Physical mechanism of current-free double layers

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Undriven double layers observed in plasmas expanding along magnetic fields are the result of a sheath instability connected with the Bohm criterion. Diverging magnetic field lines cause the presheath acceleration of ions, causing a potential jump resembling that of a double layer. The process stops when it runs out of energy. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2179393]

There have been numerous recent reports[1–5] of double layers observed in plasmas expanding along magnetic fields in laboratory simulations of plasma thrusters for space propulsion. These layers are unusual in that they occur in free space away from boundaries and are not driven by currents. We show here that the "double layers" of Charles *et al.*[1] are actually single layers and are predictable from classical sheath theory, normally applied to boundaries, with only one assumption: that of Maxwellian electrons. No complicated mechanism need be involved.

Consider the configuration of Fig. 1, in which a plasma of radius r_0, density n_0, and temperature T_e are created in a uniform field $\mathbf{B}_0$ and then injected into a large chamber with a weaker magnetic field through a region of expanding field lines. For plasma frozen to the field lines, the field $B(z)$ and the density $n(z)$ in the expansion region are related to the plasma radius r by

$$\frac{B}{B_0} = \frac{n}{n_0} = \left(\frac{r_0}{r}\right)^2. \tag{1}$$

We assume Maxwellian electrons satisfying

$$n_e = n_0 e^{-\eta}, \quad \text{where} \quad \eta \equiv -\mathrm{e}V/KT_e, \tag{2}$$

V being the potential relative to that in the source.[6] As n_e decreases along an expanding field line, V must decrease and η increase. At a certain point $z=s$, η will reach 1/2, the value at which the Bohm criterion is satisfied. The ions, assumed cold, will have fallen through a potential of $(1/2)KT_e/e$ and thus achieved a speed of $(KT_e/M)^{1/2}=c_s$. With this amount of inertia, the ions will have a density that falls more slowly than that of the electrons as η increases further,[7] and the quasineutral solution becomes unstable. A further increase in z causes n to drop and η to increase, according to Eq. (2) With $n_i > n_e$, $V''(z)$ drops rapidly, and an ion sheath must form, even in "midair." This occurs at a position where

$$\frac{n}{n_0} = e^{-1/2}, \quad \text{and} \quad \text{thus} \quad \frac{r}{r_0} = e^{1/4} = 1.28; \tag{3}$$

that is, when the plasma radius has expanded by 28%.

The further development of the sheath can be described by usual sheath theory, considering s to be the sheath edge. Normally, the ion energy of $\frac{1}{2}KT_e$ is gained in a presheath field, whose extent is governed by collisions and ionization and is therefore specific to each discharge. Here plasma expansion has taken the place of the presheath in accelerating ions to the Bohm velocity, and this happens even in a collisionless plasma. The neglect of collisions here gives results in general agreement with numerical calculations in the limit of long mean-free path.[5] As n_i and n_e separate, the sheath builds up until it reaches what would be the floating potential of a plane probe, namely[8]

$$\eta_f = -\frac{\mathrm{e}V_f}{KT_e} = \frac{1}{2}\left[1 + \ln\left(\frac{M}{2\pi m}\right)\right] \approx 5.18 \text{ in argon}. \tag{4}$$

At this point, the forward fluxes of ions and electrons are equal, and a current-free single layer is formed. This "sheath" has a thickness scaled to the Debye length λ_D, which is assumed much smaller than the scale length of ∇B, so that the change in plasma radius can be neglected henceforth. If V were to fall further, its behavior would be described by the Child-Langmuir equation for space-charge-limited ion current in a plane diode. However, the ions cannot be accelerated much further since, in the absence of a biased electrode, there is no energy source to drive them.

The directed energy of the ion stream ultimately comes from the power used to maintain the plasma in a steady state. Consider a floating plane probe or a section of the wall. It will be negative relative to the plasma interior by the amount η_f given by Eq. (4). The flux of ions at s is

$$\Gamma_i = n_0 e^{-1/2} c_s = n_0 e^{-1/2} (KT_e/M)^{1/2} \tag{5}$$

and is the same at the wall, while the flux of electrons to the wall is

$$\Gamma_e(V_f) = n_0 e^{-\eta_f} \left(\frac{KT_e}{2\pi m}\right)^{1/2}. \tag{6}$$

Equating these two leads to Eq. (4). The ions strike the wall with an energy

$$E_i = \eta_f KT_e, \tag{7}$$

so the total ion energy lost per unit area is

$$W_i = \Gamma_i E_i = n_0 e^{-1/2} (KT_e^3/M)^{1/2} \eta_f. \tag{8}$$

Being Maxwellian, the electrons in a repelling potential have a smaller density but the same temperature. Including the energy in the x and y directions,[9,10] each escaping electron

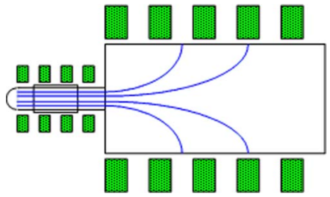

FIG. 1. Geometry of experiments in an expanding magnetic field.

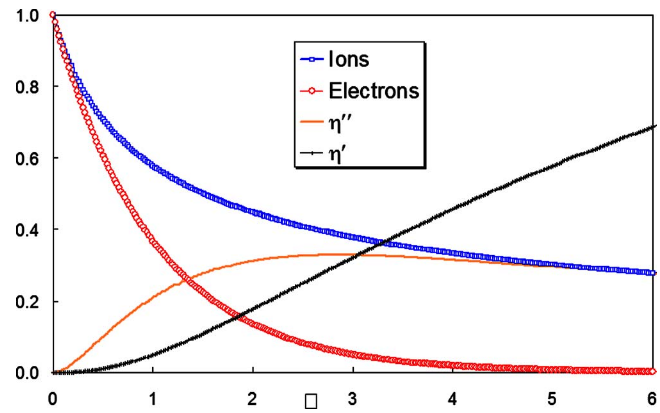

FIG. 2. Relative ion and electron densities in a sheath versus the normalized potential η, together with the corresponding first and second derivatives of η.

carries away an energy $2KT_e$. Hence, the electron energy lost per unit area is

$$W_e = n_0 e^{-\eta_f}\left(\frac{2KT_e^3}{\pi m}\right)^{1/2}. \tag{9}$$

If the only source of energy is that required to replenish the electron tail, energy conservation would require $W_i = W_e$. This results in the equation

$$\tilde{\eta}_f \ln(\tilde{\eta}_f) = \eta_f + \ln(2), \tag{10}$$

where η_f is that in Eq. (4), and $\tilde{\eta}_f$ is the floating potential required by energy conservation. For argon, $\tilde{\eta}_f$ is 4.40, compared with $\eta_f = 5.18$. An additional source of energy is required to bring η_f up to the value required by current conservation.

In maintaining a steady-state density, much more energy is supplied than that of the electron tail. Each ionization requires[11] >30 eV of energy, most of which goes into line radiation. Only a small part goes into newborn ions. When there are collisions, more energy is required to sustain the electric field in the presheath against Ohmic and charge-exchange losses. This would not be necessary in the nearly collisionless presheath considered here. In the case of helicons, there is an additional source that is not yet completely understood. Balkey *et al.*[12] have measured perpendicular ion temperatures $KT_\perp$ up to 0.5 eV, and Sun *et al.*[13] have seen parallel ion temperatures $KT_\parallel$ up to 0.9 eV. Whatever its cause, $KT_\perp$ will be converted into $v_\parallel$ in the expanding field. The point is that there is a limit to the amount of energy that can be given to the ion beam, and therefore the potential drop must stop at around $5KT_e/e$.

The ion and electron densities past $z=s$ are shown in Fig. 2, together with the derivatives η' and η'', which they produce via Poisson's equation.[7] These curves are computed for $n \approx 10^{11}$ cm^{-3} and $T_e \approx 8$ eV, conditions given by Charles *et al.*[1] Imagine a floating plate at $\eta \approx \eta_f \approx 5$. In front of it will be a normal sheath, whose thickness is much smaller than the ∇B scale length. If the imaginary barrier is now removed, $V(z)$ would continue dropping steeply because of the large charge density. However, it cannot do so because the ions would then gain energy, and there is no source for that energy. Past this single layer, the ions must retain the energy they have gained, there being no source of a reverse electric field to decelerate them, and the electrons will drift along with them, maintaining quasineutrality. Measurements[1] of $V(z)$ in a "double layer" indeed are consistent with a steep drop followed by a flat $V(z)$.

In practice, the sharp corner would be smoothed out by incidental effects. Electrons could be drawn back by the excess ion charge until there is an electron-rich layer that reverses the sign of V'' so that $V(z)$ can bend back to a horizontal line. Exactly how the single layer turns into a double layer is probably device-dependent. The electrons required to neutralize the ion charge at the layer may come from reflection at the wall sheath where the field lines end, or perhaps from a halo of electrons around the beam. Observations show[1] that the electron-rich part of the double layer is very thin, and there is only a small rounding of the sharp corner at the bottom. The structure is basically a single layer.

The maximum energy of the ion beam is then the energy-limited sheath drop, or about $5KT_e$ for argon [Eq. (4)]. For $KT_e \sim 8-10$ eV,[1] this amounts to about 45 eV, in fortuitously good agreement with the 47 eV reported by Charles *et al.*[1] One would expect that experimental deviations from the ideal theoretical model would cause the energy to be lower. Indeed, ion beams of 30 eV[2] and 15 eV[3] have been reported in other papers. Collisions should spread the ion beam in both energy and angle, and also create the cold ion background that is sometimes seen in addition to the beam. If the plasma source is a helicon discharge, part of the acceleration occurs within the discharge, causing the plasma to leave the source with a finite drift speed. The drift depends on the amount of downstream ionization. If there is no ionization downstream, there are no ions moving backwards at the source exit, and therefore the ion distribution must have been shifted forward by a parallel electric field within the discharge. The drift speed need not be as large if there is some downstream ionization caused by waves propagating out of the source. The effect of the drift is to shorten the distance before the Bohm velocity is reached and the single layer begins, but this effect is probably undetectable.

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[6]Note that Roman "e" is charge and Italic "*e*" is the base of natural logarithms.
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