Parallel Delta-Sigma Modulators for Reconfigurable ADC Design

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Why Reconfigurable ADCs ?

The standard clichés ...

- Dick Tracy's watch
- Cognitive radios

Other possibilities ...

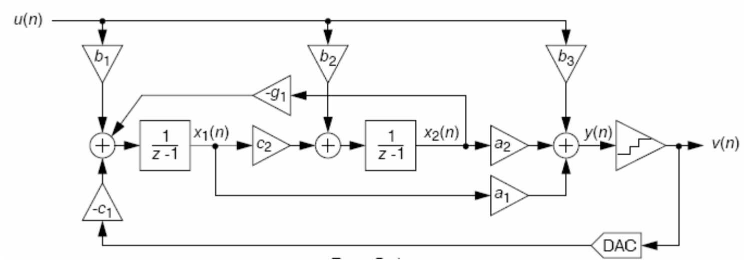
- Cross-layer radio optimization framework
- Multi-purpose sensor applications

The reality ...

- Multi-standard radios at best

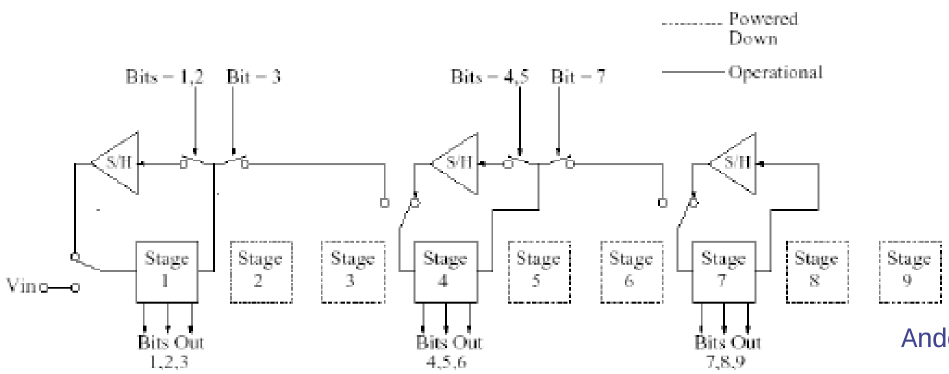
Reconfigurable ADC Design Approach #1

- Fix ADC architecture but change parameters



Delta-Sigma

Ouzunov, ISSCC '07

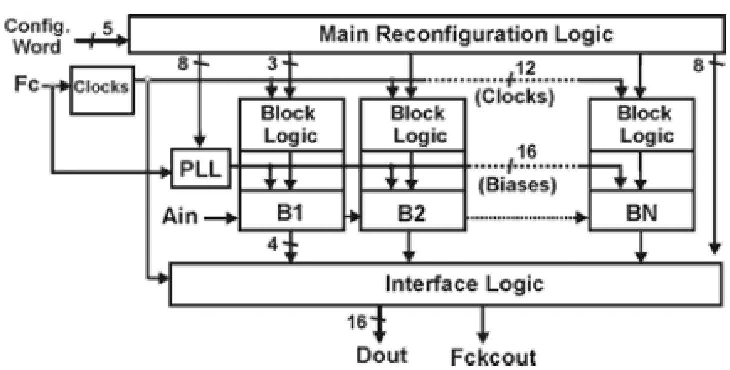


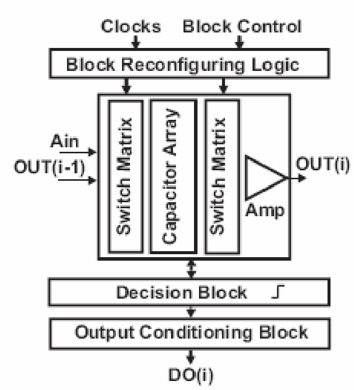
Pipeline

Andersen, VLSI '05

Reconfigurable ADC Design Approach #2

- Configure circuit blocks to change architecture
  - Can also include parameter re-configuration

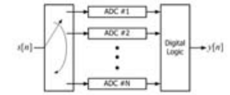




K. Gulati et. al.

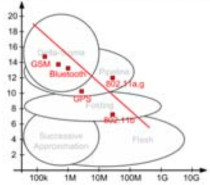
Reconfigurable ADC Design Approach #3

- Time-interleaved parallel ADCs
  - Can also include parameter re-configuration



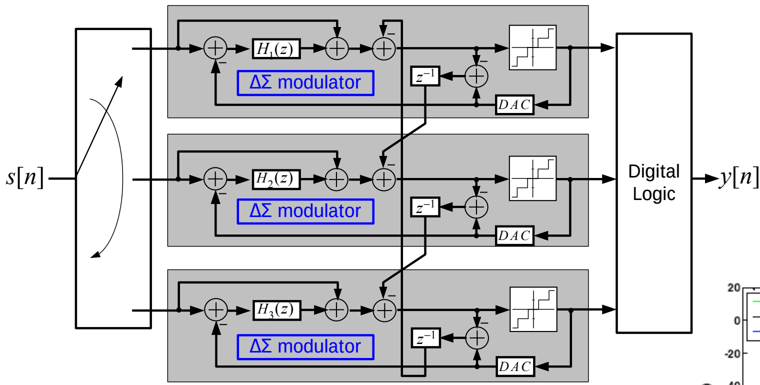
The Problems With Prior Art

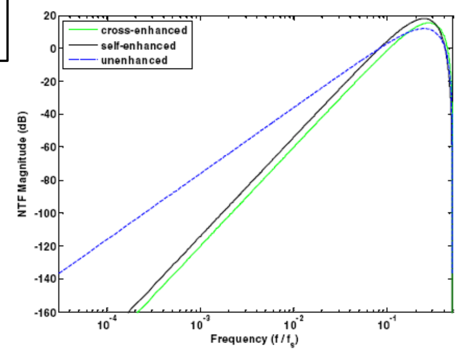
- Too wide a spec space to span
- Design effort explosion
- Circuit requirements change with architecture



Reconfigurable Parallel Delta-Sigma Modulators

- Parallel coupled delta-sigma modulators with quantization noise coupling
  - Noise coupling increases effective noise shaping
  - More general coupling possible

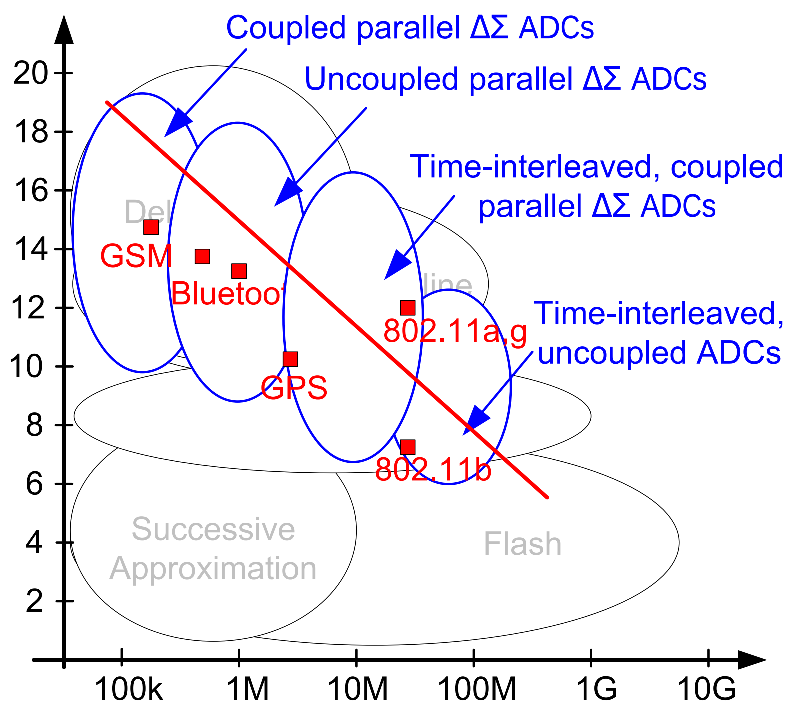




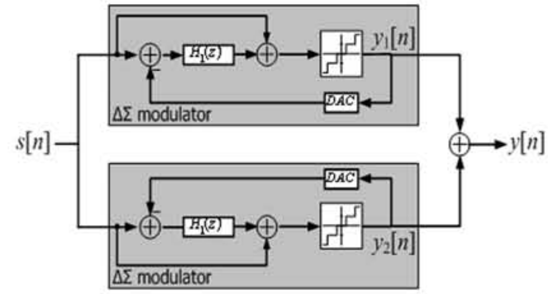
K. Lee, G. C. Temes, Electronics Letters, 2006

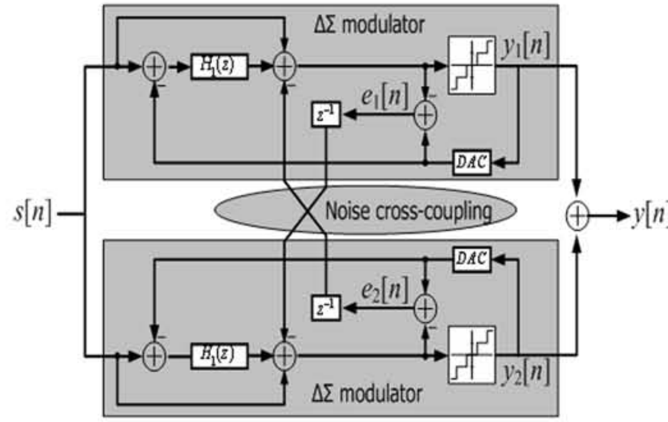
Potential Benefits – Wireless Comm Example

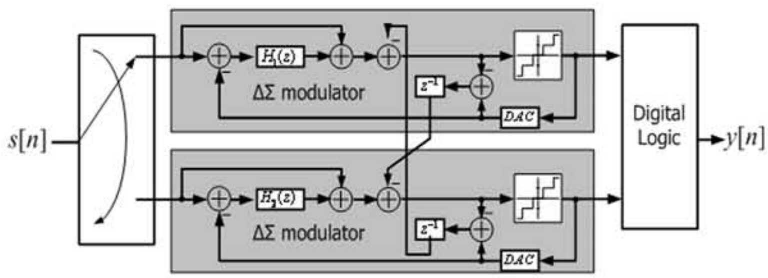
- Very easy to tradeoff power for speed and/or performance

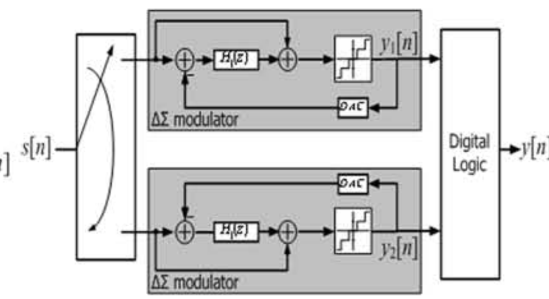


Potential Benefits …



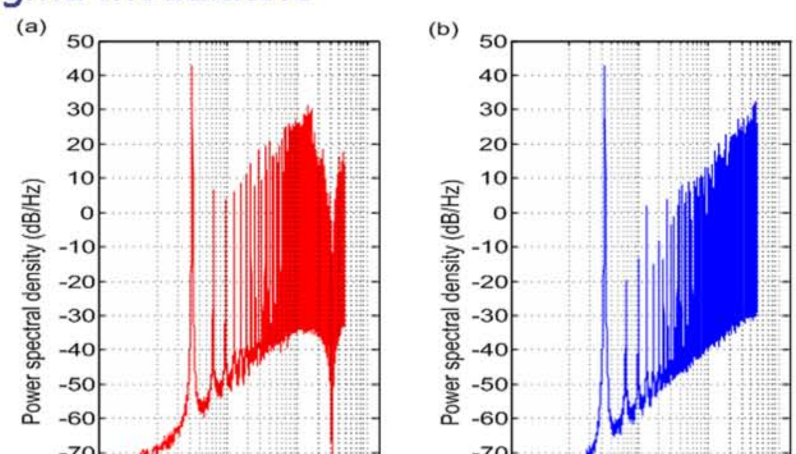






Spurious Tone Problem

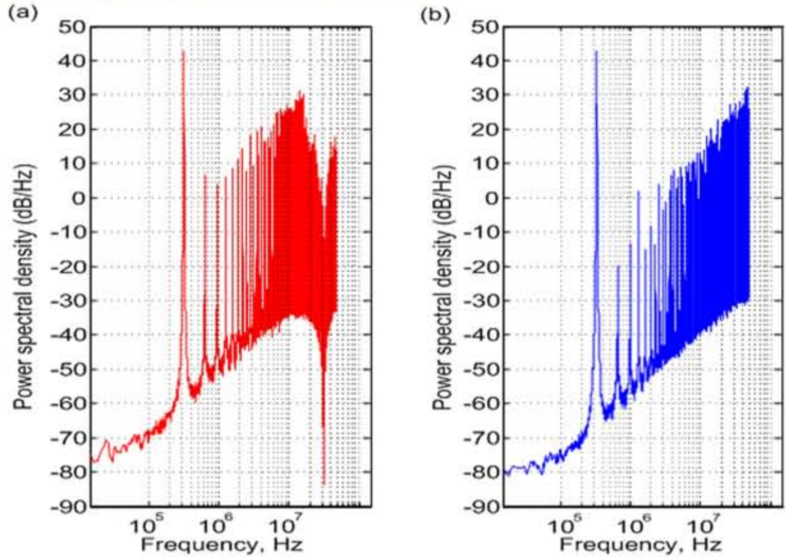
- Limit cycles in delta-sigma modulators result in spurious tones
  - Reasonably well understood in uncoupled delta-sigma modulators



Two different 3-channel time-interleaved delta-sigma modulators with net 1st order shaping

Quantization Noise Statistics Problem

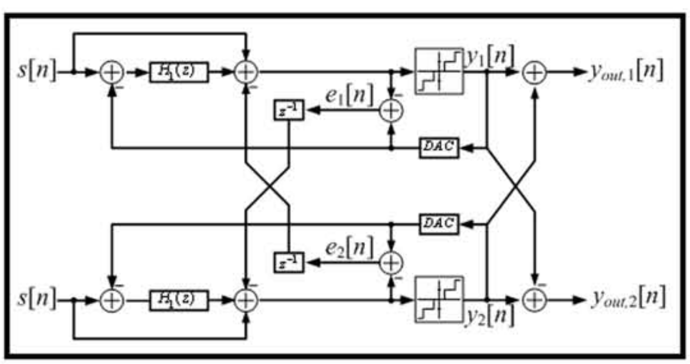
- Quantization noise typically assumed to be uniform, white, and independent of the signal
  - Enables ready analysis of delta-sigma modulators
- In general, this is NOT true!

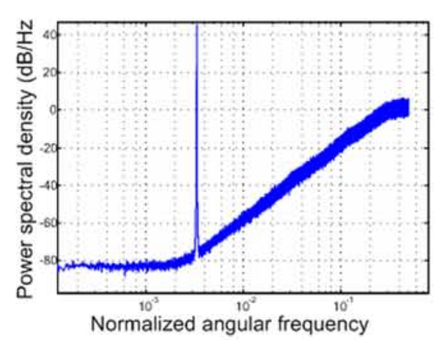


Two different 3-channel time-interleaved delta-sigma modulators with net 1st order shaping

Our Research

- Quantization noise theory for parallel ΔΣ modulators
  - With and without time-interleaving
- Developed theoretical conditions to identify parallel ΔΣ modulators that have no spurious tones

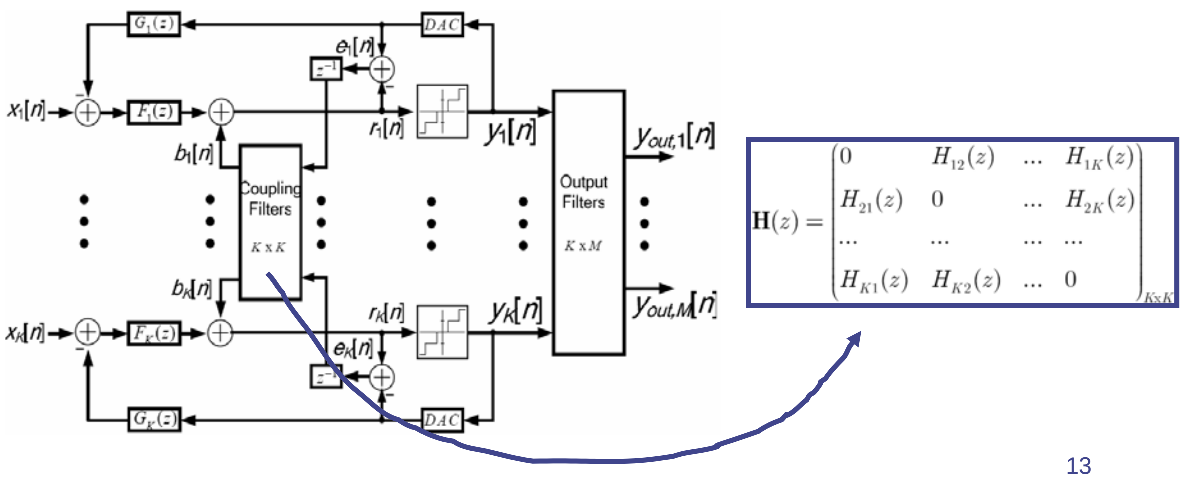




H1(z) = 0 ➔ 1st order noise shaping, but spurious tones exist;
$H_1(z) = z^{-1}(1 - z^{-1})^{-1}$ ➔ 2nd order noise shaping, and no spurious tones

Quantization Noise Theory (1)

- Analyzed in the context of a generic parallel ΔΣ modulator
- Constraints
  - Non-overloading quantizers
  - Integer valued filter responses



Quantization Noise Theory (2)

- Quantization noise properties covered by the theory
  - Probability distribution (*pdf*) of the quantization noise of the *i*^th^ channel, $e_i[n]$ ➔ <u>is $e_i[n]$ uniform or not</u> ?
  - Joint *pdf* of
    - $e_i[n]$ and $e_i[n+m]$ ➔ <u>*is $e_i[n]$ white or not*</u> ?
    - $e_j[n]$ and $e_i[n+m]$ ➔ <u>*are $e_j[n]$ and $e_i[n]$ independent (and uncorrelated) or not*</u> ?

- Implications
  - No limit cycles or spurious tones
  - Quantitative analysis becomes simple

Quantization Noise Theory (3)

- Quantization noise statistics depend only on the properties of the elements of

$$\mathbf{V}(z) = (\mathbf{I} + \mathbf{H}(z))^{-1} diag(F_1(z), F_2(z), \ldots, F_K(z))$$

- Example condition and result :
  - **<u>Condition</u>: Integer b, $1 \le b \le K$, exists such that $t_0 * v_{jb}[n] + t_1 * v_{ib}[n+m] \rightarrow 0$ only if $(t_0, t_1) = (0, 0)$**
    - **$v_{jb}[n]$ is the impulse response of $V_{ij}(z)$**
  - **<u>Result</u>: Quantization noise of the i^{th} and j^{th} channels in a parallel ΔΣ modulator are independent**
- Similar conditions imply that quantization noise is white (hence, has no spurious tones)

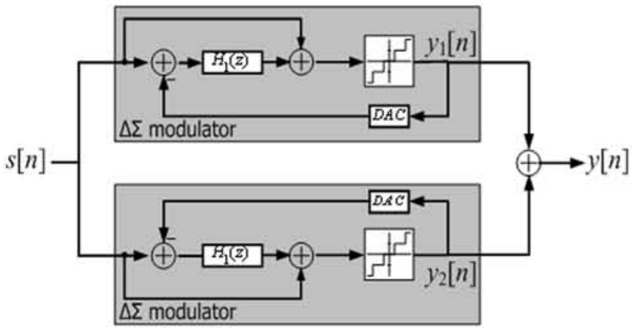
Quantization Noise Theory (4)

Full set of conditions:

- $e_i[n]$ is uniform
  - **If b exists[1] such that $t_0 * v_{jb}[n] \rightarrow 0$ only if $t_0 = 0$**
- $e_i[n]$ is independent of input signal
  - **Same condition ...**
- $e_i[n]$ is independent of $e_i[n+m]$
  - **If b exists[1] such that $t_0 * v_{ib}[n] + t_1 * v_{ib}[n+m] \rightarrow 0$ only if $(t_0, t_1) = (0, 0)$**
- $e_j[n]$ is independent of $e_i[n+m]$
  - **If b exists[1] such that $t_0 * v_{jb}[n] + t_1 * v_{ib}[n+m] \rightarrow 0$ only if $(t_0, t_1) = (0, 0)$**
- <u>Uniformity, whiteness, and uncorrelated-ness of a single realization follow from these ensemble results</u>

[1] assume b is an integer such that $1 \le b \le K$

Illustration of Application of Theory (1)



$$\mathbf{H}(z) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$H_1(z) = z^{-1}(1 - z^{-1})^{-1},\quad F_1(z) = F_2(z) = (1 - z^{-1})^{-1}$$

- Two uncoupled 1st order delta-sigma modulators
  - Would result in 1st order noise shaping
  - Expect to see strong limit cycles

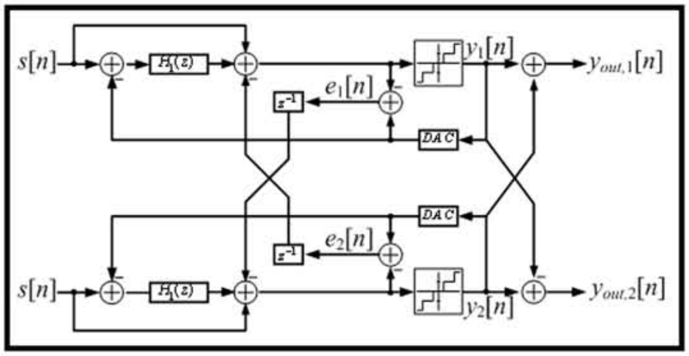
$$\mathbf{V}(z) = \frac{1}{(1 - z^{-1})}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Illustration of Application of Theory (2)

$$v_{11}[n] = v_{22}[n] = u[n],$$
$$v_{12}[n] = v_{21}[n] = 0.$$

- $e_i[n]$ is uniform
  - **If b exists[1] such that $t_0 * v_{jb}[n] \to 0$ only if $t_0 = 0$**
- $e_i[n]$ is independent of input signal
  - **Same condition ...**
- $e_i[n]$ is independent of $e_i[n+m]$
  - **If b exists[1] such that $t_0 * v_{ib}[n] + t_1 * v_{ib}[n+m] \to 0$ only if $(t_0, t_1) = (0, 0)$**
- $e_j[n]$ is independent of $e_i[n+m]$
  - **If b exists[1] such that $t_0 * v_{jb}[n] + t_1 * v_{ib}[n+m] \to 0$ only if $(t_0, t_1) = (0, 0)$**

Illustration of Application of Theory (3)



$$\mathbf{H}(z) = \begin{bmatrix} 0 & -z^{-1} \\ z^{-1} & 0 \end{bmatrix}$$

$$H_1(z) = z^{-1}(1 - z^{-1})^{-1},$$
$$F_1(z) = F_2(z) = (1 - z^{-1})^{-1}$$

- Two coupled 1st order delta-sigma modulators
  - Simple coupling ➔ would result in 2nd order noise shaping

$$\mathbf{V}(z) = \frac{1}{(1 - z^{-1})(1 - z^{-2})} \begin{bmatrix} 1 & -z^{-1} \\ z^{-1} & 1 \end{bmatrix}$$

Illustration of Application of Theory (4)

$$v_{11}[n] = v_{22}[n] = \frac{1}{2}(n+1)u[n] + \frac{1}{4}(1 - (-1)^n)u[n],$$
$$v_{12}[n] = v_{21}[n] = v_{11}[n-1].$$

- $e_i[n]$ is uniform
  - **If b exists[1] such that $t_0 * v_{jb}[n] \rightarrow 0$ only if $t_0 = 0$**
- $e_i[n]$ is independent of input signal
  - **Same condition …**
- $e_i[n]$ is independent of $e_i[n+m]$
  - **If b exists[1] such that $t_0 * v_{ib}[n] + t_1 * v_{ib}[n+m] \rightarrow 0$ only if $(t_0, t_1) = (0, 0)$**
- $e_j[n]$ is independent of $e_i[n+m]$
  - **If b exists[1] such that $t_0 * v_{jb}[n] + t_1 * v_{ib}[n+m] \rightarrow 0$ only if $(t_0, t_1) = (0, 0)$**

Illustration of Application of Theory (5)