Saturation of Beat-Excited Plasma Waves by Electrostatic Mode Coupling

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The nature of plasma waves which are resonantly excited when two laser beams beat in a rippled-density plasma is explored both theoretically and experimentally. A theoretical model is presented which, for commonly encountered experimental parameters, predicts rapid saturation of the high-phase-velocity beat wave at an amplitude below that expected for relativistic detuning. Results of experimental studies of this process are presented.

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It has been suggested that the extremely large electric fields of a relativistic ($v_\phi \cong c$) plasma wave may be used for ultrahigh gradient acceleration of particles.[1] Of fundamental importance in this acceleration scheme is the maximum electric field to which the plasma wave can be driven and the duration for which it can be made to remain coherent. Such a plasma wave has recently been excited by collinear optical mixing of two lines of a CO_2 laser in a 10^{17}-cm^{-3} plasma.[2] Thus far, relativistic detuning has been believed to be the dominant plasma-wave saturation mechanism with this excitation technique.[3] In this paper we show that if the beat-wave excitation process is considered in the presence of a weak density ripple of wave number k_i, the plasma wave, by coupling to a number of secondary electrostatic modes of frequency ω_p and wave number $k_p \pm nk_i$, can saturate at an amplitude well below the relativistic detuning limit. We then present conclusive experimental evidence for this mode-coupling process in a beat-excited plasma.

Our work differs from previous work by other authors who have studied the evolution of resonantly excited plasma waves with pump but without the ripple[3] and with the ripple but without the pump.[4,5] In this paper we consider the beat excitation of electron plasma waves in the presence of a density ripple. We begin with the Eulerian[3,6] equation for the plasma-wave electric field in the absence of collisions when driven by the ponderomotive force of the beating laser pumps,

$$\ddot{E} + \omega_p^2(1+\epsilon \sin k_l x)E + c_e^2 E'' = \tfrac{1}{2}\omega_p^2 \alpha_1 \alpha_2 \sin(k_p x - \omega_p t), \quad (1)$$

where E is the wave electric field normalized to the cold-plasma wave-breaking amplitude $E_{cold} = mc\omega_p/e$; ϵn_0 is the ripple depth (fixed ions); k_i is the ripple wave number; $c_e^2 = 3kT_e/m_e$; $\alpha_j = eE_j/m\omega_j$ is a measure of the jth laser pump intensity; and ω_p and k_p satisfy the frequency and wave-number matching conditions, $\omega_p = \Delta\omega_{lasers}$ and $\mathbf{k}_p = \pm\Delta\mathbf{k}_{lasers}$. Experimentally a ripple with $\mathbf{k}_i \cong 2\mathbf{k}_{laser}$ is produced by stimulated Brillouin scatter (SBS) of the laser light. For cold plasmas we take $c_e = 0$ in Eq. (1) and look for solutions which satisfy $E(t=0) = \dot{E}(t=0) = 0$. An exact solution is readily obtained[7] and can be written in the form

$$E(x,t) = \sum_{n=-\infty}^{+\infty} a_n(t)\exp(i\Psi_n) + \text{c.c.}, \quad (2)$$

where $\Psi_n = (k_p + nk_i) - \omega_p t$ and the coefficients $a_n(t)$ describe the slow time-scale evolution of each k (Ψ_n) component of the total electric field. From Eq. (2) it is obvious that, in addition to the beat-wave component Ψ_0, an infinite number of secondary coupled modes are excited. We expect saturation of the beat wave when the rate of energy coupling from the beat wave to the secondary modes equals the rate at which energy is supplied to the beat wave by the pumps.

The saturation mechanism is graphically illustrated in Fig. 1 which shows the numerical solutions of Eq. (1) with and without the density ripple. The beat-wave driver is incident from the left and has just

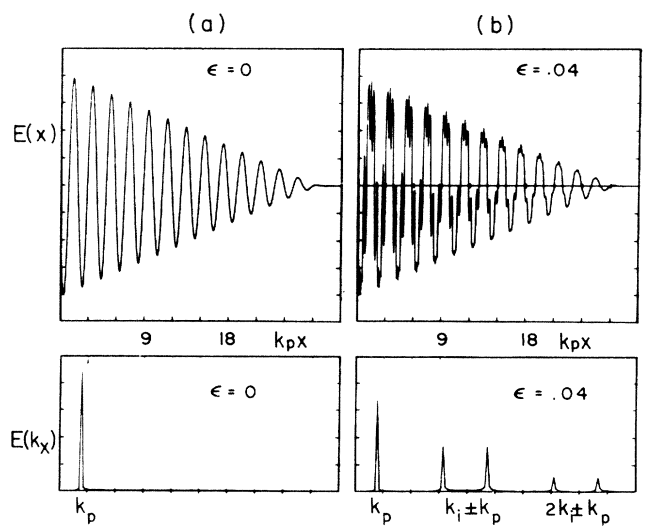

FIG. 1. Spatial evolution of the beat wave with and without a density ripple from numerical solutions of Eq. (1) with $\alpha_1\alpha_2 = 3\times10^{-4}$ and $k_i/k_p = 5$. The spectra are taken at the left-hand side of the box. (a) No ripple: $\epsilon = 0$. (b) With ripple: $\epsilon = 0.04$.

reached the right-hand boundary.[8] For the homogeneous case ($\epsilon = 0$) a coherent beat wave is seen to grow secularly in time (linearly ramped in x) and the Fourier spectrum shows a single mode at $k = k_p$ as expected. In the rippled plasma case ($\epsilon = 0.04$), higher-k components are excited as predicted by Eq. (2) and, although the peak amplitude of the wave form still exhibits secular growth with the same growth rate, the relative size of the beat-excited plasma wave (Ψ_0 component) is reduced. In Fig. 2(a) we have plotted the time histories of the various k components obtained from the exact solution of Eq. (1). The beat-wave component (Ψ_0) is observed to saturate as the higher-order modes begin to grow at the expense of the beat wave. As the ripple amplitude is increased [Fig. 2(b)] the beat wave is seen to saturate at lower amplitudes and at earlier times.

Analytically we find that the beat-wave component grows as

$$E(\Psi_0(x,t)) = (\alpha_1\alpha_2/2\epsilon)\int_0^t J_0(\epsilon\omega_p t'/2)\,d(\epsilon\omega_p t'/2), \qquad (3)$$

with saturation amplitude given by

$$E_{\text{sat}}(\Psi_0,\ \text{cold}) = 0.735(\alpha_1\alpha_2/\epsilon) \qquad (4)$$

at a time $T_{\text{sat}}(\Psi_0, \text{cold}) = 4.8(\epsilon\omega_p)^{-1}$. Similarly, the first coupled modes ($\Psi_{\pm 1}$) are found to grow as

$$E(\Psi_{\pm 1}(x,t)) = (\pm\alpha_1\alpha_2/2\epsilon)[J_0(\epsilon\omega_p t/2) - 1], \qquad (5)$$

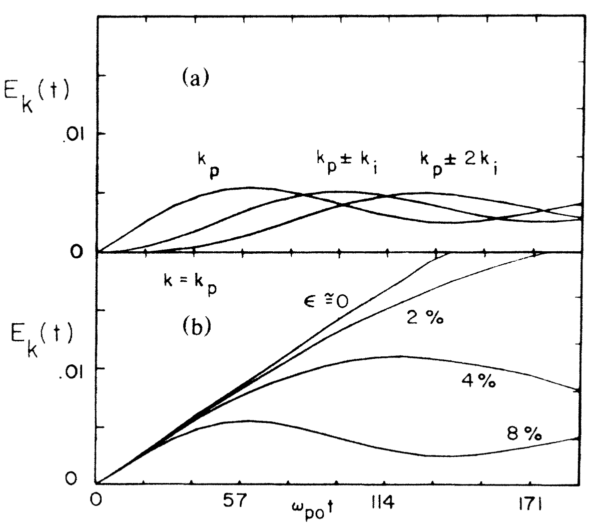

FIG. 2. Temporal evolution of the beat wave and coupled modes with $\alpha_1\alpha_2 = 3\times 10^{-4}$. (a) Evolution of beat wave and first two coupled modes (Ψ_n, $n = 0, \pm 1, \pm 2$) for $\epsilon = 0.08$. (b) Temporal evolution of beat wave for several ripple amplitudes ϵ.

with saturation amplitude given by

$$E_{\text{sat}}(\Psi_{\pm 1},\ \text{cold}) = 0.701(\alpha_1\alpha_2/\epsilon) \qquad (6)$$

at a time $T_{\text{sat}}(\Psi_{\pm 1}, \text{cold}) = 7.7(\epsilon\omega_p)^{-1}$. The results presented in Eqs. (3) and (5) remain valid for times $t \ll 2\pi(\epsilon^2\omega_p)^{-1}$ and thus accurately describe the exact numerical solutions well into saturation (Fig. 2) even for moderately large ϵ. In addition, the short time scales on which the coupling and saturation occur justify our original assumption of fixed ions for any reasonable ϵ.

To treat analytically mode-coupling saturation of beat waves in warm plasmas we assume only one mode coupling (an assumption to be justified below) and truncate the series in Eq. (2) at $n = \pm 1$. Substitution of this form of a solution into Eq. (1) yields equations for a_0 and $a_{\pm 1}$ and hence the saturation amplitudes. In particular we obtain

$$E_{\text{sat}}(\Psi_0,\ \text{warm}) = \alpha_1\alpha_2 f(p)/\epsilon, \qquad (7)$$

$$E_{\text{sat}}(\Psi_{\pm 1},\ \text{warm}) = \alpha_1\alpha_2/\epsilon, \qquad (8)$$

where $f(p) = p + (1+p^2)(2+p^2)^{-1/2}$ and p is a thermal parameter given by $p = (3/\epsilon)(k_i\lambda_d)^2$. Note that

$$\lim_{p\to 0} E_{\text{sat}}(\Psi_0) = 0.707(\alpha_1\alpha_2/\epsilon),$$

in excellent agreement with the cold-plasma estimate of Eq. (4). This is expected since, at the time of saturation of the beat wave, the higher-order modes contribute little to the total electric field as shown in Fig. 2(b). This justifies the truncation used to obtain Eq. (7), since $p = 0$ provides a worst-case check of this approximation. Neglecting pump rise times we find that saturation by mode coupling dominates over that due to relativistic detuning whenever $\alpha_1\alpha_2 < [1.6\epsilon/f(p)]^{3/2}$. For example, in the case of a modest ripple size of 4%, if we assume that $p = 2$ and $\alpha_1 = \alpha_2$ it is seen that saturation by mode coupling dominates unless $\alpha > 0.04$ or $I(CO_2) > 4\times 10^{13}$ W/cm^2. Although this requirement may be altered somewhat when pump rise times are considered it is apparent that the mode-coupling mechanism can be quite important in moderate-intensity or shorter-wavelength laser experiments.

Experiments have been performed specifically to detect and characterize the frequency and wave-number spectra of the coupled modes predicted by the above model. A CO_2 laser pulse (2-ns FWHM and 1-ns rise time) containing both the 9.56- and 10.59-μm lines with energies 4 and 8 J, respectively, is used. An $f/7.5$ ZnSe lens focuses this laser pulse onto a preformed hydrogen plasma ($n_e \cong 10^{17}$ cm^{-3}, $L \cong 2$ mm, $T_e \cong 30 \pm 10$ eV) whose plasma frequency is tuned to match the frequency separation of the two laser lines.[2] This optical configuration gives a focal spot size of

$\cong 250$ μm and a focal intensity of $\cong 10^{13}$ W/cm^2 ($\alpha_1\alpha_2 \cong 3\times 10^{-4}$). The frequency and wave-number spectra of the excited electrostatic modes are mapped out by use of ruby-laser Thomson scattering at various discrete scattering angles such that $|\mathbf{k}_{\rm scatter}-\mathbf{k}_{\rm ruby}| = |\Delta\mathbf{k}_{\rm ruby}| = |\pm n\mathbf{k}_i|$.

Figure 3(a) shows an example of a Thomson scatter frequency spectrum containing spectral lines which are shifted to the blue ($+3\mathbf{k}_i$) and shifted to the red ($-3\mathbf{k}_i$) by ω_p and $2\omega_p$, where ω_p corresponds exactly to $\Delta\omega_{\rm lasers}$. These shifts are measured relative to the central peak, which is due to scatter from the third harmonic of the ion acoustic density ripple ($3\omega_{\rm ac}\cong 0, 3k_i$). The modes with frequency $2\omega_p$ are attributed to mode coupling of the second harmonic of the beat wave.[7] Figure 3(b) summarizes schematically the modes which have thus far been observed. We have previously reported detailed Thomson-scattering measurements[2,9] of the high-phase-velocity beat-excited plasma wave at a scatter angle of 7 mrad ($\Delta\mathbf{k}_{\rm ruby}=\mathbf{k}_p$). Other modes observed include the ion acoustic density ripple (measured at $\epsilon\cong 4\%$) and its harmonics as well as a number of forward ($+|n|\mathbf{k}_i$) and backward ($-|n|\mathbf{k}_i$) propagating modes with frequencies ω_p and $2\omega_p$. The k spectrum of the beat wave, as well as that of the low-frequency modes at $n\mathbf{k}_i$, was determined to be discrete. In the discussions which follow we focus our attention on those modes which exhibit the signature of beat-wave mode coupling; that is, modes which are fundamental in ω_p and harmonic in k_i ($\mathbf{k}_p \pm n\mathbf{k}_i \cong n\mathbf{k}_i$ since $|\mathbf{k}_p| << |\mathbf{k}_i|$).

An obvious requirement for the excitation of coupled modes is the coexistence of the density ripple and the coupled modes. This condition has been verified by simultaneous time resolution of both the $\omega\cong\omega_{\rm ac}$ and $\omega\cong\omega_p$ features of the Thomson scatter spectrum at a scatter angle which is sensitive to modes with $k=k_i(\Psi_{\pm 1})$. The results of this measurement are presented in Fig. 3(c) which clearly shows the simultaneity of the SBS-induced density ripple and the first coupled mode.[10]

We have estimated the time-integrated amplitudes of the beat wave and coupled modes using first the Bragg scattering formula[2] to estimate $\tilde{n}/n_0$ and then Poisson's equation to estimate $E(\Psi_n)$. In Table I we compare the predicted and observed electrostatic field amplitudes. The theoretically predicted amplitudes follow from the saturation values calculated above for cold and warm plasmas [Eqs. (4), (6), (7), and (8)] where, for warm plasmas, we have taken $p=2$ and $\alpha_1\alpha_2 = 3\times 10^{-4}$ as in the experiment. For reference we also compare the predicted beat-wave saturation amplitude for relativistic detuning. The measured amplitude of the beat wave is $\cong(3\text{–}9)\%$.[2] Theoretically we expect a saturation amplitude of $\cong 8\%$ for relativistic detuning (including pump rise time[11]) and $\cong 3\%$ for mode coupling. Thus the measured beat-wave amplitude is consistent with both the relativistic detuning and mode-coupling saturation models. However, observations of the coupled modes and the fair agreement between their theoretically predicted and experimentally measured amplitudes is strongly suggestive of the importance of the contribution of the mode-coupling saturation mechanism in our experiment.

Another possible contribution to the observed electrostatic modes is the mode coupling of plasma waves produced by counterpropagating optical mixing (CPOM).[7] CPOM can occur when one incident laser line mixes with the SBS from the other laser line. This situation can be modeled by replacement of the driver in Eq. (1) by the CPOM equivalent. In this case the directly excited mode is the $\Psi_{\pm 1}$ plasma wave, and the first coupled mode is a beat wave (Ψ_0). The saturation field amplitudes scale as $\alpha_j'\alpha_k k_i/2k_p$ where α_j' is

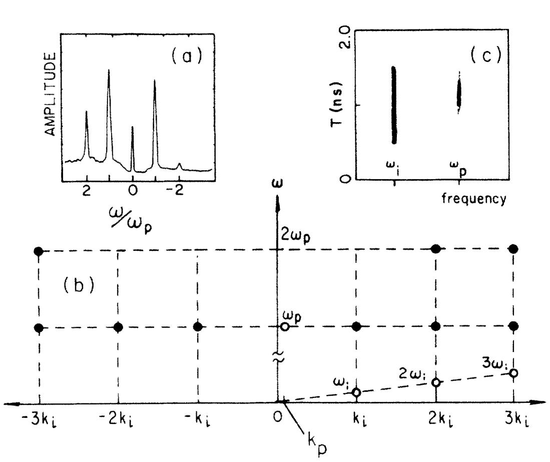

FIG. 3. (a) Frequency spectrum of Thomson-scattered ruby-probe beam light at $\Delta\mathbf{k}_{\rm ruby} = \pm 3\mathbf{k}_i$. (b) Summary of electrostatic modes thus far observed. These modes exhibiting the signature of coupled modes are indicated by solid dots. (c) Time-resolved Thomson scatter frequency spectrum of the SBS density ripple (left-hand side) and the first coupled mode (right-hand side).

TABLE I. Summary of theoretically predicted and experimentally estimated electrostatic field amplitudes for the beat wave (Ψ_0) and first coupled mode (Ψ_{+1}) for $T_e = 30$ eV and $\epsilon = 0.04$.

	$E(\Psi_0)$ (%)	$E(\Psi_1)$ (%)
Experiment	3–9	0.09–0.9
Relativistic detuning	8	. . .
Cold-plasma mode coupling	0.6	0.5
Warm-plasma mode coupling	3	0.8

the v_{osc}/c associated with the Brillouin-scattered pump component. This gives CPOM saturation fields which are about four times as large (measured SBS reflectivity $\simeq 4\%$, $k_l/k_p \cong 20$) as in the collinear optical mixing case. However the experimental measurements, $E(\Psi_{+1}) \cong (0.09\text{–}0.9)\%$, are below this prediction. This is perhaps due to incoherence of the Brillouin backscattering "pump,"[12] thermal effects, or overestimation of CPOM driver amplitude due to the relatively large size of the SBS scatter volume compared to the beat-wave excitation region. Since theoretically we expect $E(\Psi_{+1}) \cong 4E(\Psi_0)$, the CPOM contribution to the beat wave is believed to be negligible.

Beat-wave saturation by mode coupling can be avoided by elimination of the ripple (short laser pulses or $T_e \cong T_i$ plasmas) or by thermal quenching of the coupling ($p >> 1$). In cold-plasma, long-pulse experiments, the mode-coupling saturation mechanism can undergo self-stabilization. As the coupled modes grow, plasma heating can occur through wave-particle interactions, thereby quenching the coupling. Ultimately relativistic detuning will dominate when the plasma is heated to the point where $p \cong 0.9 \times \epsilon/(\alpha_1\alpha_2)^{2/3}$. This self-stabilization mechanism has been observed in particle simulations.[7] For early times the beat wave grows much as seen in the numerical solutions shown in Fig. 1(b), and the k spectrum clearly shows the excitation of coupled modes. However, at later times we observe abrupt quenching of the coupling concomitant with the onset of particle heating observed in electron phase space.

Mode coupling can also be important in laser fusion experiments where ion- and plasma-wave instabilities are simultaneously excited. The coupling between the ion and plasma waves can lead to quenching of high-frequency instabilities,[13] which are a source of pellet preheat as a result of the generation of hot electrons.

Saturation by mode coupling is not expected to affect the excited-wave amplitude in nonresonant schemes such as the plasma wake-field accelerator.[2] However, the high–phase-velocity plasma-wave energy can rapidly couple out ($T \cong 2/\epsilon\omega_p$) to the higher-$k$ modes,[4,5] thus limiting the number of wavelengths available for acceleration.

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[6]The Lagrangean equivalent of Eq. (1) yields an electric field which does not differ appreciably from the Eulerian result when we consider electron excursions consistent with our experimental parameters.

[7]Further details will be presented in a forthcoming publication.

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