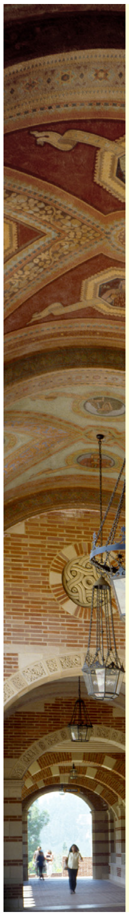

Nonlinear turbo codes: Even Closer to Shannon Limit.

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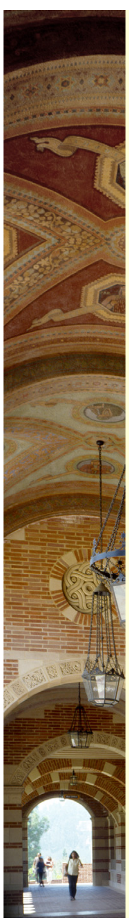
Annual Research Review
28th January 2008



Motivation

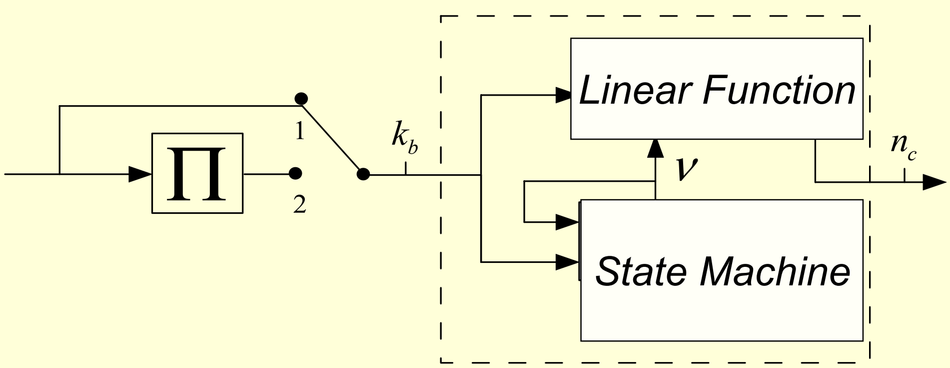
- History:
  - Shannon limit: “Given any channel, pick (at random) ANY error correcting code that produces in the output the the optimal distribution and let the size of the code go to infinity. The probability of making an error tends to zero”.
  - Until the 90s: “All the codes except the ones we design are capacity achieving”.
    - Problem: decoding complexity.
  - Capacity achieving codes:
    - Berrou: turbo codes.
    - Gallager: Low-Density Parity-Check (LDPC).
  - Can we get even closer to Shannon Capacity with same decoding complexity?

Nonlinear turbo codes

- The answer is YES, using nonlinear turbo codes in some applications.

1. Higher-order modulations over AWGN.
2. Applications where a non-uniform distribution in the transmitted signal is required.

Traditional Linear Turbo Codes



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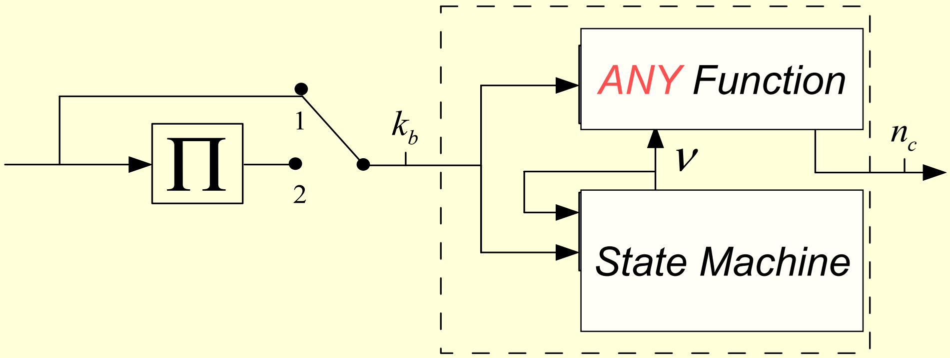
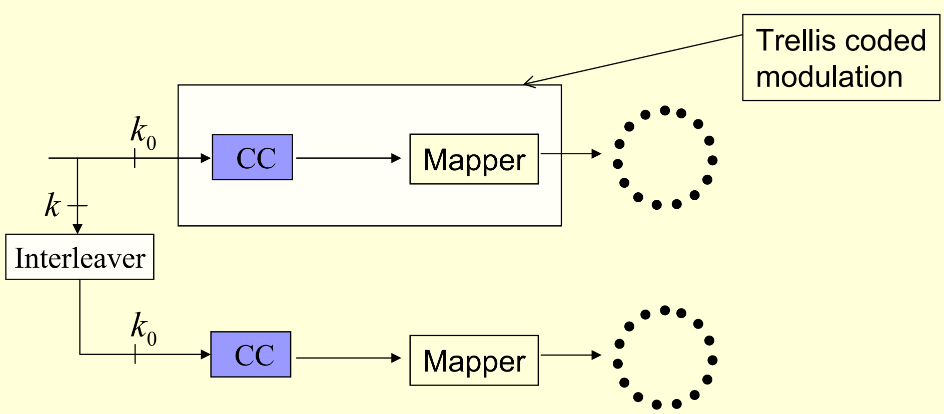
Nonlinear Turbo Codes



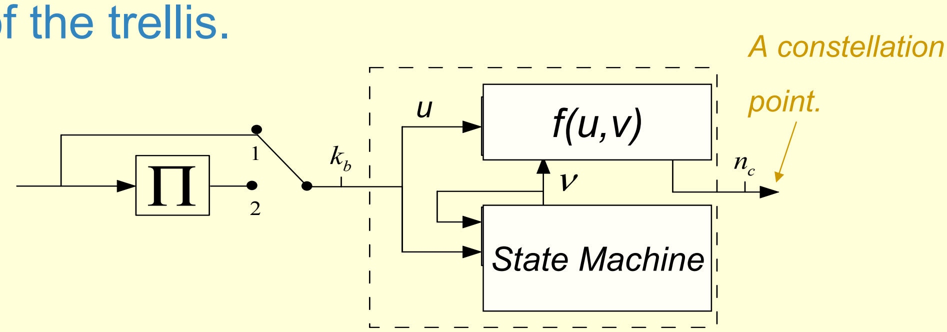
1) Higher-order modulations

- Higher-order modulations:



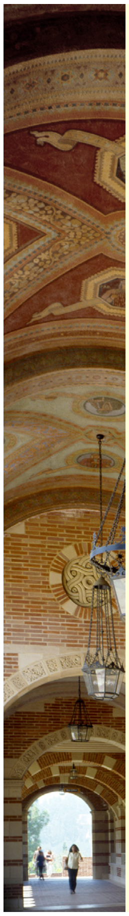
Higher-order modulations

- The serial concatenation of a convolutional code with a mapping could be too restrictive.
- We can use any function that assigns constellation points to each of the branches of the trellis.



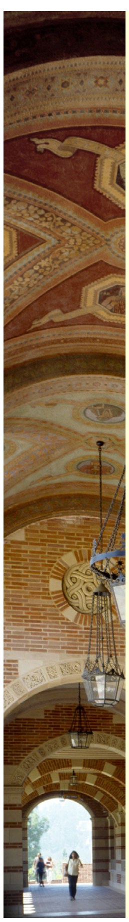
Analytical Bit Error Rate bound

- We provide a method to predict the BER of parallel concatenated non-linear codes over asymmetric channels, in particular the Z-Channel, under Maximum Likelihood decoding.
- We extend the uniform interleaver analysis proposed in [Benedetto '96].
- Uniform interleaver: given the two constituent codes, average over all possible interleavers of a certain length *K.*
- Key difference: **nonlinearity** of the constituent codes. We cannot assume that the all-zero codeword is transmitted. We need to average over all possible codewords.



Analytical Bit Error Rate bound (2)

- A new probabilistic device:
- Uniform interleaver for linear codes (Benedetto): “A uniform interleaver of length *k* is a probabilistic device that maps a given input word of Hamming weight *w* into all distinct permutations of it with equal probability”.
  - This definition is useful only when assuming the all-zero codeword is transmitted.
- New definition for ANY code: “A uniform interleaver of length *k* is a probabilistic device the chooses between all possible position permutations with equal probability. Then, for each position, the value of the symbol can be changed to any other value with equal probability”.
  - This new definition includes the linear case and leads to similar conclusions for both linear and nonlinear constituent codes.



Trellis structure and effective free distance

- The new uniform interleaver analysis allows to show that a recursive encoder is required for nonlinear codes as well.
- Benedetto showed that an important metric to maximize in the constituent linear codes is the effective free distance. We extend this notion for nonlinear codes:
- **Effective free distance:** minimum distance in the output produced by any two possible input words with Hamming distance 2.
- This definition includes the linear case.

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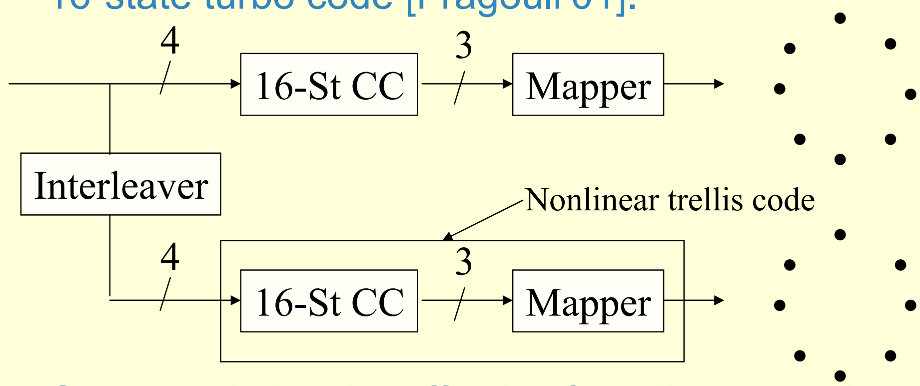
Example: 2 bits/s/Hz 8PSK

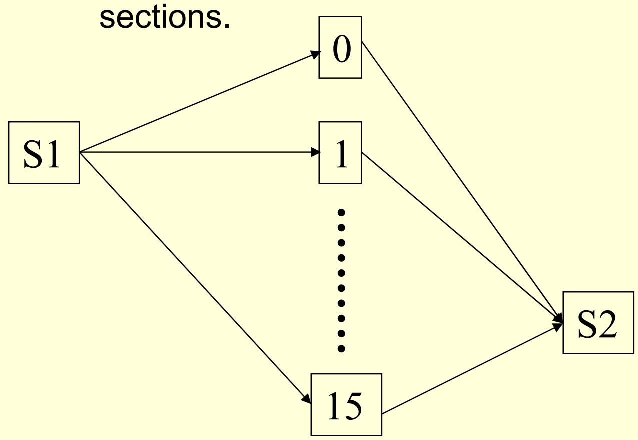
- Constrained capacity: 2.8 dB.
- We compare against the best previously published 16-state turbo code [Fragouli'01].

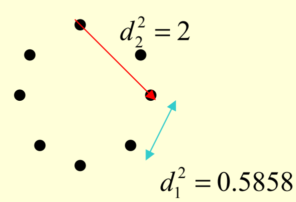


- Goal: maximize the effective free distance, where the distance in this case is the squared Euclidean distance.

Example: 2 bits/s/Hz 8PSK

- Achieving greater effective free distance than linear codes:
  - This is a fully connected trellis: given any two states S1 and S2, there are 16 trellis paths from S1 to S2 after two trellis sections.

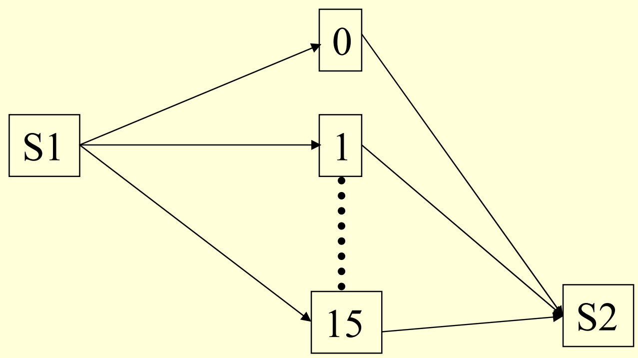


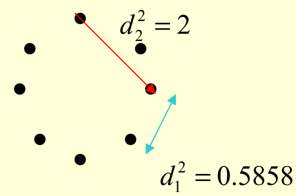


  - Using a linear convolutional code and a mapper, all the searches we and previous works have done, have at least on pair of those paths that have a distance of $2d_1^2 = 1.171573$

Example: 2 bits/s/Hz 8PSK

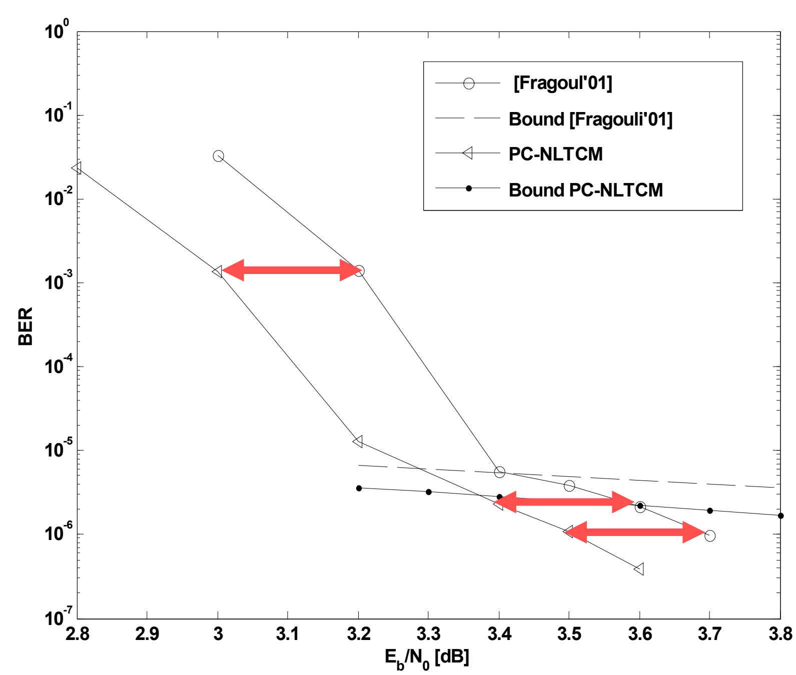
- Achieving greater effective free distance than linear codes:





  - We found a nonlinear labeling that guarantees that for each pair of those paths, there is at least one trellis sections where the constellation points have squared Euclidean distance 2.
  - There could still be a path in more than 2 trellis sections with distance less than 2. By a search in the trellis structure we avoid that.
  - The resulting nonlinear constituent code has an effective free distance equal to 2.

0.2 dB gain for rate-2/3, 16-states over the best published 10,000 bit code.

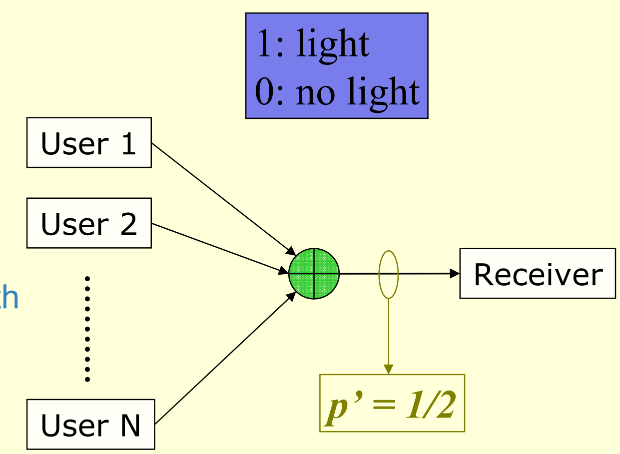


Observations

- Interleaver design plays an important role in the code's performance as shown in [Fragouli'01]. The uniform interleaver analysis averages over all possible interleaver, which explains the difference in the error flour between simulations and the bound.
- However, the BER bounding analysis helps in the design process.
- The gain in performance comes almost at no extra cost.

2) The OR Multiple Access Channel (OR-MAC)

- Simple model for multiple-user optical channel with non-coherent combining.
- 0+X=X, 1+X=1
- N users, all transmitting with the same ones density p: P(*X*=1)=*p*, P(*X*=0)=*1-p*.

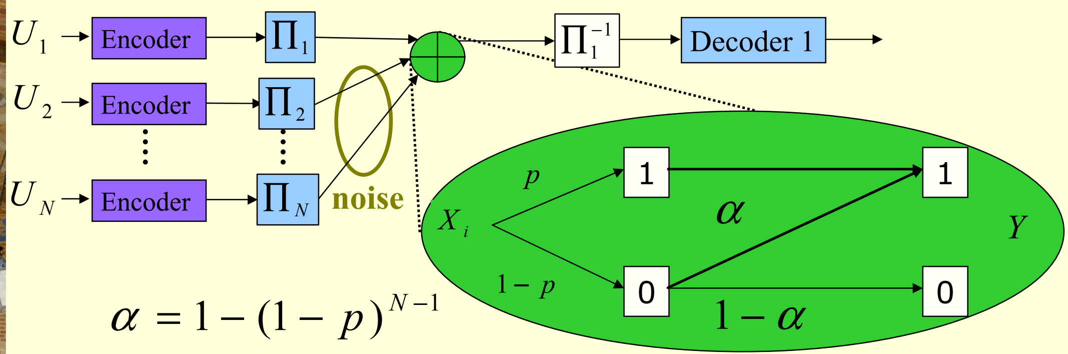


- Theoretically: Sum-rate = 1 (100% efficiency) can be achieved with a ones density in the transmission of

$$p(N) = 1 - (1/2)^{1/N} \approx \frac{\ln(2)}{N}$$

Single-user decoding: Z-Channel

- A practical alternative is to treat all but a desired user as noise.
- When treating other users as noise in an OR-MAC, each user "sees" a Z-Channel.

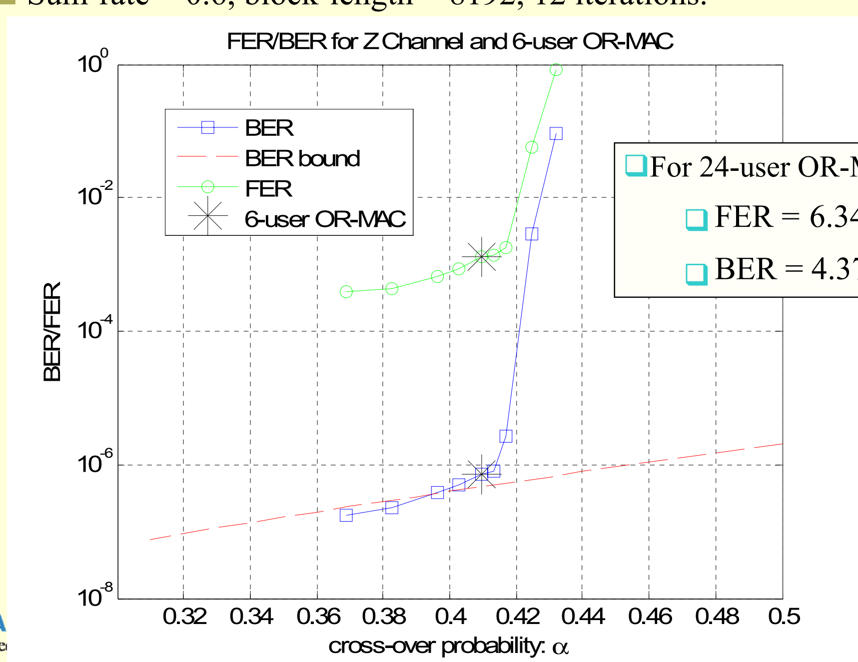


$$\alpha = 1 - (1-p)^{N-1}$$

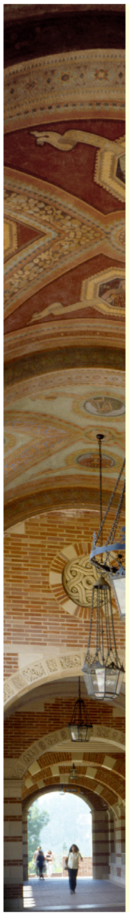
- The achievable sum-rate is lower bounded by ln(2) (around 70%), for any number of users.

Results for 6-user OR-MAC

- Parallel concatenation of 8-state NLTCs.
- Sum-rate = 0.6, block-length = 8192, 12 iterations.



- For 24-user OR-MAC:
  - FER = 6.34×10^{-4}
  - BER = 4.37×10^{-7}



Conclusions

- We have found the nonlinear turbo codes can outperform linear codes in certain applications, getting even closer to Shannon capacity with the same complexity:
  - Higher-order modulations
  - Applications requiring non-uniform distributions in the transmission.
- We have proved that the analytical tools developed for linear turbo codes, and the design criteria, can be extended to nonlinear turbo codes.
- Future work:
  - The state-machine is still binary and linear. There could be some gain using non-binary state-representations, with extra decoding complexity.
  - More examples.