Rewriting Codes for Flash Memory

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1. Background
   - Introduction to Flash Memory
   - Modeling the Memory Device
   - Recent Work
   - The Problem
2. Constructions
   - Binary Expansion
   - Non-Binary Concatenation
   - Spider Codes
3. Conclusion and future work

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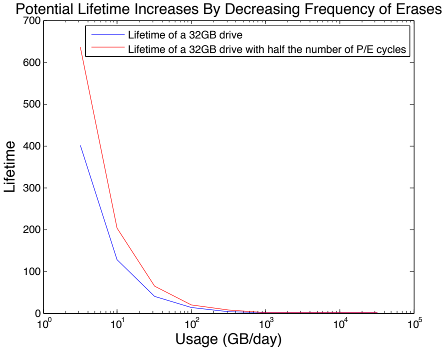
- This information is then compared against some threshold and a discrete value is read.
- Programming is performed at the granularity of an individual cell while erasing is performed on blocks of cells ($\approx 10^6$ cells).
- This need for frequent erases speeds up the aging process drastically [1].

[1] A.R. Olson and D.J. Langlois, "Solid State Drives Data Reliability and Lifetime," *Imation White Paper* April 2008.

Impact of program and erase cycles on lifetime



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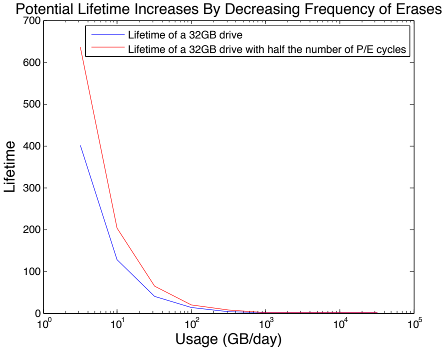

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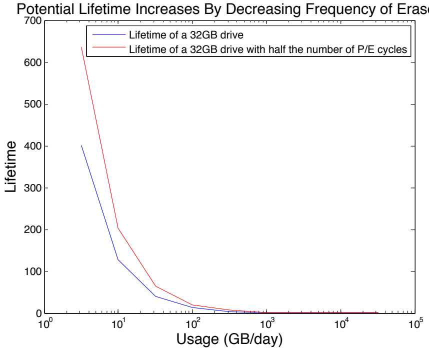

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- Goal is to prolong the lifetime of the memory by reducing the number of erases.

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Model

Memory Model

The memory state is modeled as a vector $\mathbf{y}^j$ of length n where j is the current write (or generation). Each element y_i^j, $1 \leq i \leq n$, takes values in the set $\{0, 1, 2, \ldots, q-1\}$. On write j, the encoder writes one of M_j messages to the memory by updating $\mathbf{y}^{j-1}$ to $\mathbf{y}^j$ while satisfying the WOM-constraint $\mathbf{y}^j \geq \mathbf{y}^{j-1}$. Decoder sees $\mathbf{y}^j$.

Sum-Rate

Definition

If M_j codewords can be represented at generation j, then generation j has rate $\frac{1}{n}\log(M_j)$. Sum rate is the sum of rates across generations.

Rivest and Shamir two write code [1]

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Previous work

- Early capacity results
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- Flash Memory Codes
  - Yaakobi *et al.*, "Codes for Write-Once Memories", 2012.
  - Huang *et al.*, "Error-Correcting Codes for Flash Coding", 2011.
  - Shpilka, "Capacity achieving multiwrite WOM codes", 2012.

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  - Also achieved capacity for non-binary WOM-codes (any number of writes) .
  - Codes have block lengths that grow exponentially as the gap to capacity becomes smaller.

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- The goal is to provide flexible high-rate *non-binary* strategies for *multilevel* WOM.
- The contribution of this work is to provide the best known non-binary WOM-codes with block lengths that need not grow exponentially with the block length of the code.

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- Construct a new non-binary code based on these constituents.

Construction 1

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- For any collection of codewords $\mathbf{c_1} \in C^1_{t,n,q^{'}}, \mathbf{c_2} \in C^2_{t,n,q^{'}}, \ldots, \mathbf{c_k} \in C^k_{t,n,q^{'}}$, a codeword $\mathbf{c} \in C_{t,n,q}$ is formed through the operation $\mathbf{c} = \sum_{j=0}^{k-1} (q^{'})^j \mathbf{c_j}$.

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- Resultant maximum sum-rate is $\log_2(\binom{q^{'}+t-1}{t}) \log_{q^{'}}(q)$.

An example of Construction 1 using Rivest and Shamir write-twice code

Write number	*Data bits*	*Encoding by the base-code $\mathcal{C}_2$*	*Encoded values in the 8-ary cell*
1	*(11,01,10)*	*(100,001,010)*	*(4,1,2)*
2	*(00,11,01)*	*(101,111,110)*	*(5,7,6)*

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- Represent information sequences as the ordered pair $(\mathbf{c_{j,q^{'}}}, \mathbf{v})$ where $\mathbf{c_{j,q^{'}}}$ can be any codeword from $C_{t,n,q^{'}}$ and $\mathbf{v}$ is a vector of length n with elements $\{0, 1, 2, \ldots, \frac{q}{q^{'}+t-1} - 1\}$.

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- Resultant maximum sum-rate $t\log_2(\frac{q}{q^{'}+t-1}) + \log_2(\binom{q^{'}+t-1}{t})$.

A simple example

Construct a 2-write code using 9 levels from a $t = 2$ write code of $q' = 2$ levels.

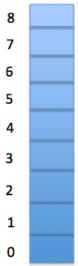

Figure: Cell with $q = 9$ levels.

First write

Encode $(\mathbf{c}_{0,2}, \mathbf{v})$ where $\mathbf{c}_{0,2}$ is first generation codeword from a write-twice binary codebook and $\mathbf{v}$ is a vector with elements $\{0, 1, 2\}$.

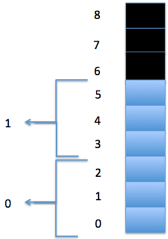

Figure: Cell with $q = 9$ levels after 1st generation.

Second write

Encode $(\mathbf{c}_{1,2}, \mathbf{v})$ where $\mathbf{c}_{1,2}$ is a second generation codeword from a write-twice binary codebook and $\mathbf{v}$ is a vector with elements $\{0, 1, 2\}$.

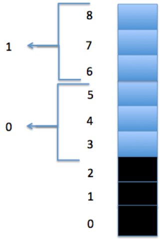

Figure: Cell with $q = 9$ levels after 2nd generation.

Construction 3

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- In the first generation an equal weight sequence is sent.
- In the second generation, the second generation of a collection of binary codebooks will be used to encode more information.

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  - Stage 2: Update the memory further by using a codebook that only increments the elements in the codeword vector with value 0.
  - Stage 3: Update the memory further by using a codebook that only increments the elements in the codeword vector with value 1.

Summary - Rates achieved for 2 writes

q	B.E.	N.B.C.	S.C.	Capacity
4	2.9856	-	3.2970	3.3219
8	4.4784	4.3299	5.0506	5.1699
16	5.9712	6.3299	6.8875	7.0875
32	7.4640	8.3299	8.8059	9.0444
64	8.9568	10.3299	10.733	11.0224
128	10.4496	12.3299	12.7010	13.0112

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- Future Work
  - Enhancement with error correction/detection capabilities.
  - Constructing better codes for the expected case.

Thank You

Thank you for your attention