NETWORKED FAULT-TOLERANT CONTROL AND DISTRIBUTED WATER SYSTEMS

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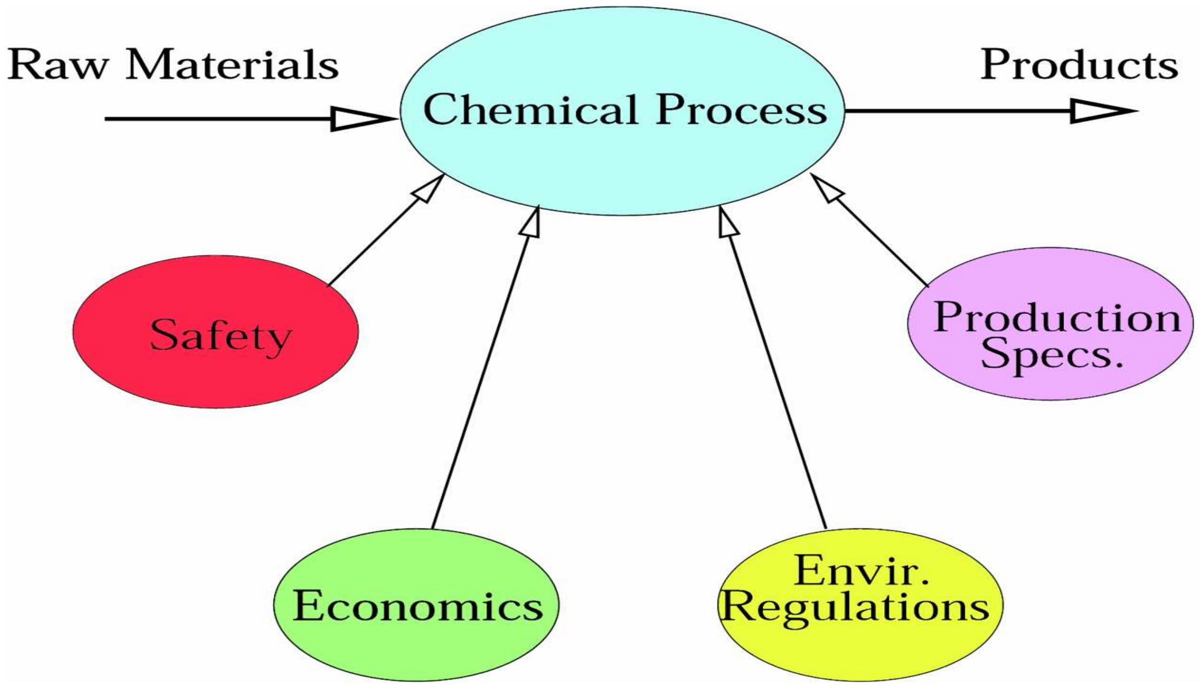
PROCESS CONTROL: INCENTIVES & OBJECTIVES

- Incentives for chemical process control



◇ Need for continuous monitoring and external intervention (control)

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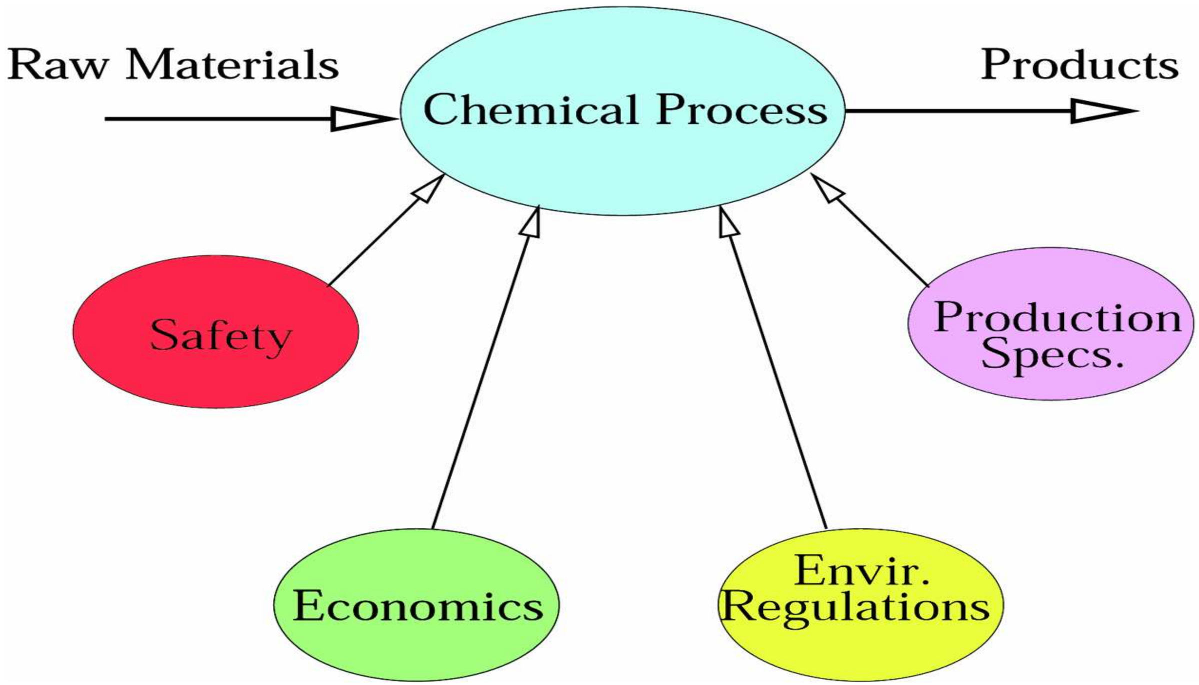
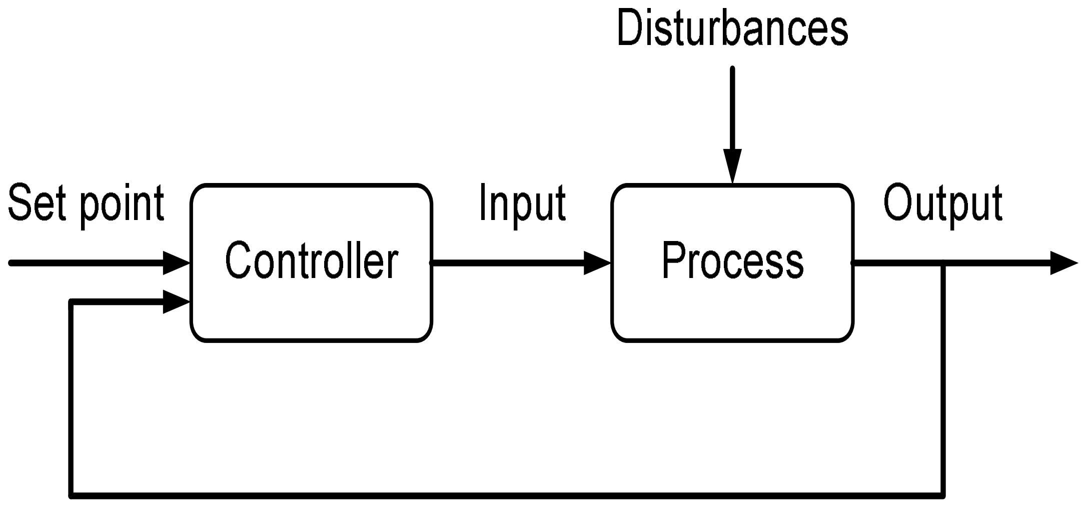


  - ◇ Need for continuous monitoring and external intervention (control)

- Objectives of a process control system
  - ◇ Ensuring stability of the process
  - ◇ Suppressing the influence of external disturbances
  - ◇ Optimizing process performance

MODEL-BASED APPROACH TO CONTROLLER DESIGN

- Selection of inputs/outputs - Feedback control loop



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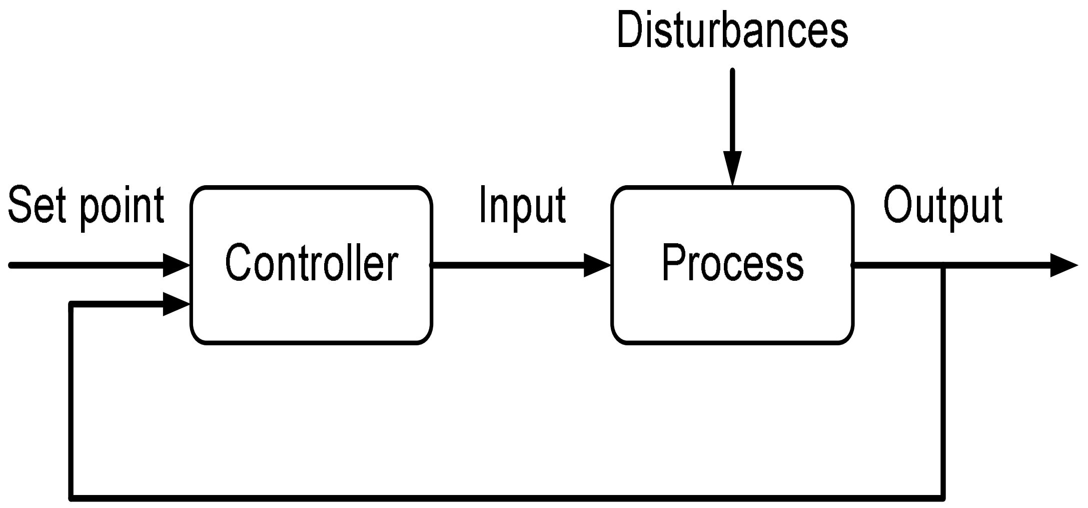
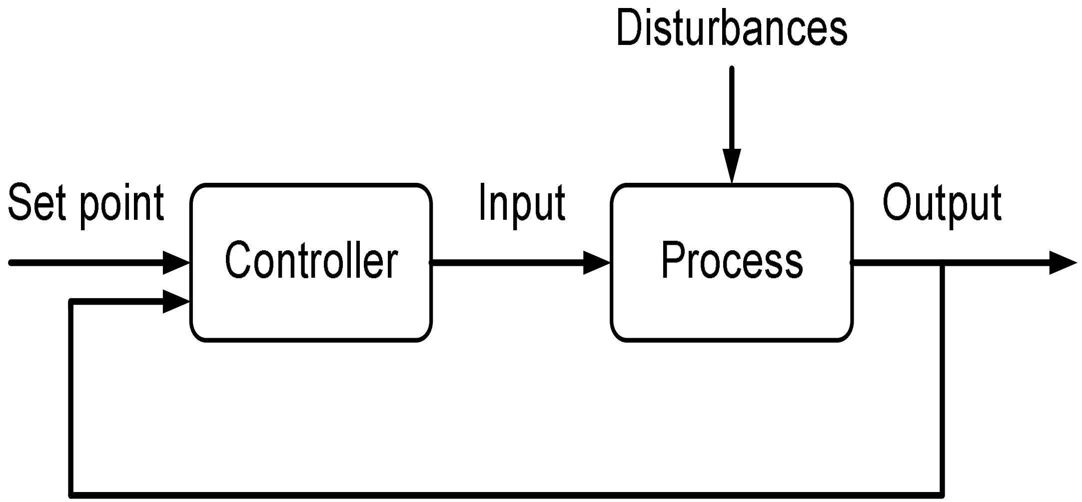


Controller synthesis based on a process model

MODEL-BASED APPROACH TO CONTROLLER DESIGN

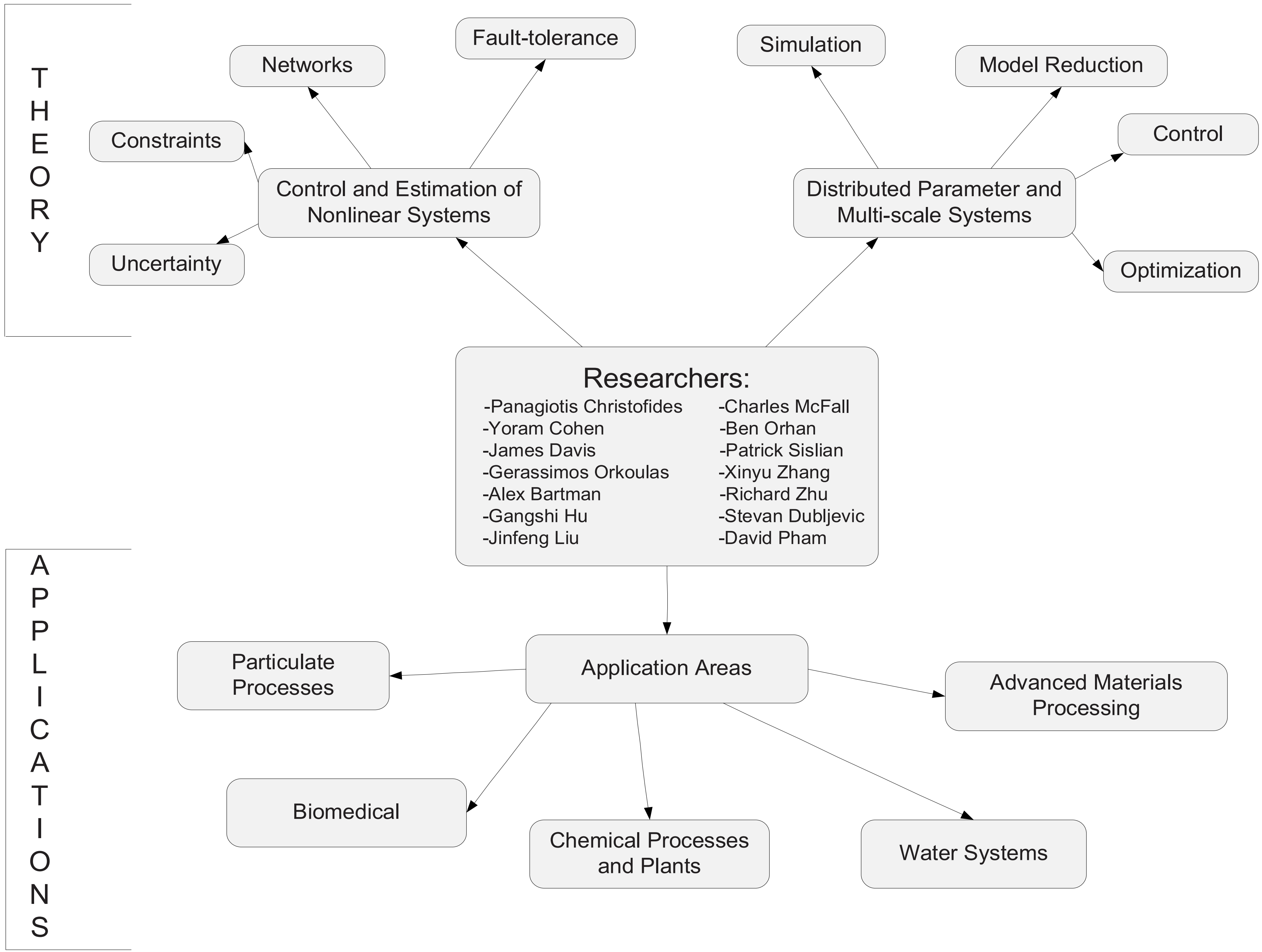
- Selection of inputs/outputs - Feedback control loop



Controller synthesis based on a process model

- Model construction: First-principles / System identification
  - ◇ Possibility of improved closed-loop performance
    - ▷ Model accounts for inherent process characteristics (e.g., nonlinearity, spatial variations)
  - ◇ Characterization of limitations on achievable closed-loop stability, performance and robustness

CONTROL SYSTEMS RESEARCH IN OUR GROUP



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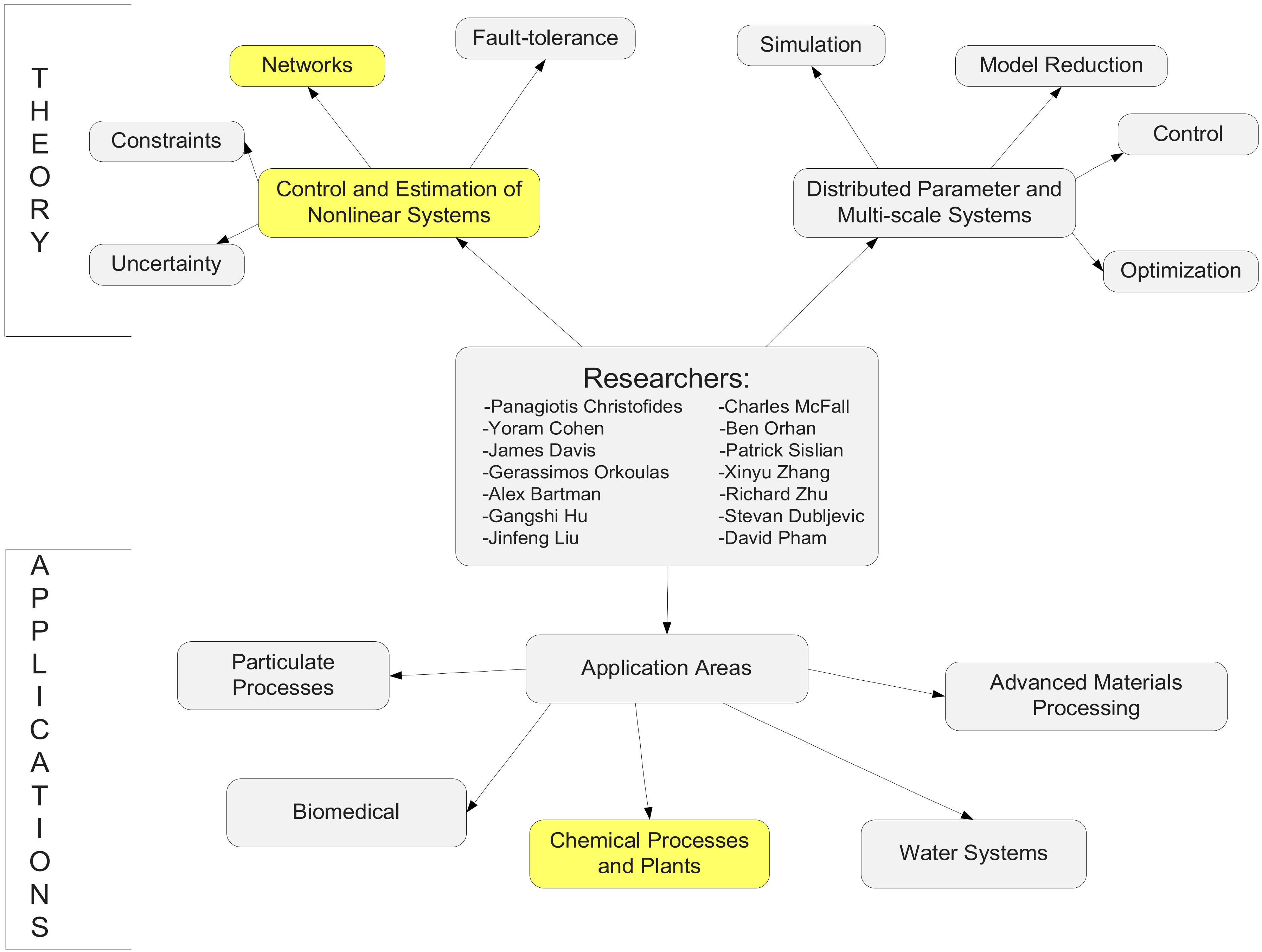
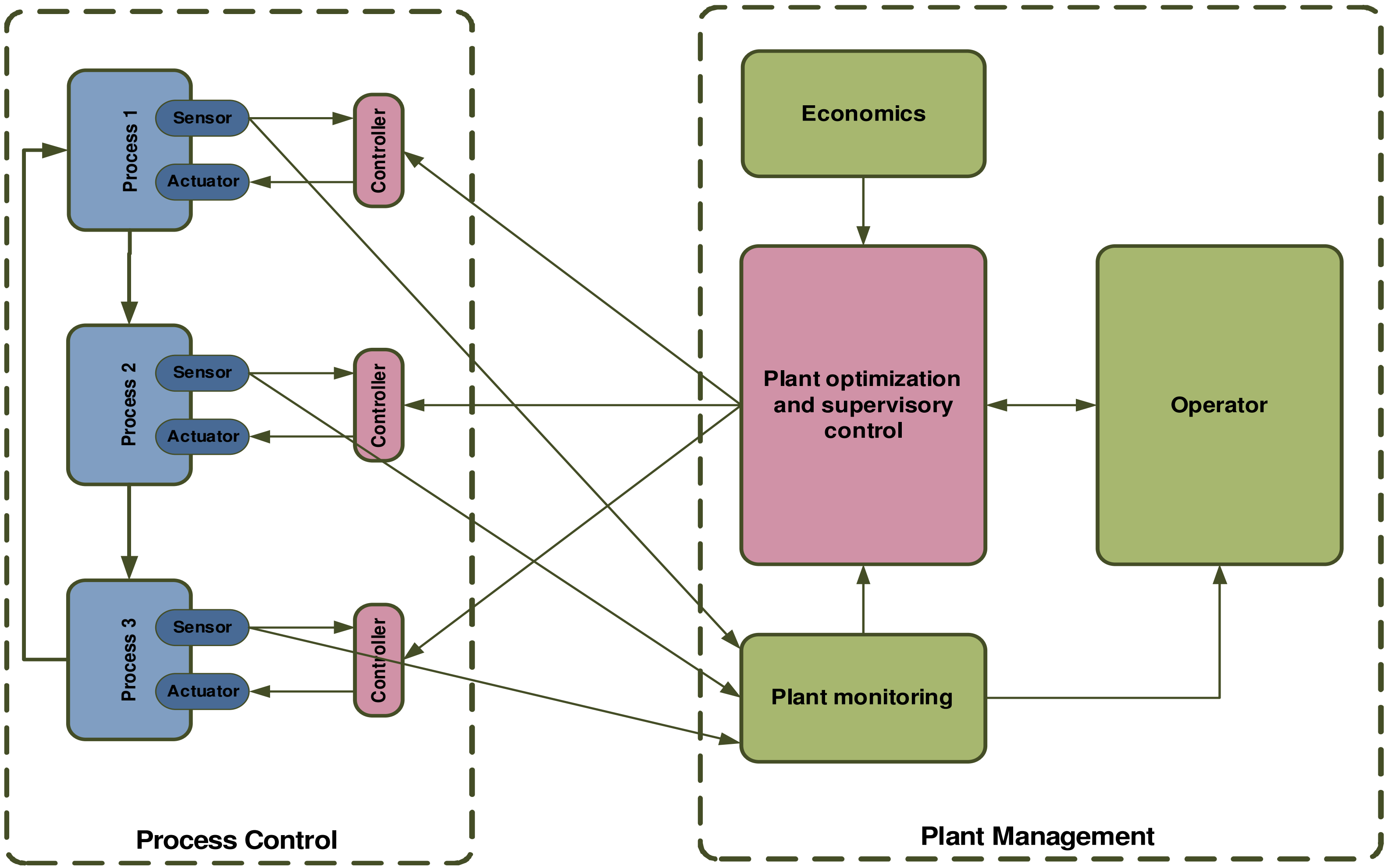
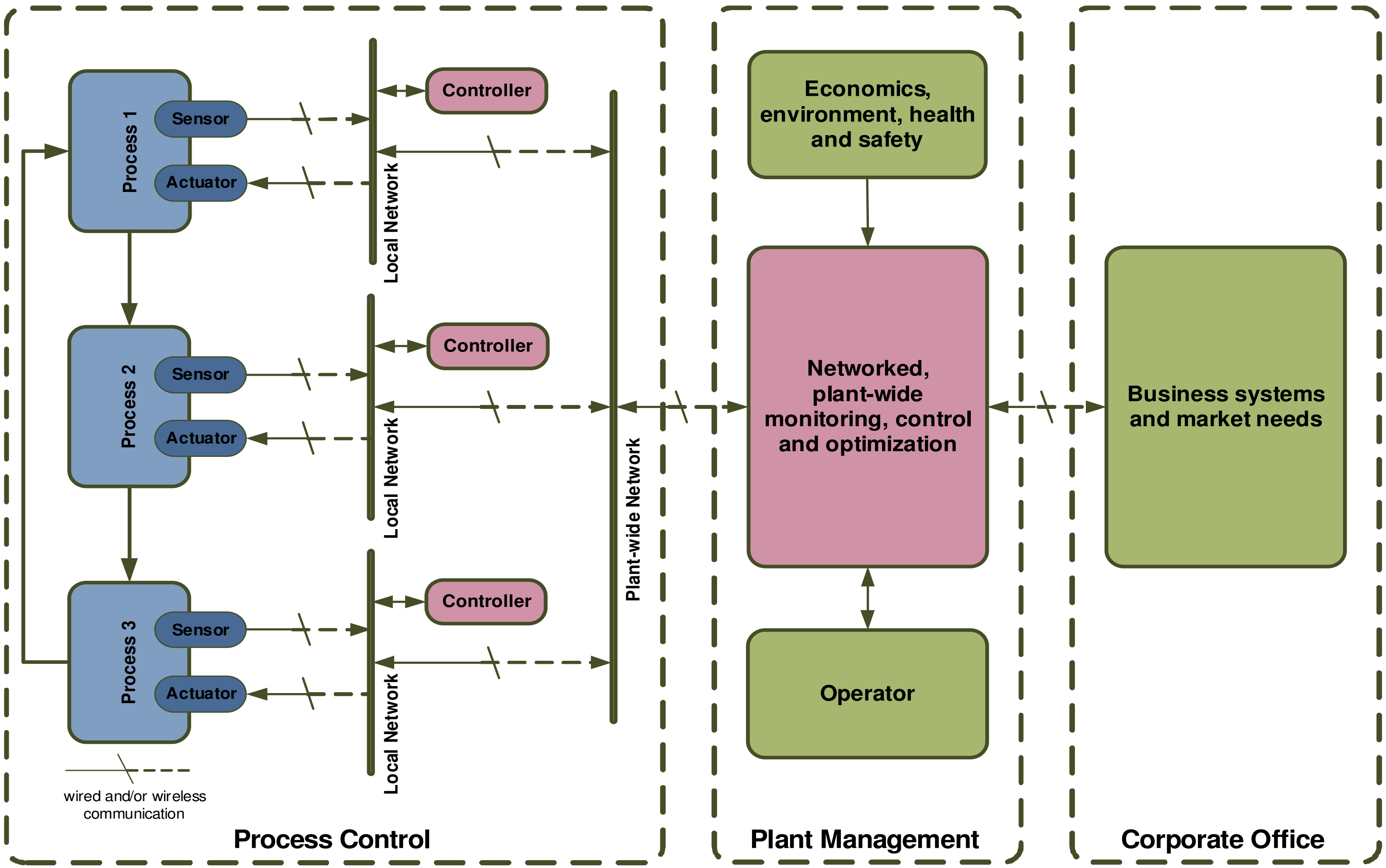


TRADITIONAL PLANT OPERATIONS PARADIGM



"SMART PLANT" OPERATIONS PARADIGM

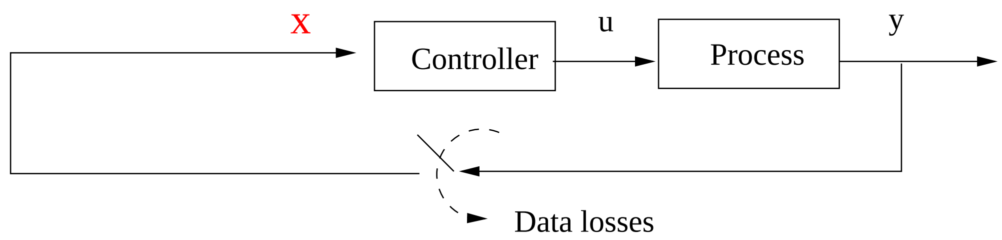
(Christofides, Davis and co-workers, AIChE J., 2007)



- Networked process control, monitoring and operations.

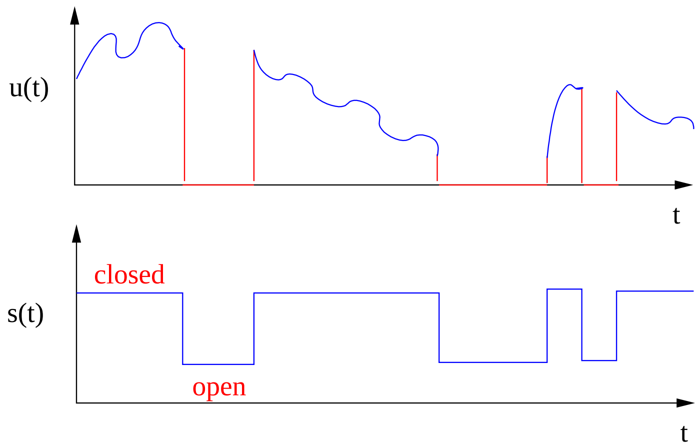
NETWORKED PREDICTIVE CONTROL: INTRODUCTION

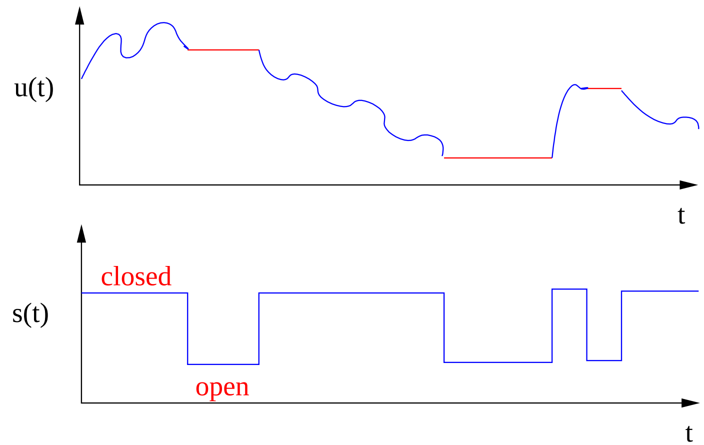
- Data losses due to interference and asynchronous measurement sampling



  - Feedback is lost (sensor data is not available or controller cannot communicate with actuator)
  - Actuator must take decision without new information

- Potential decisions that the control actuator can take
  - Zero input - Last available input approaches





- Input variation based on a pre-determined trajectory - model predictive control

NETWORKED PREDICTIVE CONTROL: SCOPE & MAIN RESULT

- We consider nonlinear systems with uncertainty and data losses and propose a controller design that takes into account data losses (D. Munoz dela Pena and P. D. Christofides, SCL 2008a and 2008b; IEEE TAC, 2008)
- Lyapunov-based model predictive control (LPMC) (P. Mhaskar, N. H. El-Farra, and P. D. Christofides, IEEE TAC, 2005; SCL 2006)
- When data is lost the actuator implements in open-loop the input sequence computed by an appropriately designed LMPC
- Properties
  - Guarantees practical stability in the absence of data losses
  - Guarantees that the stability region is an invariant set for the closed-loop system under data losses if the maximum time in which the loop is open, is shorter than a given constant that depends on the parameters of the system and the Lyapunov-based controller that is used to formulate the optimization problem

PRELIMINARIES

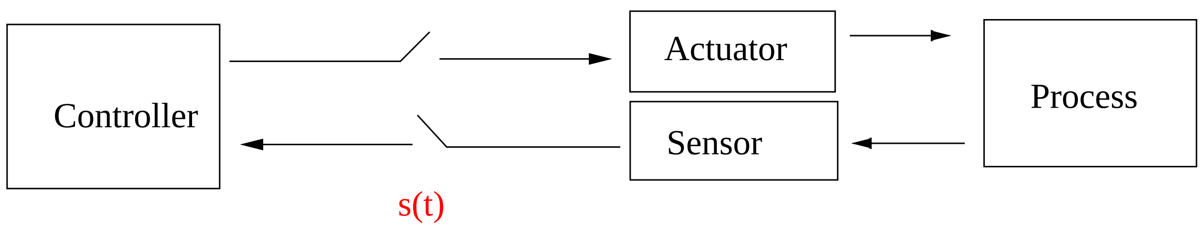
- Nonlinear systems

$$\dot{x}(t) = f(x(t), u(t), w(t)) \quad (1)$$

  - The state is $x(t) \in R^{n_x}$, the input is $u(t) \in R^{n_u}$ and the disturbance is $w(t) \in W$ with $W := \{w \in R^{n_w} \text{ s.t. } |w| \leq \theta, \theta > 0\}$

- Modelling data losses
  - Discrete random signal $s(t_k)$ (recall MPC is discrete time controller) determined by a random variable P with a uniform probability distribution.
  - When $s(t_k) = 1$, at sampling time t_k the full state is available and the actuator receives the control input. When $s(t_k) = 0$, either the state is not available, or the control input is lost.
  - We consider systems where there is a limit on the maximum number of consecutive sampling times in which data is lost.

$$N_o \geq \max j - i \text{ s.t. } s(t_i) = s(t_j) = 1,\ s(t_k) = 0,\ k \in (i, j).$$



PRELIMINARIES

- Model Predictive Control (MPC)

$$\begin{aligned}
\min_{u \in S(\Delta)} & \int_{t_k}^{t_{k+N}} [\tilde{x}(\tau)^T Q_c \tilde{x}(\tau) + u(\tau)^T R_c u(\tau)] d\tau \\
\text{s.t.} \quad & \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0) \\
& \tilde{x}(t_k) = x(t_k) \\
& \text{Set of constraints on the state and the input}
\end{aligned}$$

  - $S(\Delta)$ is the family of piece-wise constant functions with sampling period Δ
  - Nominal model ($w(t)$ is set to zero)

- Stability of MPC controllers
  - Recursive feasibility
  - Existence of a Lyapunov function for the closed-loop system (in general the cost function)
  - Use of appropriate constraints and cost function

PRELIMINARIES

- Lyapunov-based Model Predictive Control (LMPC) (P. Mhaskar, N. H. El-Farra, and P. D. Christofides, IEEE TAC, 2005; SCL 2006)
  - Based on feedback $h(x)$ and Lyapunov function $V(x)$
  - Asymptotically stable closed-loop system

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|), \ \frac{\partial V}{\partial x} f(x, h(x), 0) \leq -\alpha_3(|x|), \ |\frac{\partial V}{\partial x}| \leq \alpha_4(|x|) \qquad (2)$$

  - Nominal sampled trajectory $\hat{x}(t)$

$$\dot{\hat{x}}(t) = f(\hat{x}(t), h(\hat{x}(t_k)), 0), \ t \in [t_k, t_{k+1}], \ \hat{x}(t_0) = x(t_0)$$

  - Optimization problem

$$\begin{aligned} \min_{u \in S(\Delta)} & \int_{t_k}^{t_{k+N}} [\tilde{x}(\tau)^T Q_c \tilde{x}(\tau) + u(\tau)^T R_c u(\tau)] d\tau \\ \text{s.t.} \quad & \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0) \\ & \tilde{x}(t_k) = x(t_k) \\ & V(\tilde{x}(t)) \leq V(\hat{x}(t)), \ \forall t \in [t_k, t_{k+1}] \end{aligned}$$

PRELIMINARIES

- Contractive constraint

$$V(\tilde{x}(t)) \leq V(\hat{x}(t)), \ \forall t \in [t_k, t_{k+1}]$$

  - Depends on $V(x(t)$ and the nominal sampled trajectory $\hat{x}(t)$ for $h(x)$
  - Must hold only in the first time step
  - Due to the receding horizon scheme and based on the properties of the Lyapunov-based controller $h(x)$, if no data losses are present
    * Practical stability is guaranteed
    * Recursive feasibility is guaranteed

- When data losses occur, the properties are lost
  - Stability is no longer guaranteed
  - The actuator strategy has to be defined
  - The contractive constraint has to be modified in order to take into account data losses

LMPC FOR SYSTEMS WITH DATA LOSSES

- Proposed Lyapunov-based Model Predictive Controller:

$$\begin{aligned} \min_{u \in S(\Delta)} & \int_{t_k}^{t_{k+N}} [\tilde{x}(\tau)^T Q_c \tilde{x}(\tau) + u(\tau)^T R_c u(\tau)] d\tau \\ \text{s.t.} \quad & \dot{\tilde{x}}(t) = f(\tilde{x}(t), u(t), 0) \\ & \tilde{x}(t_k) = x(t_k) \\ & V(\tilde{x}(t)) \leq V(\hat{x}(t)), \; \forall t \in [t_k, t_{k+N}] \end{aligned} \tag{3}$$

- Contractive constraint

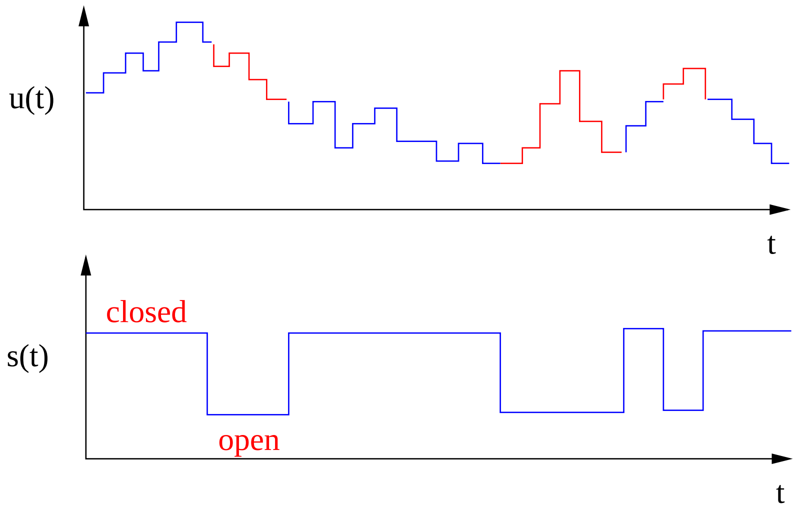
$$V(\tilde{x}(t)) \leq V(\hat{x}(t)), \; \forall t \in [t_k, t_{k+N}]$$

  - Depends on $V(x(t))$ and the nominal sampled trajectory $\hat{x}(t)$ for $h(x)$
  - Must hold along the whole prediction horizon

LMPC FOR SYSTEMS WITH DATA LOSSES

- When data is lost the actuator keeps applying the last optimal input trajectory computed

  - Proposed receding horizon scheme
    - If $s(t_k) = 1$ then solve (3) and obtain $u_k^*(t)$
    - If $s(t_k) = 0$ then $u_k^*(t) = u_{k-1}^*(t)$
    - Apply $u(t) = u_k^*(t), \; t \in [t_k, t_{k+1}]$
    - Obtain new sample



- This control scheme not only modifies the controller, but also the hardware
  - Actuator must be able to store whole input trajectories and take decisions
  - Controller must send whole future input trajectories (whether MPC or other control law)

LMPC FOR SYSTEMS WITH DATA LOSSES

- Theorem 1. No data losses. (P. Mhaskar, N. H. El-Farra, and P. D. Christofides, IEEE TAC, 2005; SCL, 2006)

Consider the trajectory $x(t)$ of system (1) with no data losses in closed-loop with LMPC controller (3). Let $\Delta, \epsilon_s, \epsilon_w > 0$ and $\rho > \rho_w > \rho_s > 0$. Then, if $x(t_0) \in \Omega_\rho$, $x(t)$ is ultimatelly bounded in Ω_{ρ_w}.

- Guarantees practical stability in the absence of data losses

- Theorem 2. System subject to data losses. (D. Munoz dela Pena and P. D. Christofides, SCL 2008a and 2008b; IEEE TAC, 2008)

Consider the trajectory $x(t)$ of system (1) subject to data losses in closed-loop with LMPC controller (3). Let $\Delta, \epsilon_s, \epsilon_w > 0$ and $\rho > \rho_w > \rho_s > 0$ and N be defined as in Proposition 2. If $x(t_0) \in \Omega_\rho$ and the maximum time without data is less or equal to N for all times, then $x(t) \in \Omega_\rho$, $\forall t$.

- Guarantees that the stability region is an invariant set for the closed-loop system under data losses if the maximum time in which the loop is open, is shorter than a given constant that depends on the parameters of the system and the Lyapunov-based controller that is used to formulate the optimization problem.

APPLICATION TO A CHEMICAL REACTOR

- Well mixed, non-isothermal continuous stirred tank reactor (P. Mhaskar, A. Gani, C. McFall, P. D. Christofides, and F. J. Davis, AIChE J., 2006)
  - Three parallel irreversible elementary exothermic reactions
  - $A \rightarrow B$, $A \rightarrow C$ and $A \rightarrow D$
  - Feed of pure A at flow rate F, temperature T_{A0} and molar concentration $C_{A0} + \Delta C_{A0}$ where ΔC_{A0} is an unknown time varying uncertainty
  - A jacket is used to remove/provide heat Q to the reactor (control input)
- First principles model

$$\begin{aligned}
\frac{dT}{dt} &= \frac{F}{V_r}(T_{A0} - T) - \sum_{i=1}^{3} \frac{\Delta H_i}{\sigma c_p} k_{i0} e^{-E_i/RT} C_A + \frac{Q}{\sigma c_p V_r} \\
\frac{dC_A}{dt} &= \frac{F}{V_r}(C_{A0} + \Delta C_{A0} - C_A) + \sum_{i=1}^{3} k_{i0} e^{-E_i/RT} C_A
\end{aligned} \tag{4}$$

APPLICATION TO A CHEMICAL REACTOR

- Process parameters

F	4.998 $[m^3/h]$	k_{10}	3*10^6 $[h^{-1}]$
V_r	1$[m^3]$	k_{20}	3*10^5 $[h^{-1}]$
R	8.314 $[KJ/kmol \cdot K]$	k_{30}	3*10^5 $[h^{-1}]$
T_{A0}	300 $[K]$	E_1	5*10^4 $[KJ/kmol]$
C_{A0}	4 $[kmol/m^3]$	E_2	7.53*10^4 $[KJ/kmol]$
ΔH_1	-5.0*10^4 $[KJ/kmol]$	E_3	7.53*10^4 $[KJ/kmol]$
ΔH_2	-5.2*10^4 $[KJ/kmol]$	σ	1000 $[kg/m^3]$
ΔH_3	-5.4*10^4 $[KJ/kmol]$	c_p	0.231 $[KJ/kg \cdot K]$

- Three steady-states (two locally asymptotically stable and one unstable)

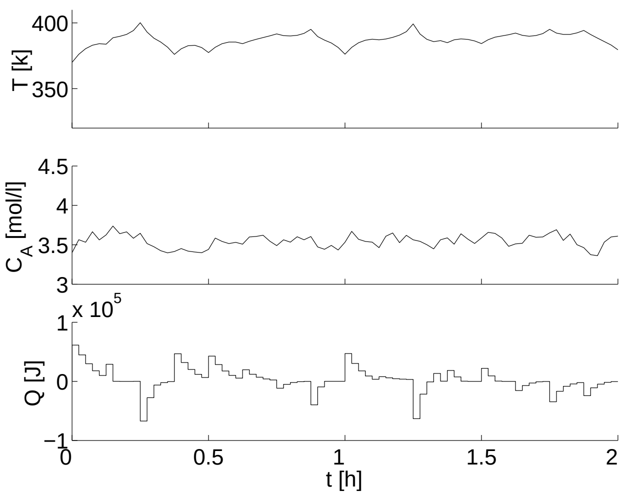
- Control objective: Stabilize the system at the open-loop unstable steady state at:

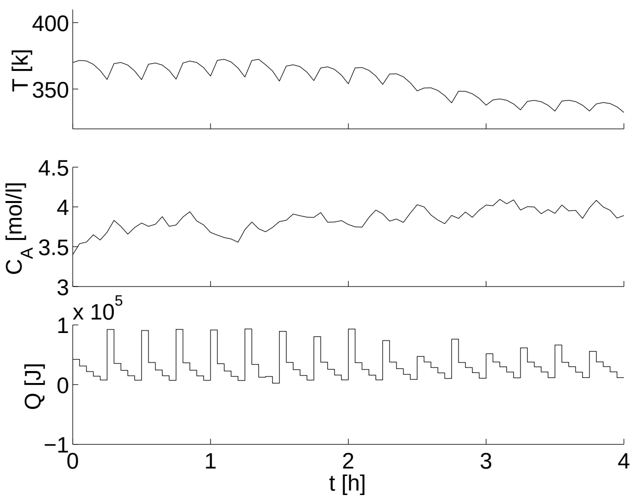
  – $T_s = 388K, \; C_{As} = 3.59 mol/l$

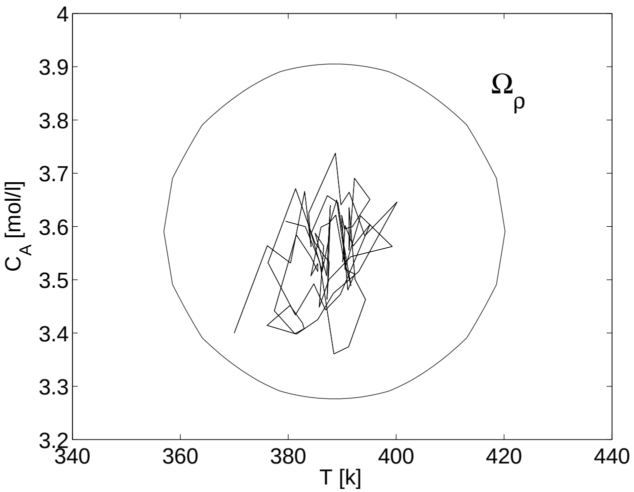
- Poisson sequence with appropriate bounds is used to model data losses / LMPC with appropriate constraints

APPLICATION TO A CHEMICAL REACTOR

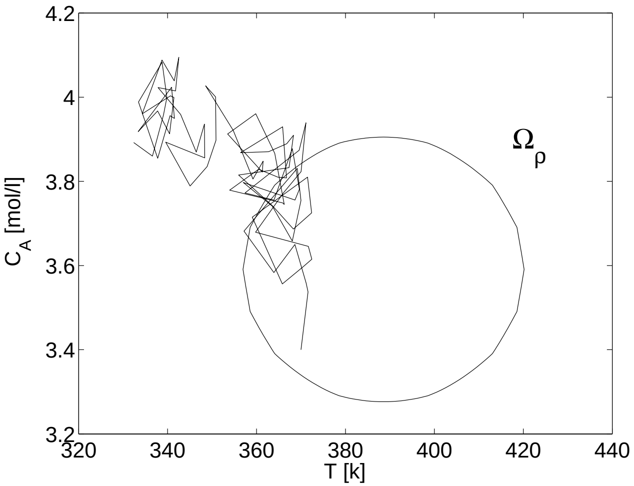
- Comparison with original LMPC schemes for worst case admissible data losses







$V(\tilde{x}(t)) \leq V(\hat{x}(t)),\ \forall t \in [t_k, t_{k+N}]$



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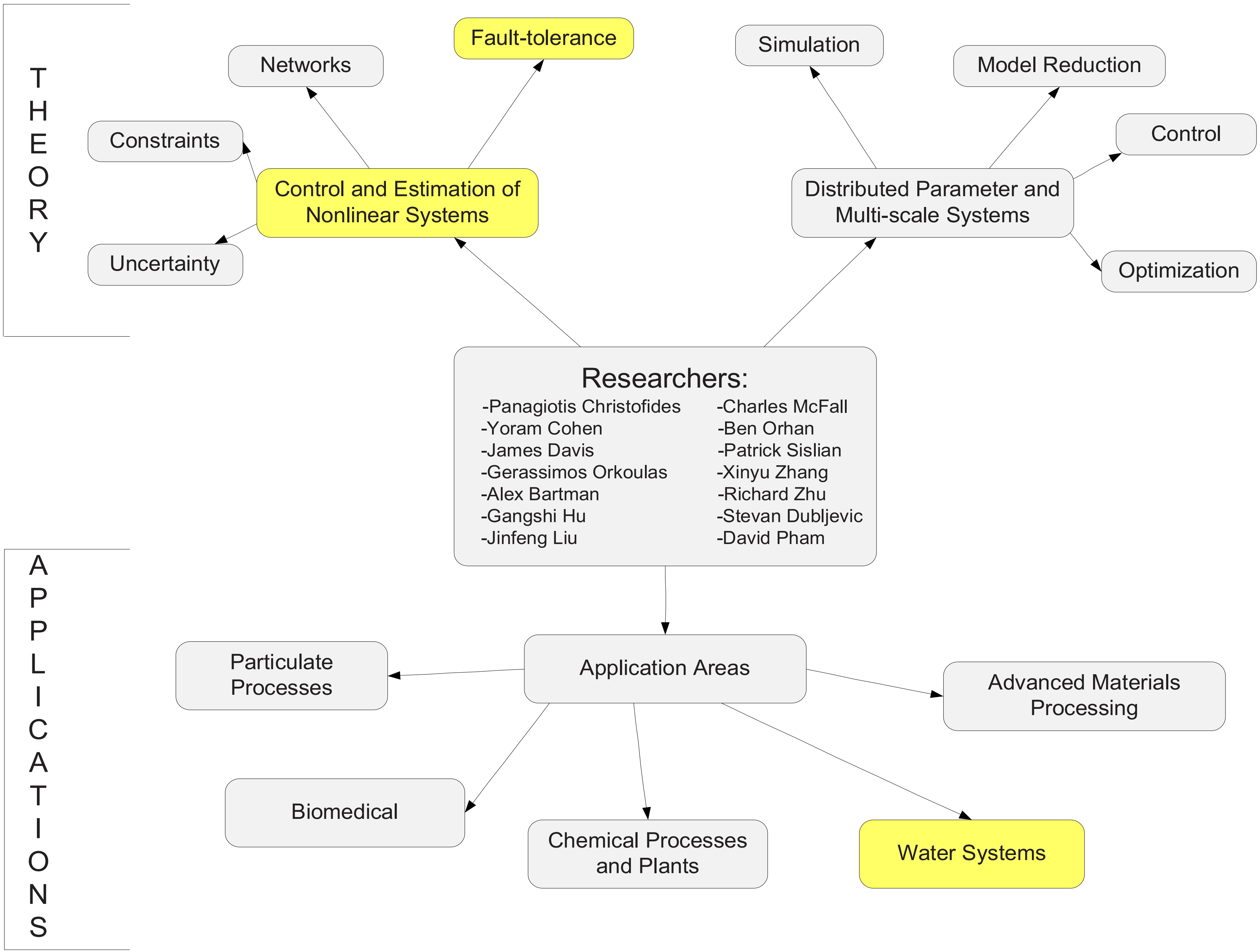
CONTROL SYSTEMS RESEARCH IN OUR GROUP



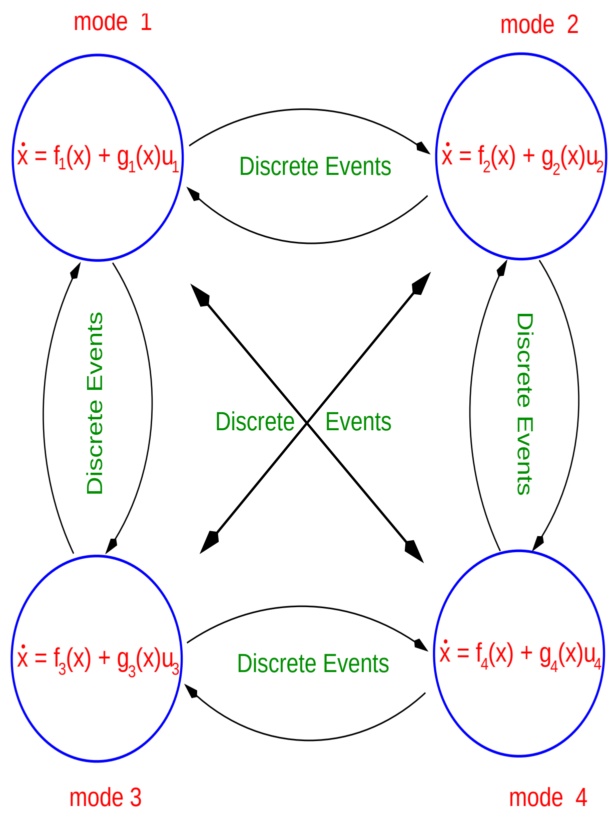
MOTIVATION FOR FAULT-TOLERANT CONTROL

- **Process control system failure:**
  - ◇ Typical sources:
    - ▷ Failure in control algorithm
    - ▷ Faults in control actuators and/or measurement sensors
- **Motivation for fault-tolerant control:**
  - ◇ Preserve process integrity & dependability
  - ◇ Minimize negative economic & environmental impact:
    - ▷ Raw materials waste, production losses, personnel safety, $\cdots$, etc.
- **Characteristics of chemical processes:**
  - ◇ Nonlinear behavior
    - ▷ Complex reaction mechanisms
    - ▷ Arrhenius reaction rates
  - ◇ Input constraints
    - ▷ Finite capacity of control actuators

A HYBRID SYSTEMS FRAMEWORK FOR FAULT-TOLERANT PROCESS CONTROL

- **State-space description** (El-Farra and Christofides, AIChE J., 2003)**:**

$$\dot{x}(t) = f(x(t)) + g_i(x(t))(u_i(t) + m_i(t))$$
$$y = h(x),\ i(t) \in \mathcal{I} = \{1, 2, \cdots, N < \infty\}$$
$$u_{i,min} \leq \|u_i(t)\| \leq u_{i,max}$$



◇ $x(t), y(t) \in \mathbb{R}^n, \mathbb{R}^m$: continuous process state/ measured variables

◇ $u_i(t),\ m_i(t) \in \mathbb{R}^m$: manipulated inputs/fault for i-th mode

◇ $i(t) \in \mathcal{I}$: discrete variable controlled by supervisor

◇ N : total number of control configurations

◇ $f(x),\ g_i(x)$: sufficiently smooth nonlinear functions

FAULT-TOLERANT CONTROL PROBLEM FORMULATION

- **Integrated fault-detection and fault tolerant control design:**
  - ◇ Design of fault-detection and isolation filter
    - ▷ 'Residuals' dedicated to each manipulated input

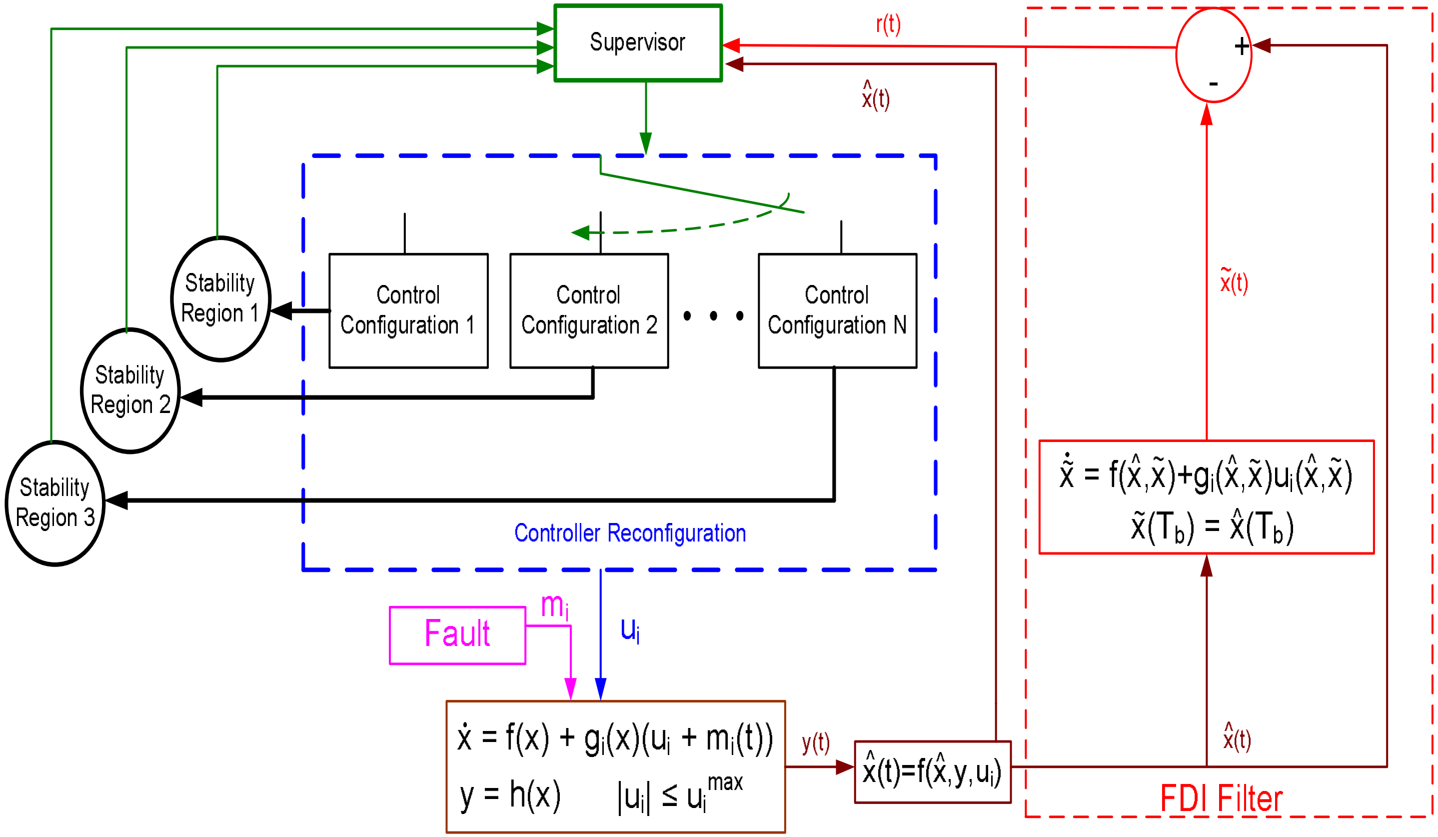
$$r_{i,k}(t) = f_f(y, u_i^k)$$

  - ◇ Synthesis of a family of stabilizing feedback controllers
    - ▷ Model for each control configuration: $\dot{x} = f(x) + g_i(x)u_i$
    - ▷ Magnitude of input constraints: $\|u_i\| \leq u_{i,max}$
    - ▷ Family of Lyapunov functions: $V_i, \quad i = 1, \cdots, N$
  - ◇ Design of supervisory switching laws that orchestrate mode transitions

$$i(t) = \phi(r(t), y(t),\ i(t^-),\ t)$$

- **Objective:**
  - ◇ Maintain closed-loop stability under failure situations

OUTPUT FEEDBACK FAULT-DETECTION & ISOLATION & FAULT-TOLERANT CONTROL STRUCTURE

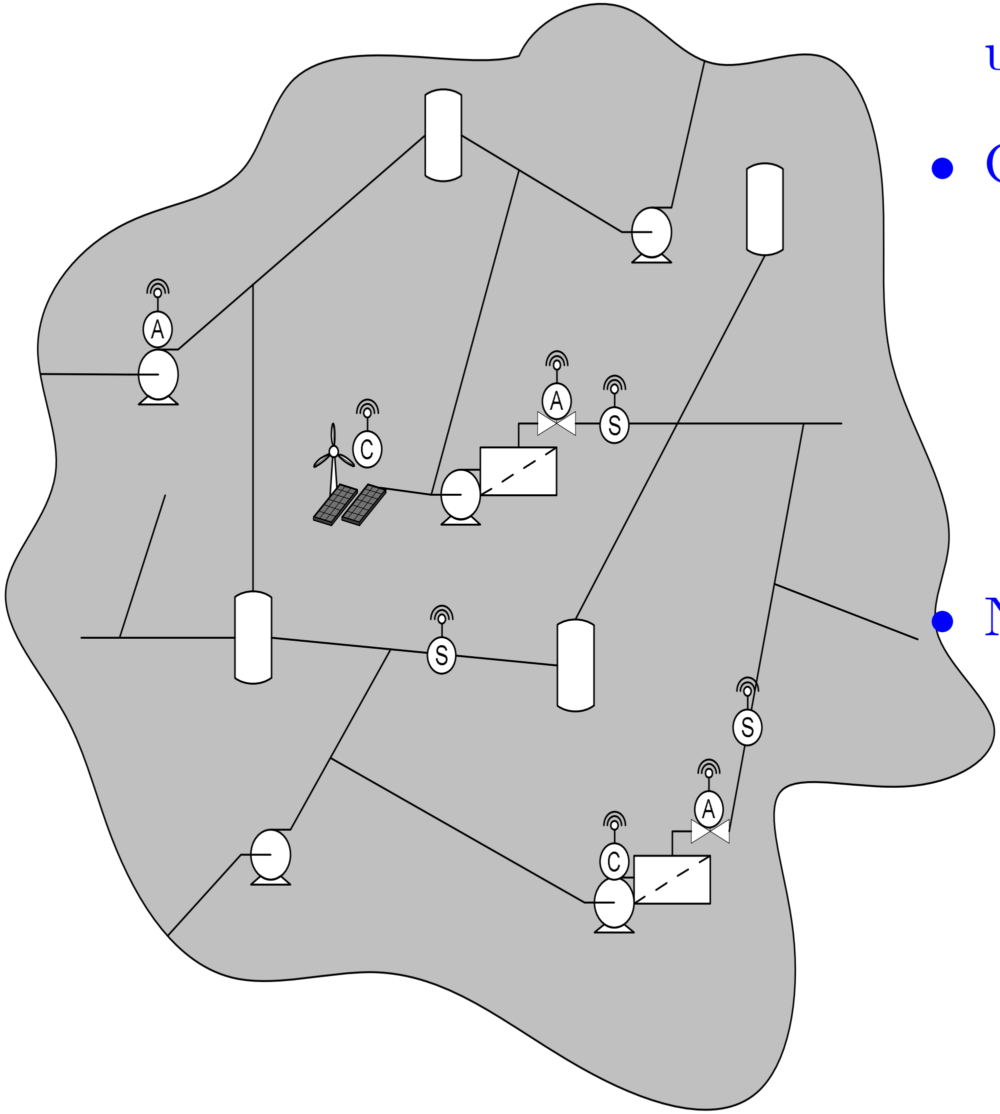


(Mhaskar et al., Automatica, 2008)

- Components:
  - ▷ State estimator
  - ▷ Control configurations
  - ▷ Fault-detection filter
  - ▷ Supervisor

DISTRIBUTED "SMART" WATER SYSTEMS

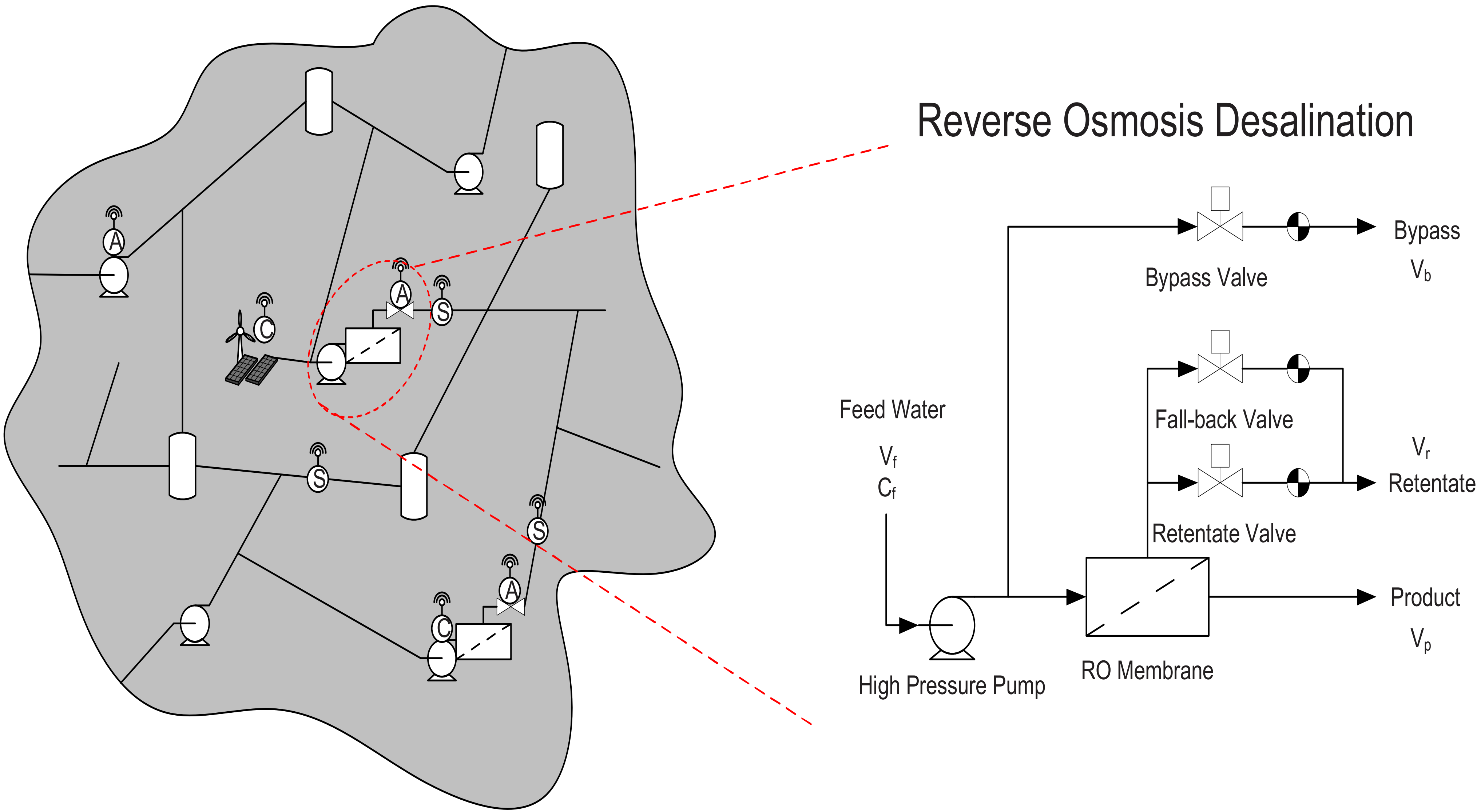
(Kaiser, Cohen, Pottie and Christofides)



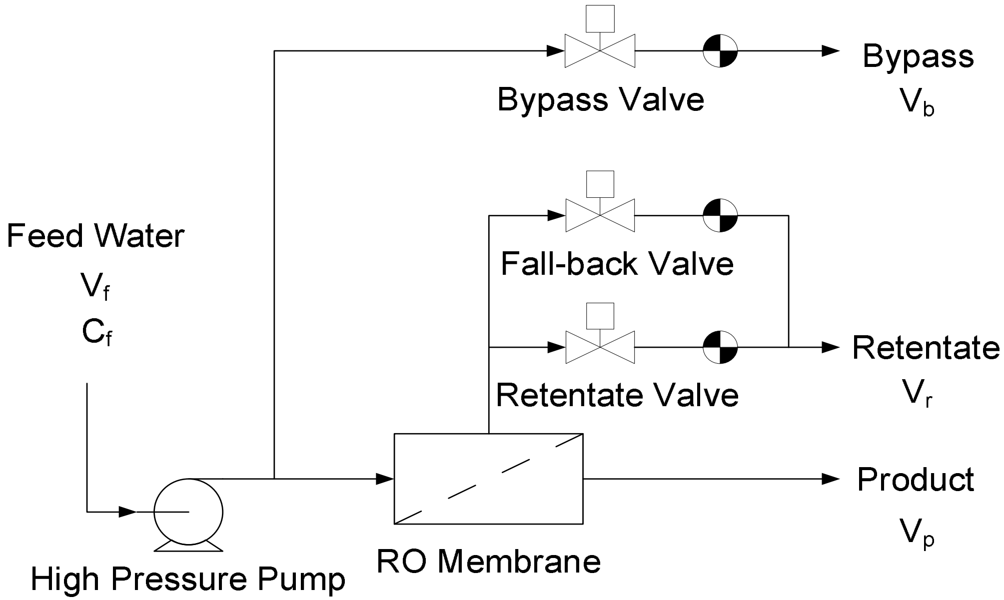
- Distributed water systems can be deployed in urban, agricultural and industrial settings
- Growing water demands motivate:
  - ◇ Efficient use of available water resources
  - ◇ Reclamation of water through desalination
  - ◇ Robust operation at the network level and at each individual unit
- Networked monitoring and feedback control:
  - ◇ Fault-tolerance at the individual unit level to prevent component failure
  - ◇ Provide operation at the network level that is robust to component failures
  - ◇ Coordinate network-wide operation in an optimal fashion

DISTRIBUTED "SMART" WATER SYSTEMS

(Kaiser, Cohen, Pottie and Christofides)



RO PROCESS MODEL DESCRIPTION

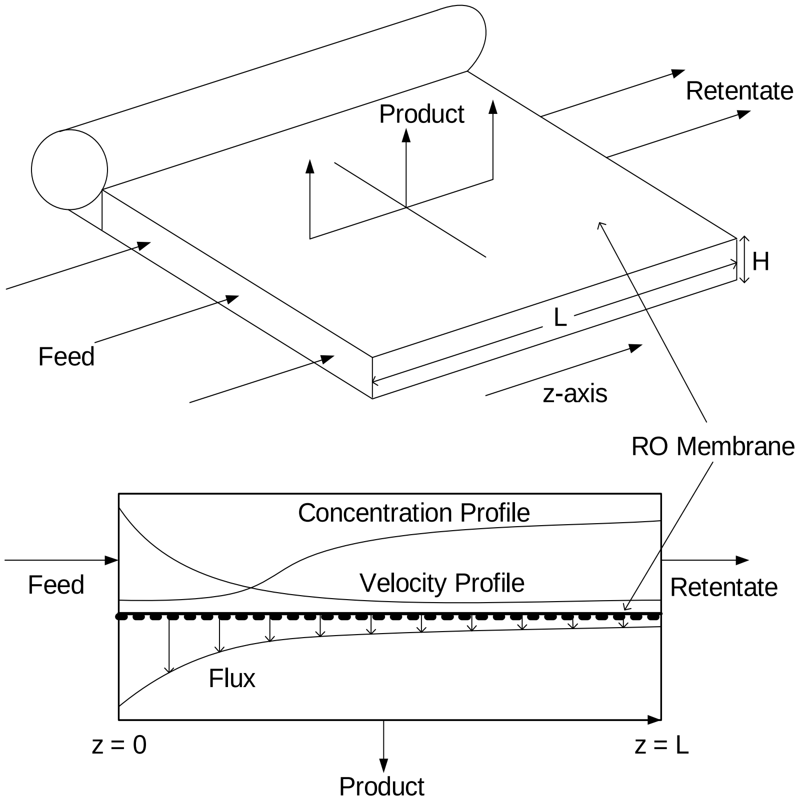


- **Process dynamic model:**

$$
\begin{aligned}
\frac{dv_b}{dt} &= \frac{A_p}{\rho V}\left(P - \frac{1}{2}v_b^2 e_{v1}\right) \\
\frac{dv_r}{dt} &= \frac{A_p}{\rho V}\left(P - \frac{1}{2}v_r^2 e_{v2}\right)
\end{aligned}
$$

- **Features of this model**
  - ◇ Pressure, P, is a function of v_b, v_r, and C_f (coupling occurs through P)
  - ◇ Primary control configuration: actuation on (e_{v1}, e_{v2})
  - ◇ Fall-back control configurations: (e_{v1}, e_{v2}^{fb}), (v_f, e_{v2}^{fb})

RO PROCESS MODEL DESCRIPTION



$$\begin{aligned}
\frac{dC_z}{dz} &= \frac{C_z}{v_z}\frac{K_m(P-K_{\Delta\pi}C_z)}{\rho H} \\
\frac{dv_z}{dz} &= -\frac{K_m(P-K_{\Delta\pi}C_z)}{\rho H} \\
C_z(z=0) &= C_f \\
v_z(z=0) &= \alpha v_{mf} \\
v_z(z=L) &= \alpha v_r
\end{aligned}$$

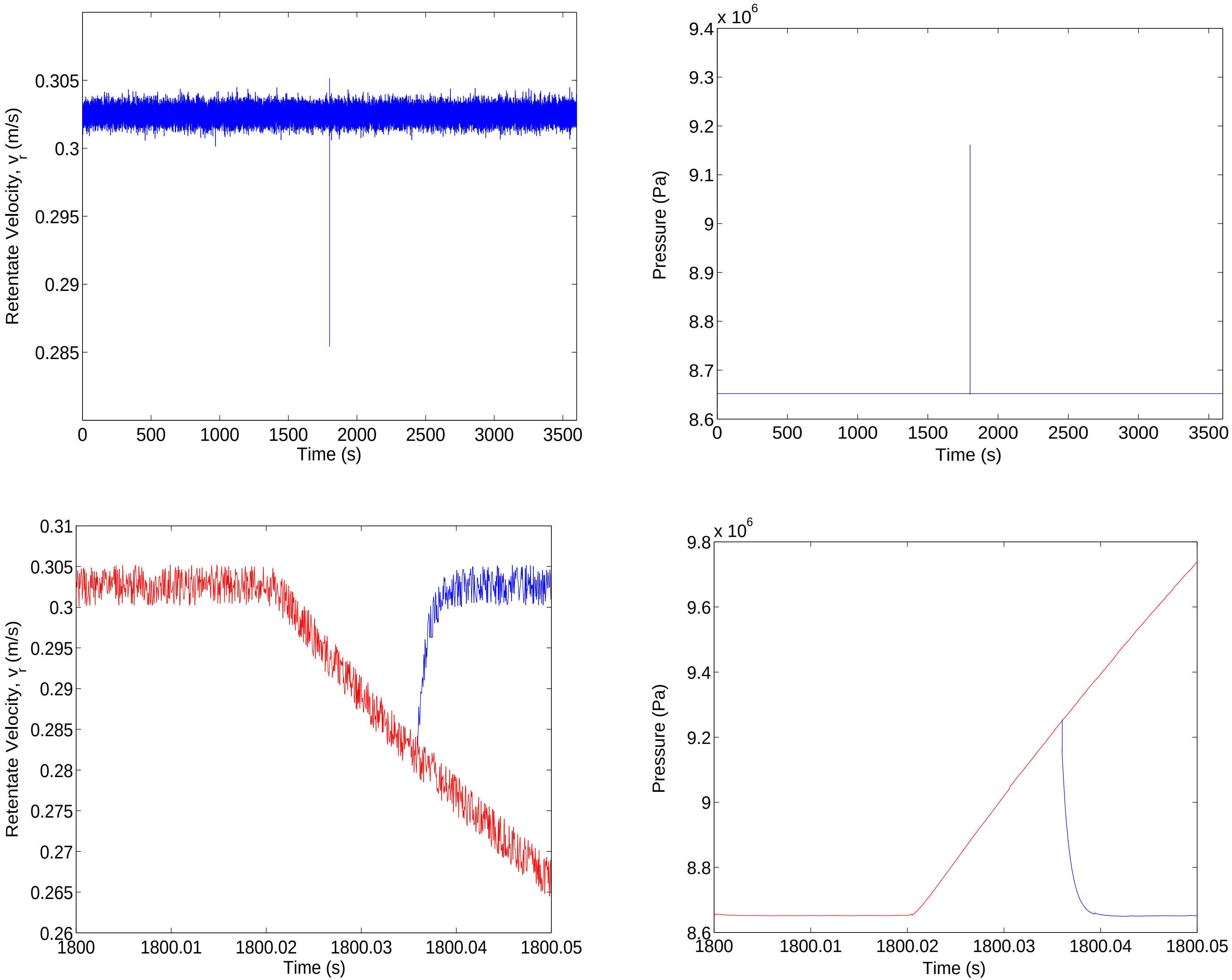
- **Features of this model**
  - ◇ Given v_b, v_r, and C_f this problem has three boundary conditions
  - ◇ Pressure, P, can be determined via shooting method
  - ◇ Shell balance assumes radially well mixed plug flow

FDIFTC OF RO SYSTEM

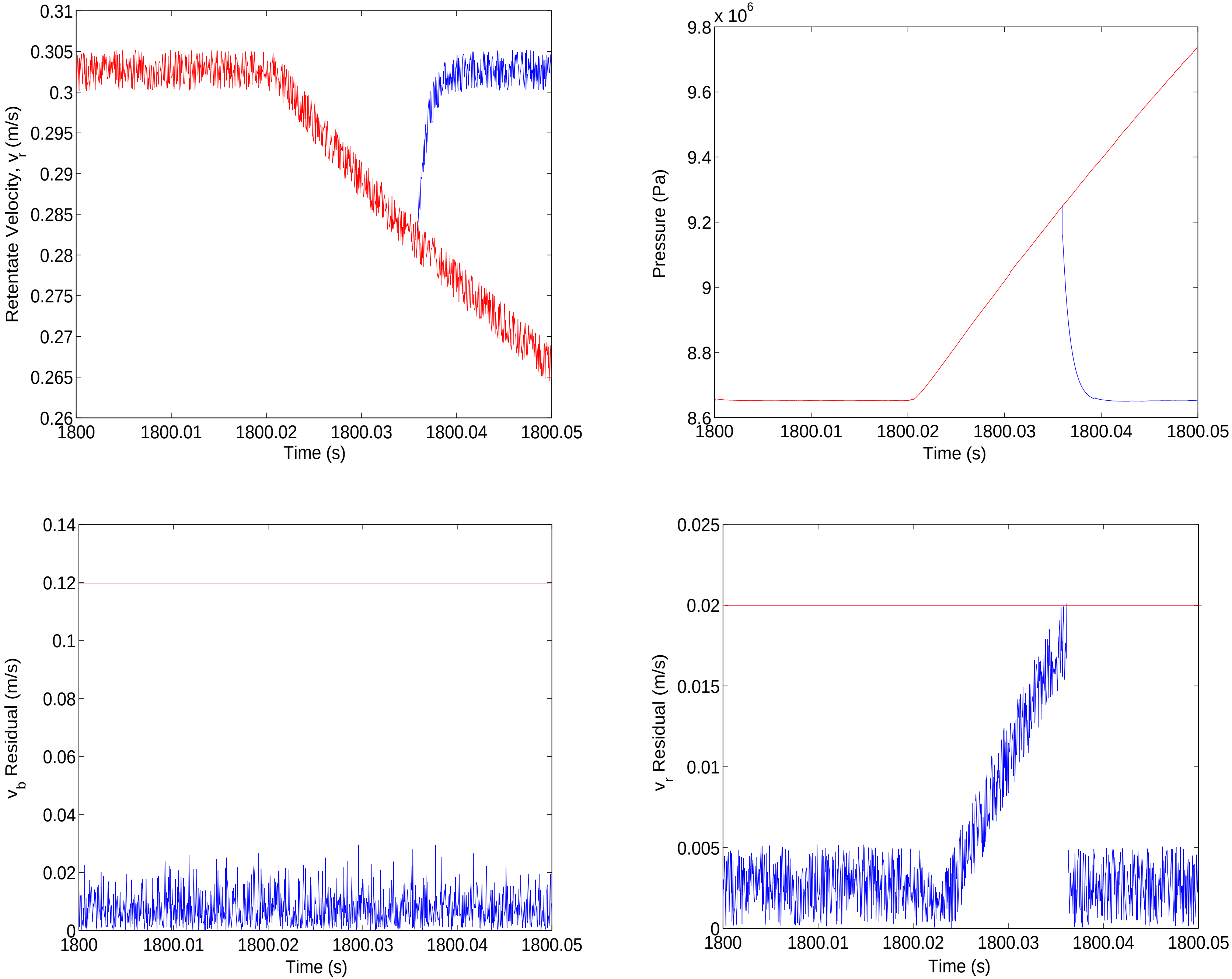
(McFall et al., IECR, 2008; JPC, 2008)

- **Real RO systems may encounter actuator failures, noise, and parameter mismatch**
- **Accounting for noise**
  - ◇ v_b measurements have gaussian noise with standard deviation 0.01 $\frac{m}{s}$
  - ◇ v_r measurements have gaussian noise with standard deviation 0.005 $\frac{m}{s}$
  - ◇ Measurements are sampled at a rate of 20 kHz
  - ◇ Control is Sample-and-hold
- **Accounting for parameter mismatch**
  - ◇ A_m in the system is 9 m^2, while A_m in the model is 13 m^2
- **Creeping failure in the retentate valve**
  - ◇ The value e_{v2} increases linearly with time after $t \approx 30\ min$
- **Bounded Lyapunov-based control**
  - ◇ Quadratic control Lyapunov function $V_i = x^T P_i x$

FDIFTC OF RO DESALINATION



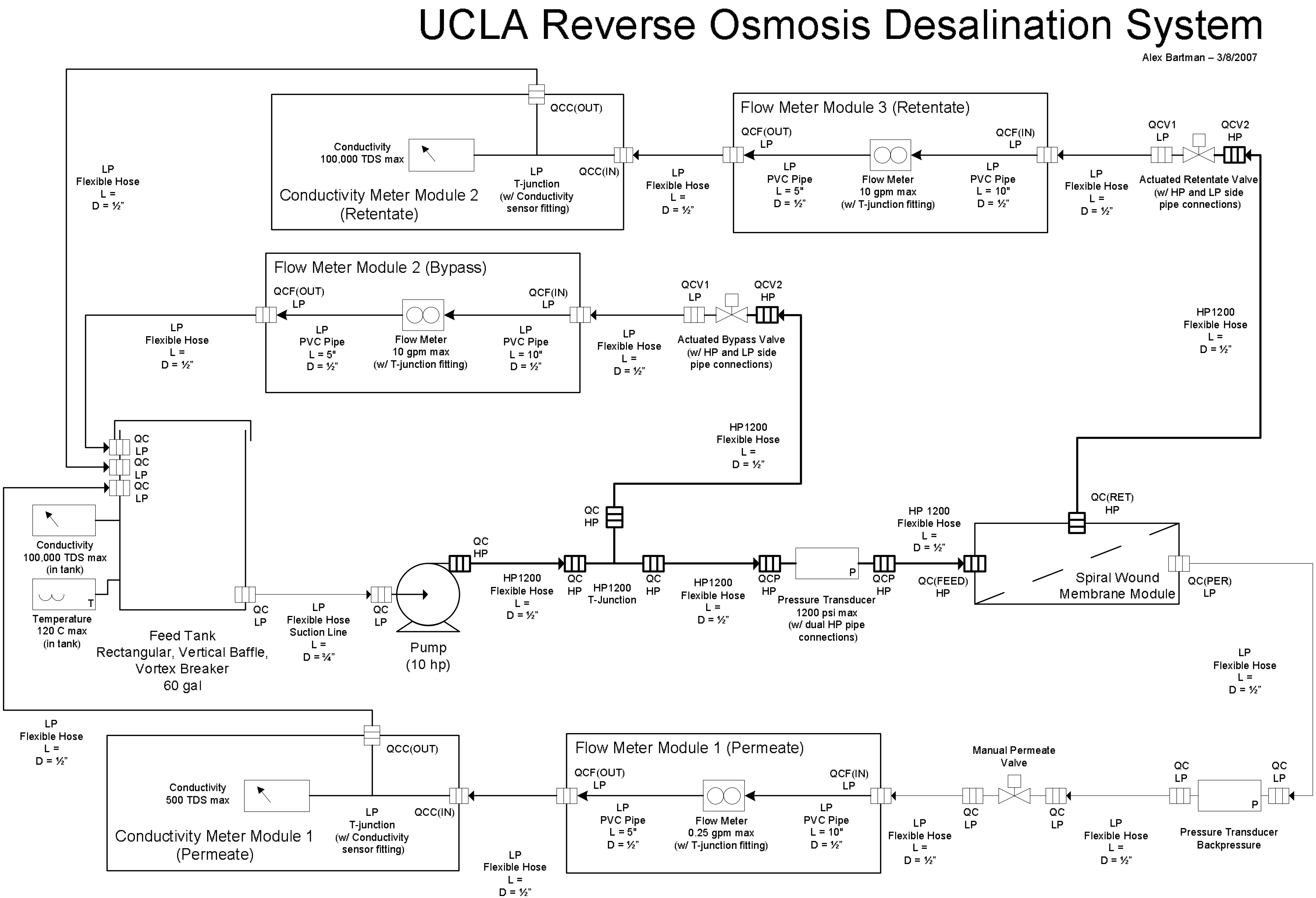
FDIFTC OF RO DESALINATION



CURRENT WORK: EXPERIMENTAL IMPLEMENTATION

- **Lab and field scale reverse osmosis system**
  - ◇ Design: feed flows of 2 to 10 GPM at up to 1000 psi
- **Flexible system with interchangeable components**
  - ◇ Can be reconfigured easily for:
    - ▷ brackish or sea water, wide TDS range
    - ▷ Straight through or circulating feed
    - ▷ Various control configurations and strategies
  - ◇ System and vehicle will eventually provide portable field use
    - ▷ Enables testing of various source waters
    - ▷ Remote web based control and communication
  - **Interchangeable modular units consist of:**
    - ◇ Pretreatment filters, tanks, actuated pumps, actuated valves, gauges/meters, RO membranes, post treatment, control system

DIAGRAM OF RO PROCESS



CONCLUSIONS

- **Chemical process systems with:**
  - ▷ Nonlinear dynamics
  - ▷ Data losses
  - ▷ Input constraints
  - ▷ Control system failures
- **Networked predictive control:**
  - ◇ Guarantees practical stability in the absence of data losses
  - ◇ Guarantees the stability region is invariant
  - ◇ Contractive constraint must hold along the whole prediction horizon
- **Integrated fault-detection & isolation and fault-tolerant control:**
  - ◇ Design of nonlinear fault-detection filters
  - ◇ Design of constrained nonlinear feedback controllers
  - ◇ Design of fault-tolerant supervisory switching laws
- **Applications to chemical reactor and RO process**

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