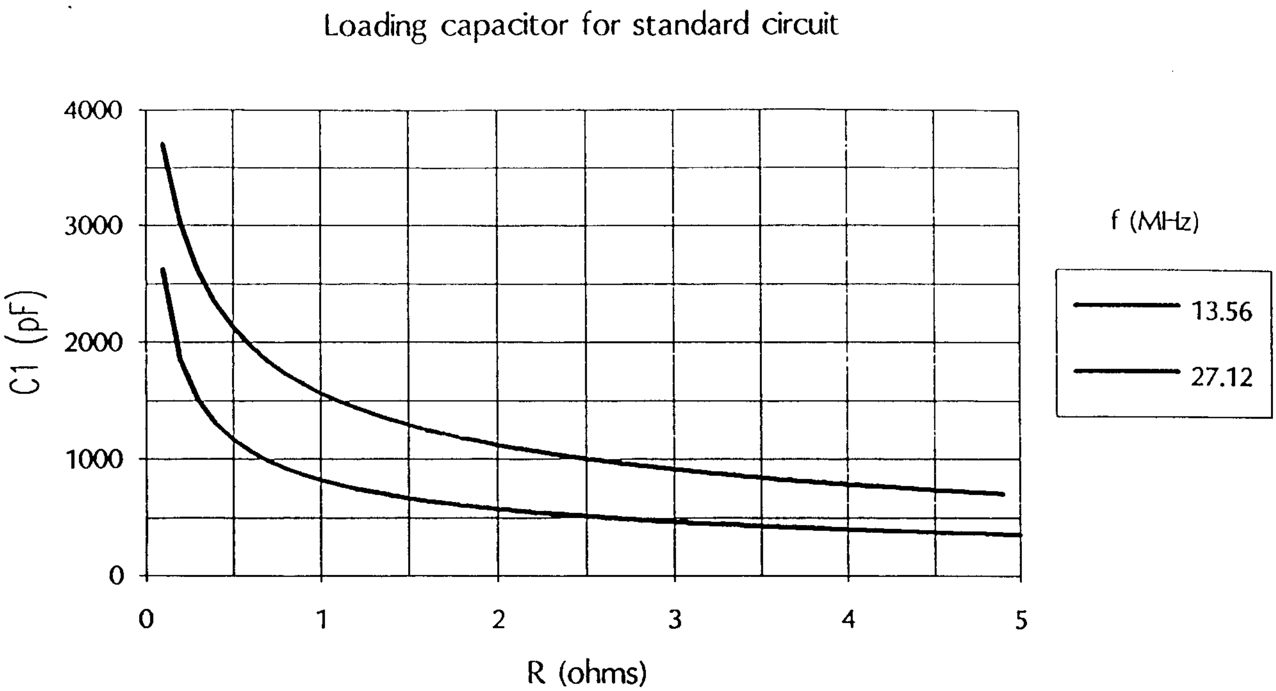


Figure 3: Standard circuit: Loading capacitance vs. load resistance, in practical units, for two frequencies.

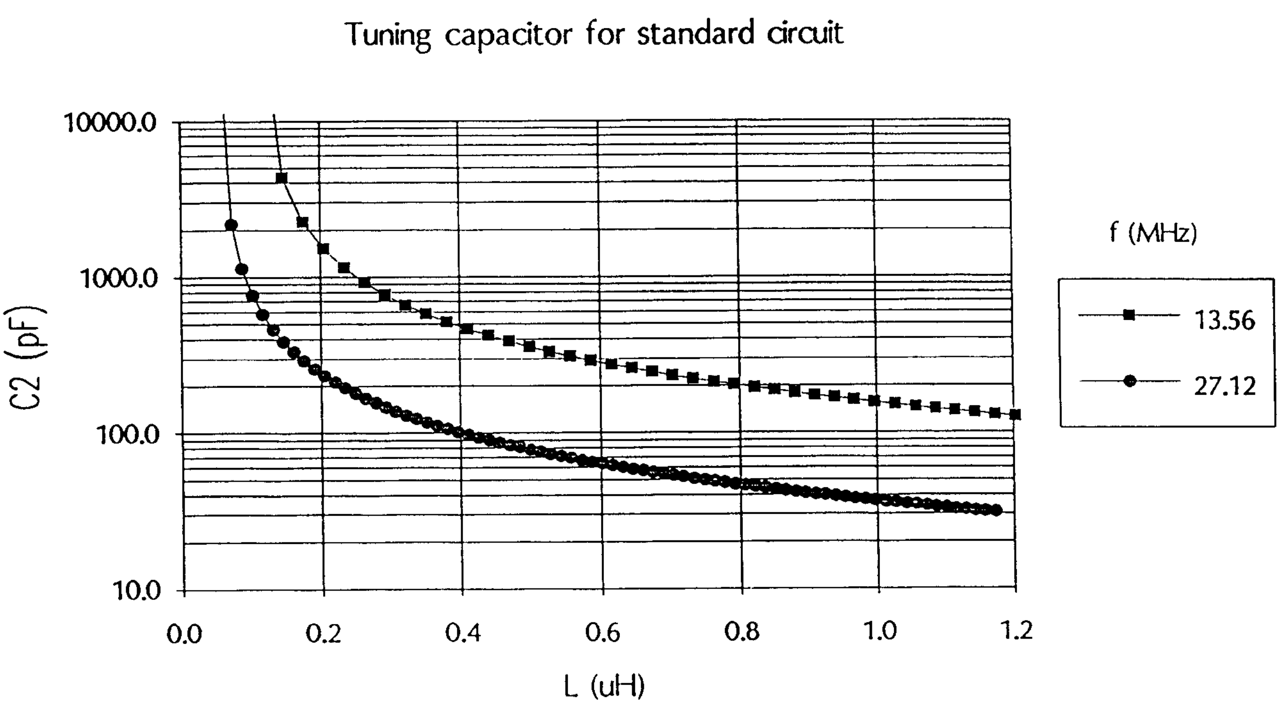


Figure 4: Standard circuit: Tuning capacitance vs. load inductance, in practical units, for two frequencies.

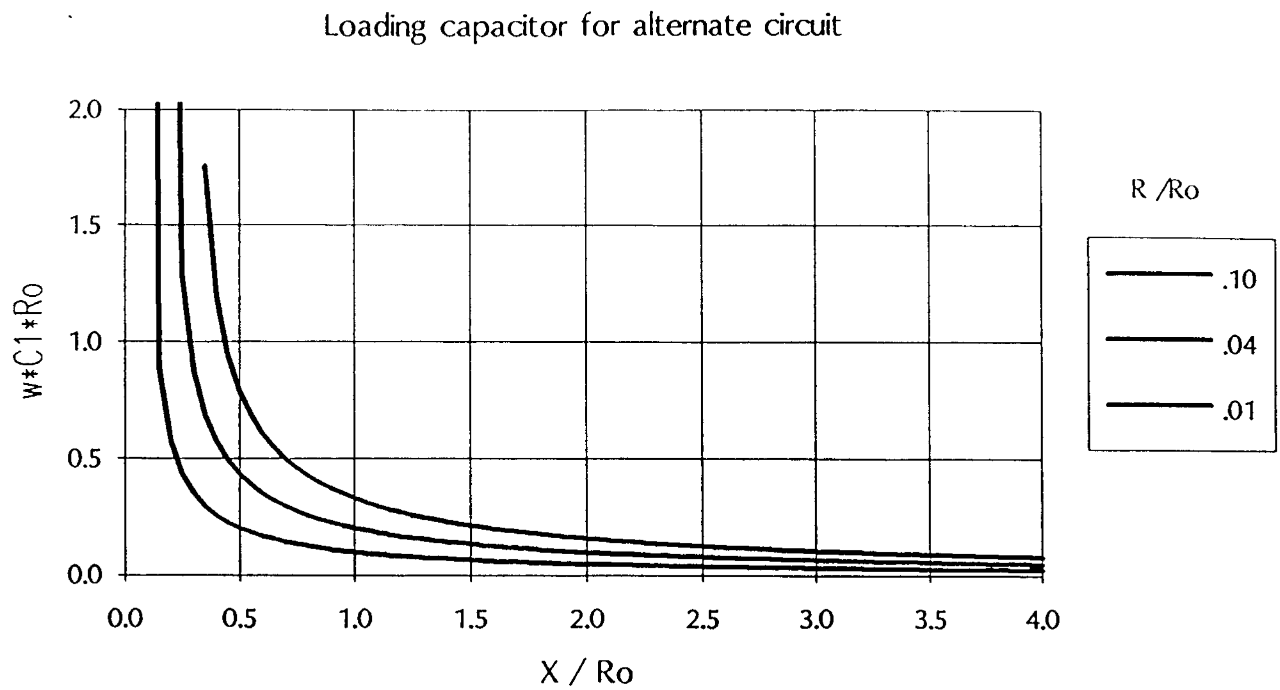


Figure 5: Alternate circuit: normalized loading capacitance vs. normalized load inductance, for three values of normalized load resistance.

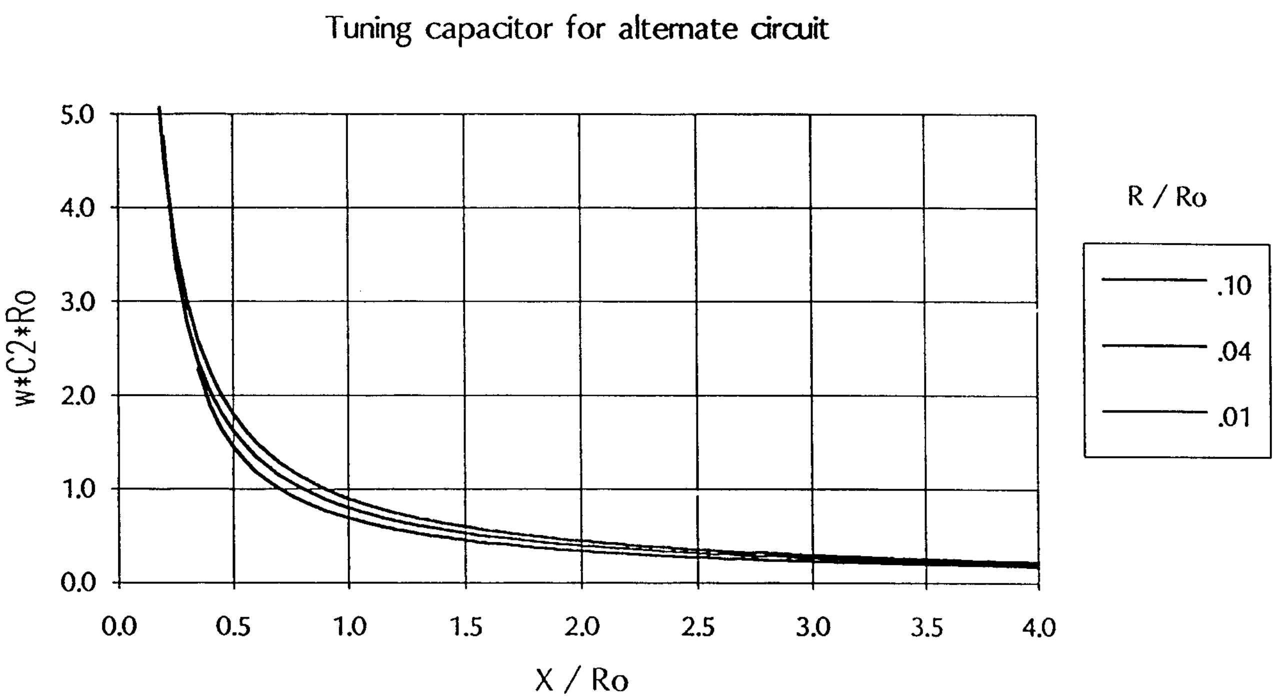


Figure 6: Alternate circuit: normalized tuning capacitance vs. normalized load inductance for three values of normalized load resistance.

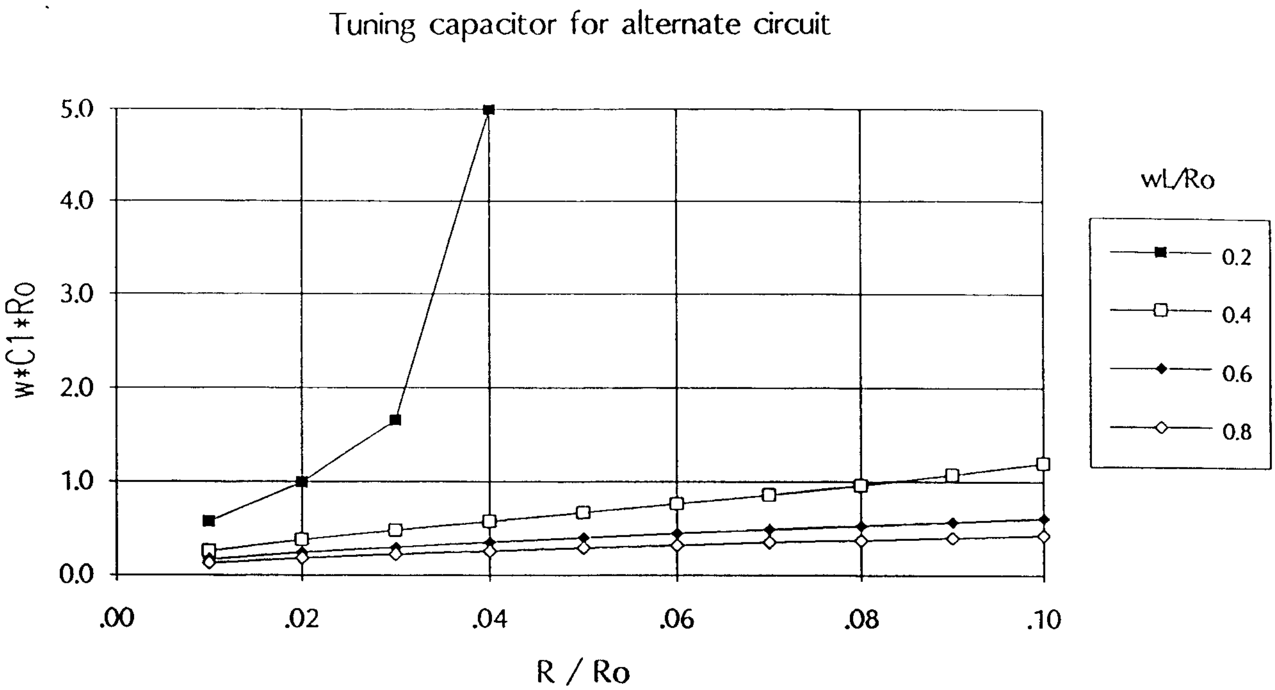


Figure 7: **Alternate circuit: normalized tuning capacitance vs. normalized load resistance, for four values of normalized load inductance.**

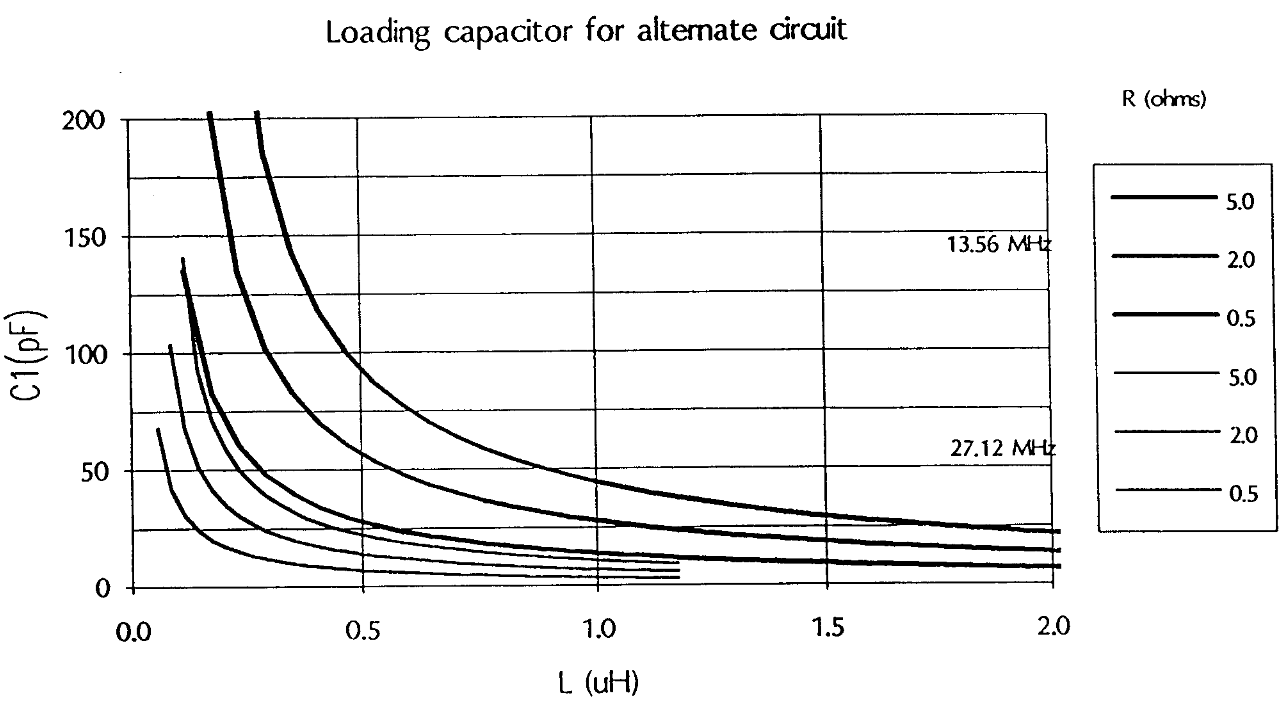


Figure 8: Alternate circuit: loading capacitance vs. load inductance, in practical units, for three values of load resistance and two frequencies.

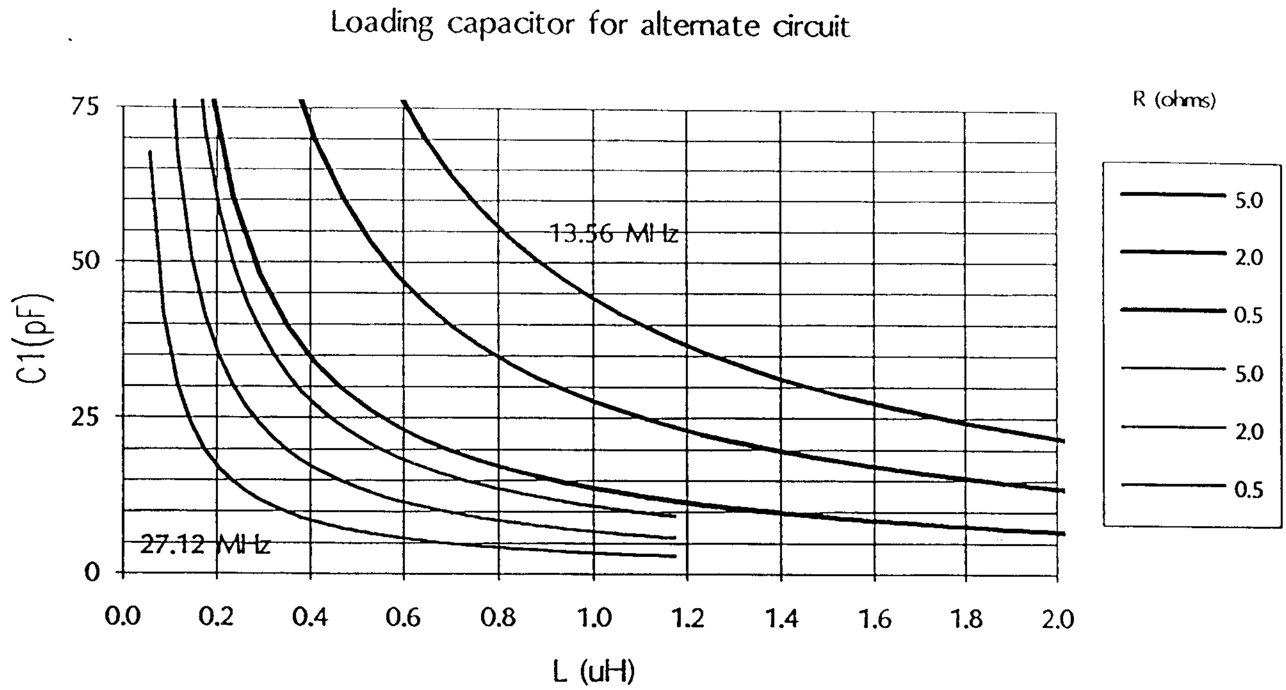


Figure 9: Same as Fig. 7, on an expanded vertical scale.

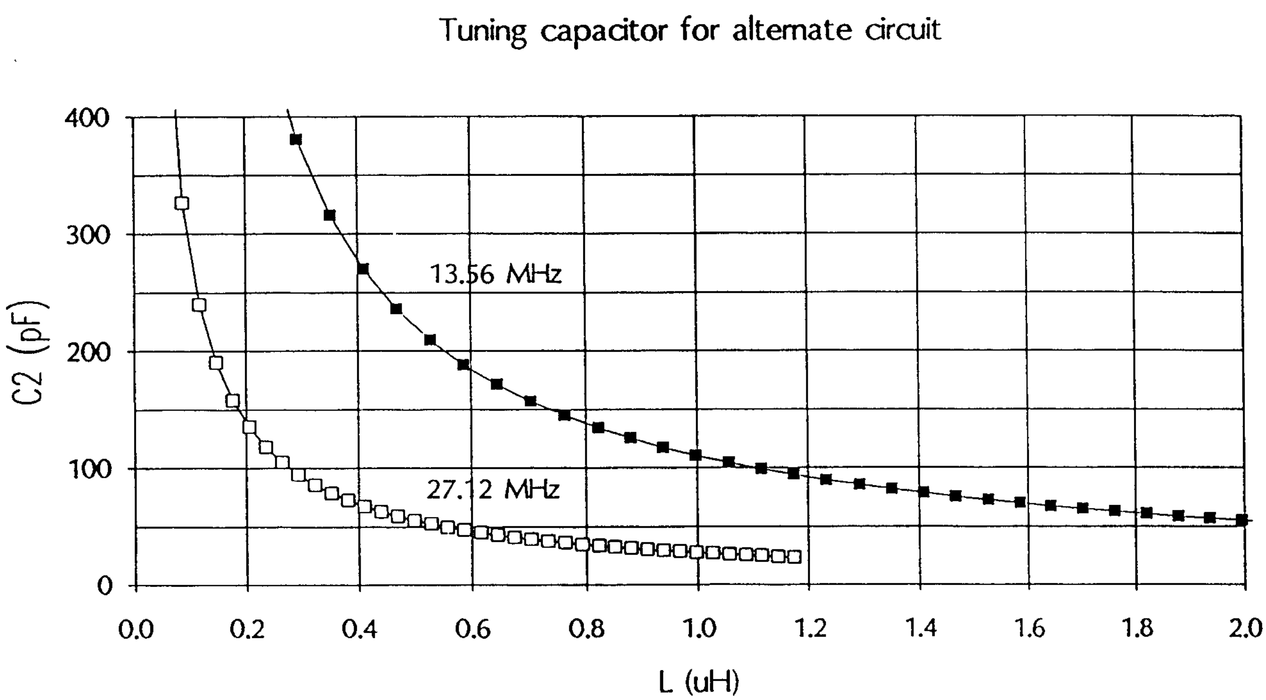


Figure 10: Alternate circuit: tuning capacitance vs. load inductance, in practical units, at two frequencies, for $R = 2$ ohms.

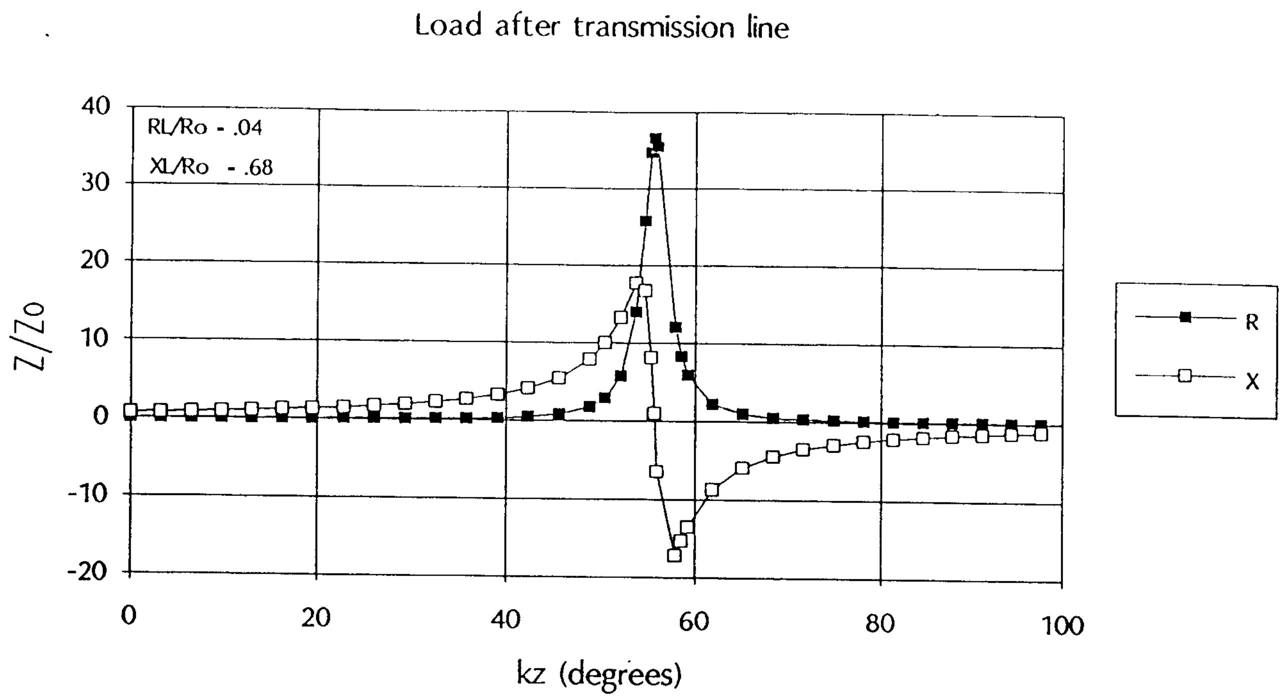


Figure 11: Effective load impedance vs. transmission line length, in normalized units, for a normalized antenna impedance corresponding to 2 ohms and 0.2 uH at 27.12 MHz.

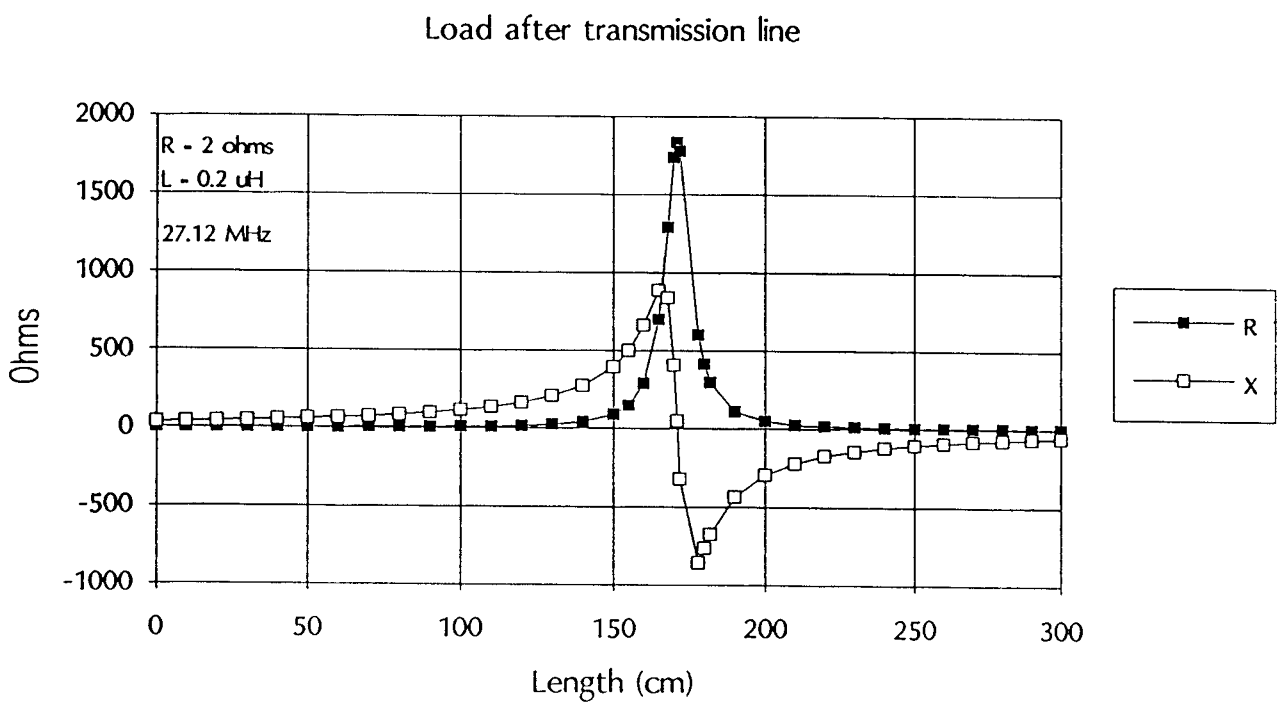


Figure 12: Same as Fig. 11, but in practical units.

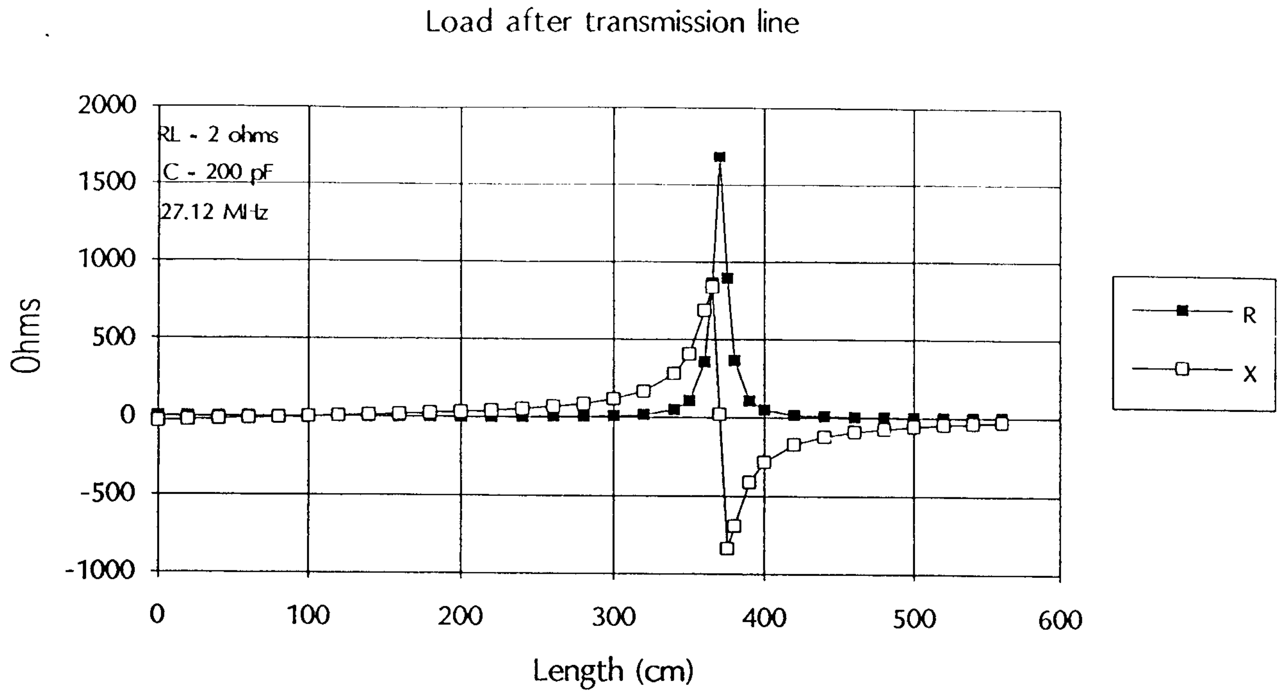


Figure 13: Same as Fig. 12, but for a capacitive load of 200 pF.

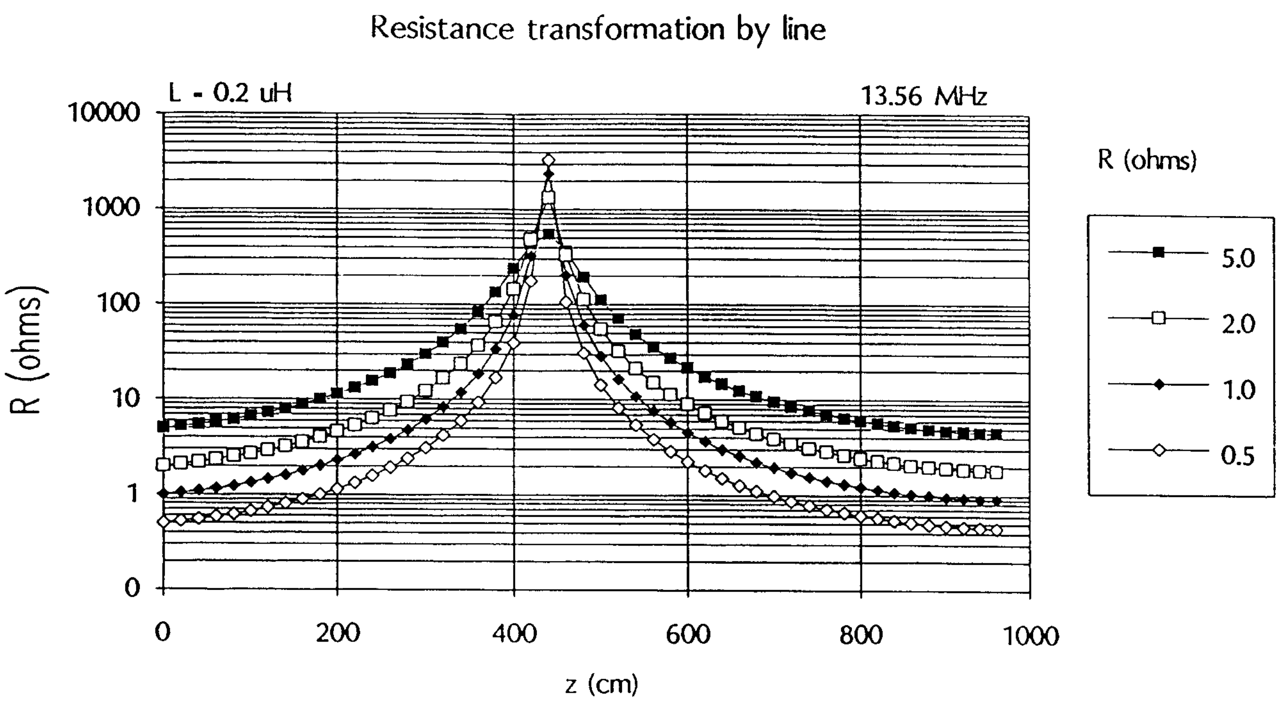


Figure 14: Effective load resistance vs. transmission line length, in practical units, for an antenna inductance of 0.2 uH at 13.56 MHz, and for four values of real load resistance. Note that the effective R increases greatly, but the position of the peak is insensitive to R_L.

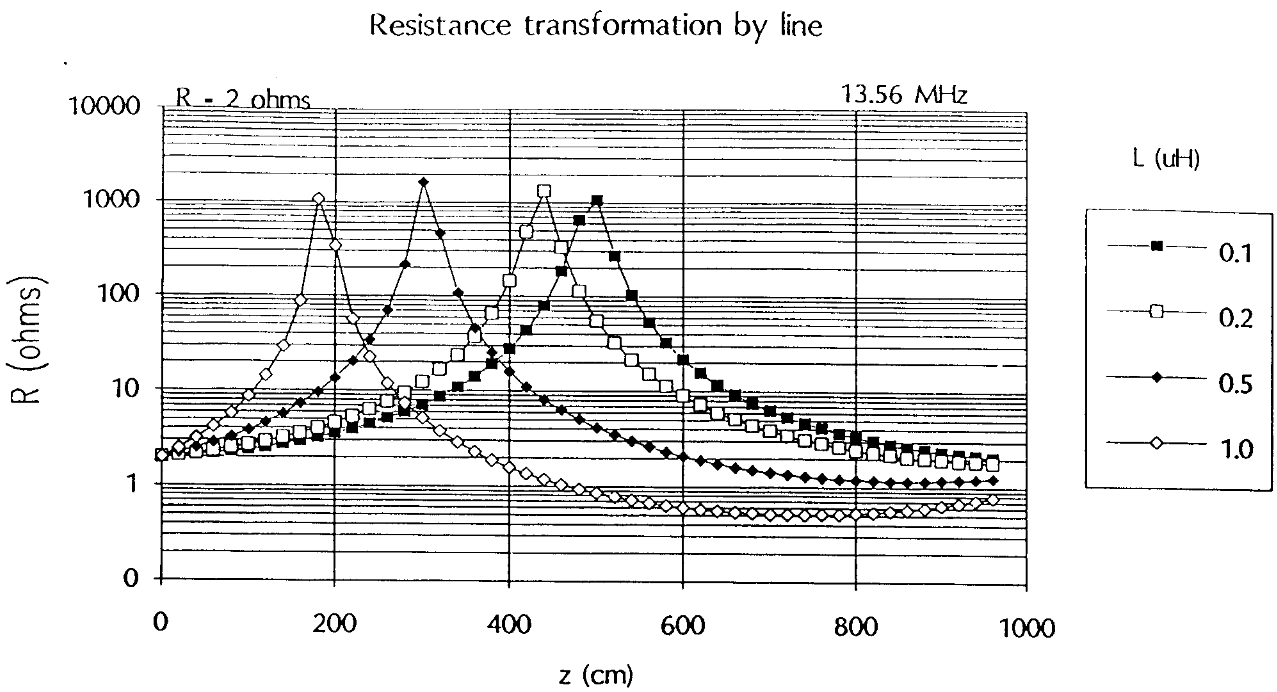


Figure 15: Effective load resistance vs. transmission line length, in practical units, for a real load resistance of 2 ohms at 13.56 MHz, and for four values of antenna inductance. Note that the position of the peak depends on L.

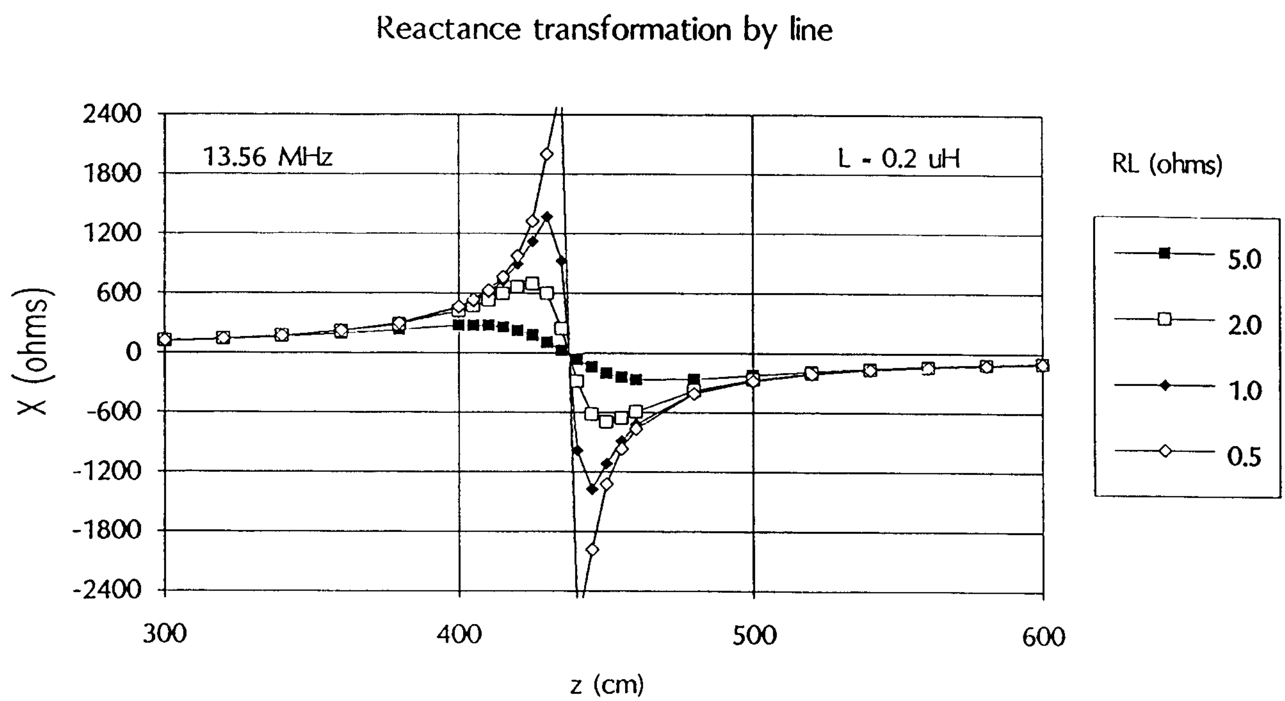


Figure 16: Effective load reactance vs. transmission line length, in practical units, for an antenna inductance of 0.2 uH at 13.56 MHz, and for four values of real load resistance. Note that the effective X changes sign at a critical length which is insensitive to R_L.

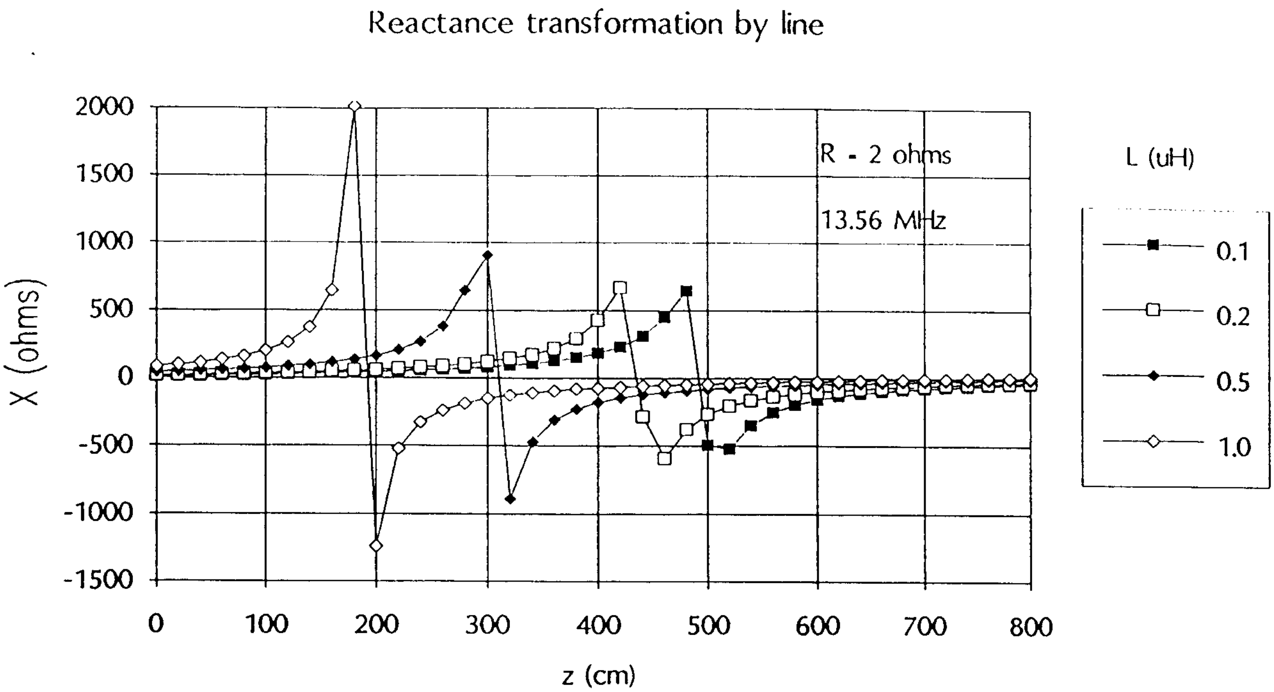


Figure 17: Effective load reactance vs. transmission line length, in practical units, for a real load resistance of 2 ohms at 13.56 MHz, and for four values of antenna inductance. Note that the critical length depends on L.

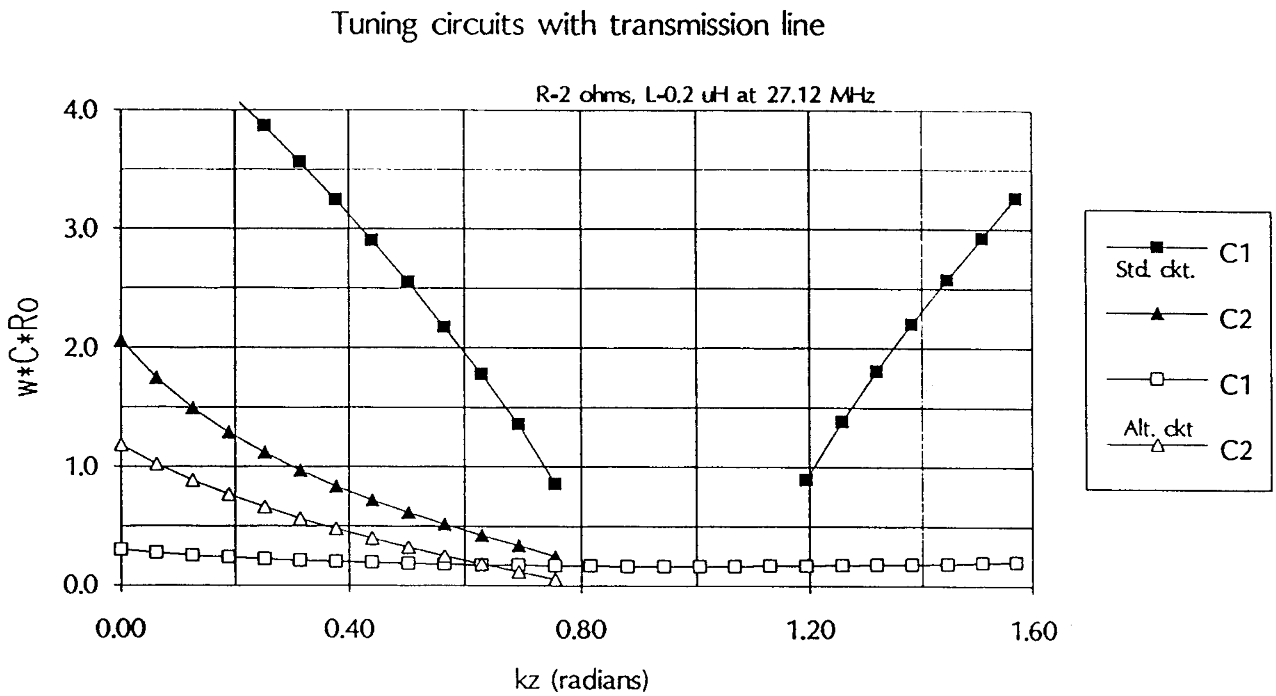


Figure 18: Normalized loading and tuning capacitances vs. normalized line length, for both the standard and alternate circuits, for a normalized antenna impedance which corresponds to 2 ohms and 0.2 uH at 27.12 MHz.

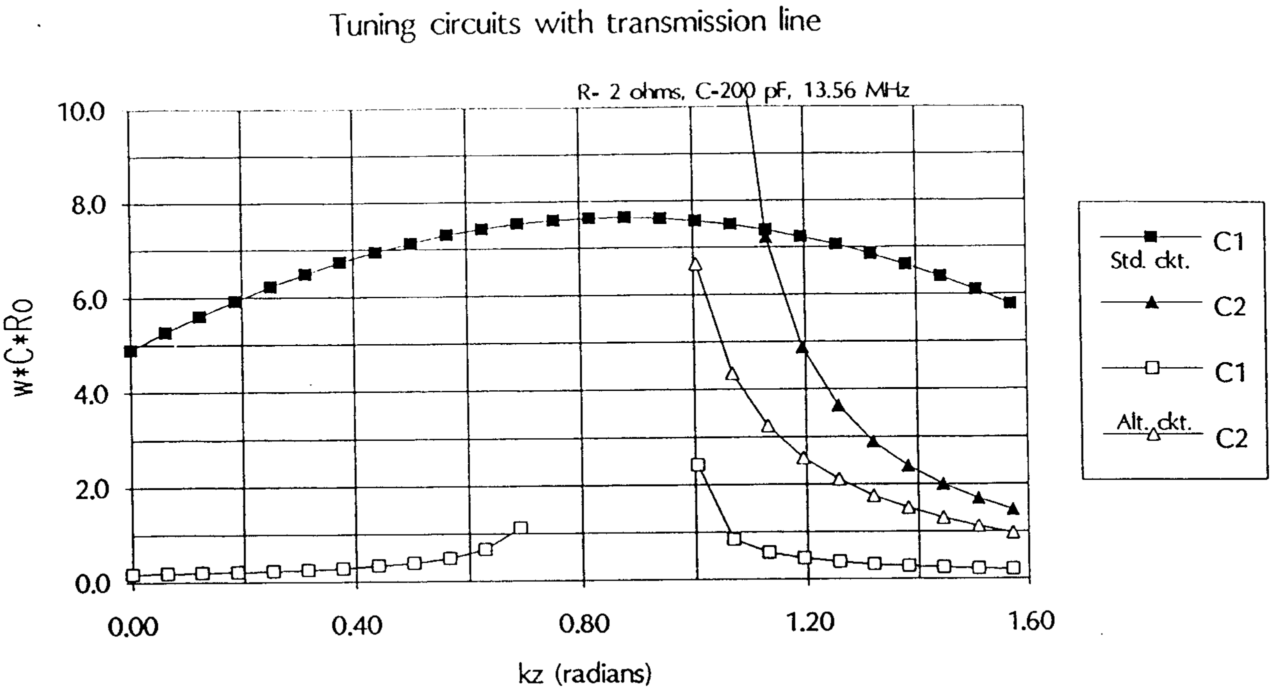


Figure 19: Normalized loading and tuning capacitances vs. normalized line length, for both the standard and alternate circuits, for a normalized antenna impedance which corresponds to 2 ohms and 200 pF at 13.56 MHz. Note that a line length of at least one radian is required to match to the capacitive load.