Decision Fusion in Sensor networks for Spectrum Sensing based on Likelihood Ratio Tests

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April 2009

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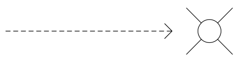
Detection and Decision Fusion in Fading Environment

- Detection by Single sensor
  - Hypothesis test
  - Cognitive radio
- Decision Fusion using Multiple Sensors
- Detection by Single Sensor under Fading
- Multi-Sensor Decision Fusion under Fading
- Sequential Likelihood Ratio Test for Spectrum Sensing

Hypothesis Test

- Hypothesis Test applications
  - Surveillance
  - Target Detection
  - Spectrum Sensing



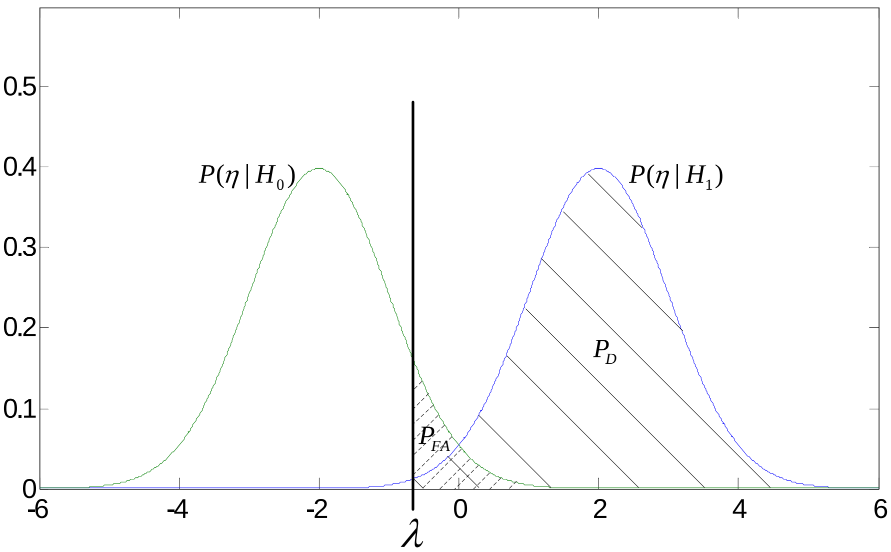
- Example-Matched Filter Detection
  - Signal model

$$X = \begin{cases} S_0 + N, H_0 \\ S_1 + N, H_1 \end{cases}$$

  - $\eta = \langle X, (S_1 - S_0) \rangle$

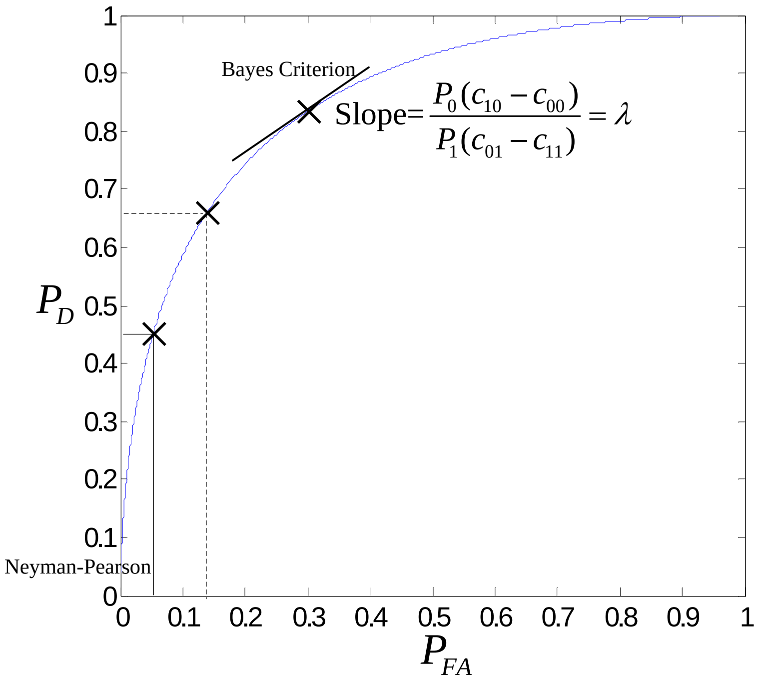
$$= \begin{cases} \langle S_0, (S_1 - S_0) \rangle + \langle N, (S_1 - S_0) \rangle, H_0 \\ \langle S_1, (S_1 - S_0) \rangle + \langle N, (S_1 - S_0) \rangle, H_1 \end{cases}$$

  - $Decision = \begin{cases} \hat{H}_0, \eta \le \lambda \\ \hat{H}_1, \eta > \lambda \end{cases}$



Receiver Operating Curve

- Receiver Operating Curve: P_{FA} v.s. P_D
- Setting Threshold
- Criteria
  - Neyman-Pearson
    - P_{FA} upper-bounded
  - Bayes
    - Priors and costs



Spectrum Sensing in Cognitive Radio

- Wireless communications rely on spectra.
  - Current usage model: frequency bands are licensed.
  - The licensed bands are often vacant- low utilizations.
- Cognitive Radio-to increase the spectrum utilization.
  - Allows secondary user to access the spectrum when it is vacant.
  - Secondary users sense the spectrum before accessing.
  - Accuracies of the spectrum sensing is crucial.
    - Formulated as binary hypothesis test problem
      - H_0: Spectrum Vacant
      - H_1: Spectrum Occupied

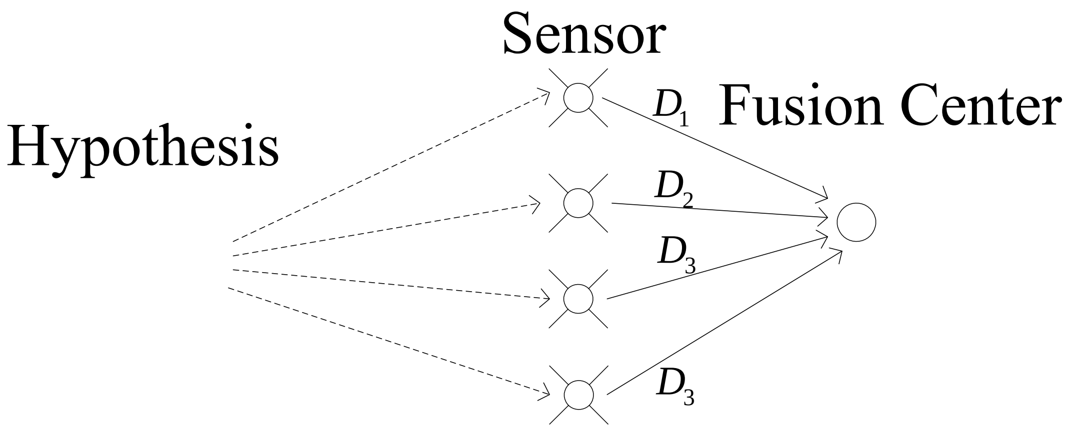
[S. Haykin, "Cognitive radio: brain-empowered wireless communications," IEEE JSAC. 2005.]

Detection Criteria and Implications in Cognitive Radio

- Interpretations of P_D and P_{FA} in cognitive radio
  - H_1 : Spectrum used by the primary users
  - Secondary users access the spectrum if decision is H_0
  - Channel conflict: Decision H_0 under the truth H_1
  - Miss of the spectrum opportunity: Decision H_1 under the truth H_0
- Neyman-Pearson
  - Upper-bound probability of false alarm while maximizing probability of detection
  - Protect the spectrum opportunities of the secondary users while minimizing the channel conflicts
- Lower-Bounded Probability of Detection (LBPD) [Chung 08]
  - Lower-bound probability of detection while minimizing probability of false alarm
  - Protect the primary users while maximizing the spectrum opportunities for the secondary users

Decision Fusion Framework

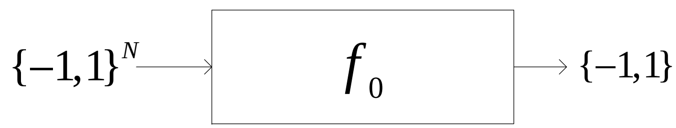
- Sensors make binary decisions.
  - Many applications require binary decisions.
  - Accuracy of a single sensor is limited.
  - Fusion of multiple decisions increases accuracies.



[R. Viswanathan and P. K. Varshney, "Distributed Detection with Multiple sensors I. Fundamentals," Proceedings of the IEEE, 1997.]

Decision Fusion Framework

- N sensors make binary decisions.
  - Probability of False Alarm $\{P_{FAi} \mid i = 1, 2, ..., N\}$
  - Probability of Detection $\{P_{Di} \mid i = 1, 2, ..., N\}$
  - Sensor decisions $\{D_i \mid i = 1, 2, ..., N\}$
- The fusion center makes final decision.
  - Fusion Rule: $f_0 : \{-1, 1\}^N \to \{-1, 1\}$



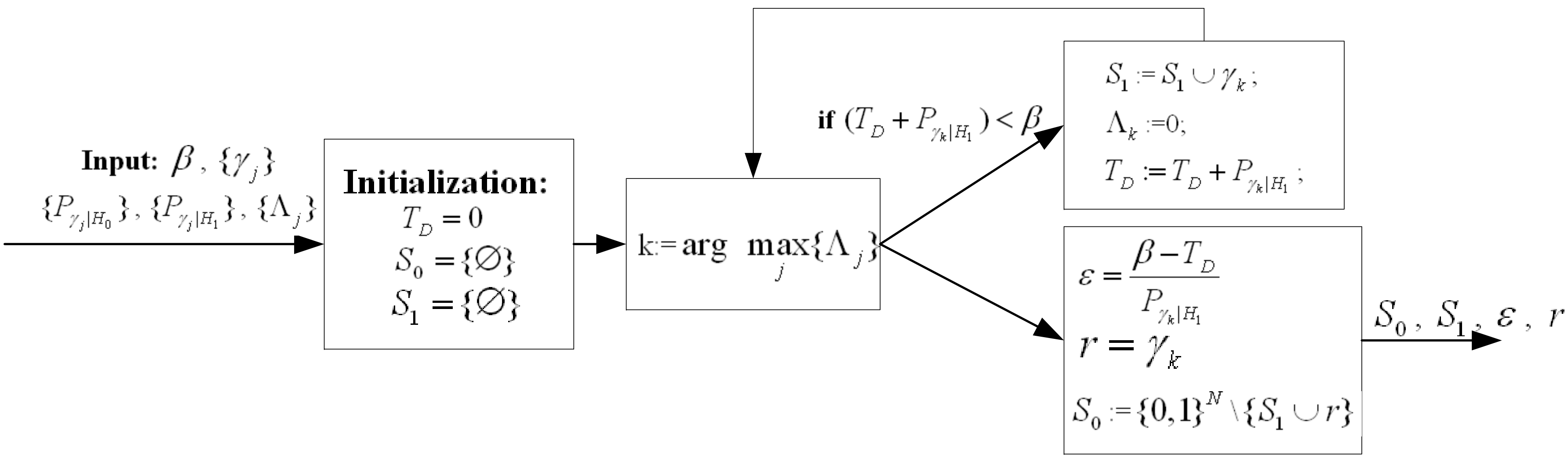
  - Fusion Rule with random strategy:

$$f_0(\{D_i\}) = \begin{cases} -1, \text{ when } \{D_i\} \in S_0 \\ +1, \text{ when } \{D_i\} \in S_1 \\ \begin{cases} +1 \text{ with probability } \varepsilon \\ -1 \text{ with probability } 1-\varepsilon \end{cases}, \text{ when } \{D_i\} \equiv r. \end{cases}$$

  - Solve the parameters of the Fusion Rule: $S_0, S_1, r, \varepsilon.$

Algorithm for Computing the Fusion Rule

- For each element $\gamma_j \in \{-1,1\}^N$, we denote
  - $P(\gamma_j \mid H_0)$ by $P_{\gamma_j|H_0}$
  - $P(\gamma_j \mid H_1)$ by $P_{\gamma_j|H_1}$
- The likelihood ratio, associated with γ_j , is defined as $\Lambda_j = P_{\gamma_j|H_1} \big/ P_{\gamma_j|H_0}$



[W. Chung and K. Yao, "Decision Fusion in Sensor Networks for Spectrum Sensing based on Likelihood Ratio Tests," Proceedings of SPIE, 2008.]

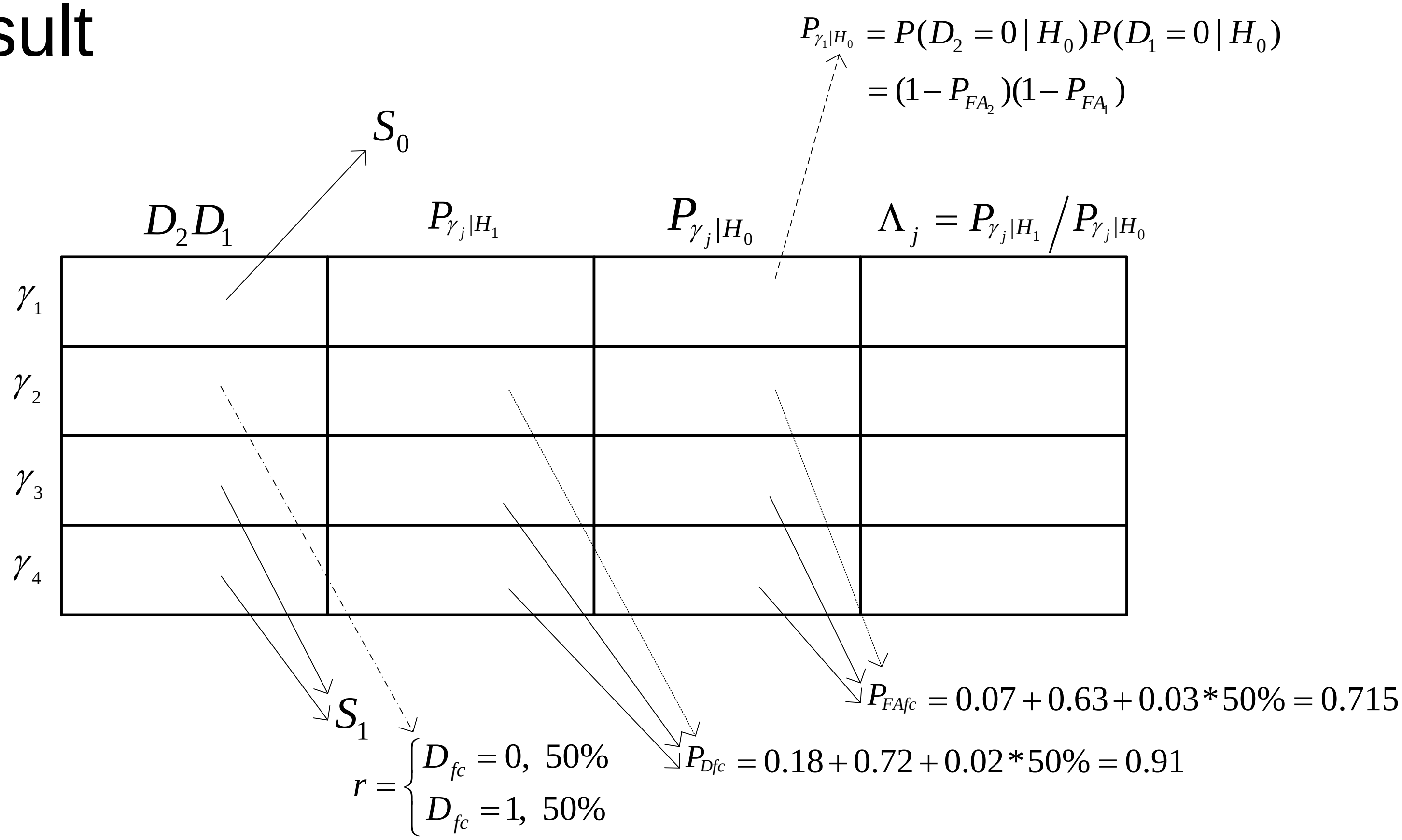
Fusion of Two Sensors

- Two Sensors
  - Operating points

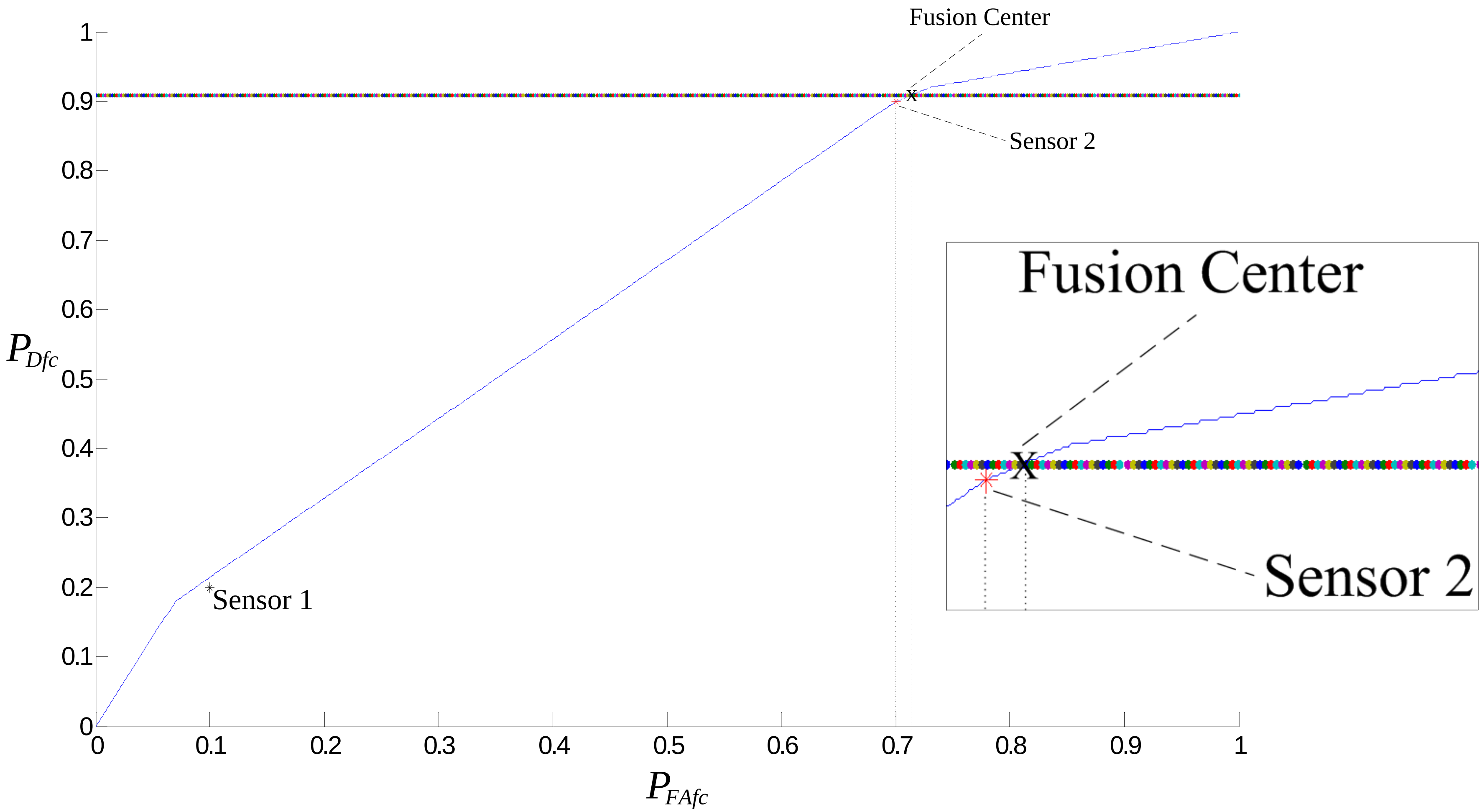
	P_D	P_{FA}
Sensor 1	0.2	0.1
Sensor 2	0.9	0.7

  - Goal: (Lower-Bounded Probability of Detection Criterion) Minimizing P_{FAfc} while P_{Dfc} is lower bounded by 0.91

- Result

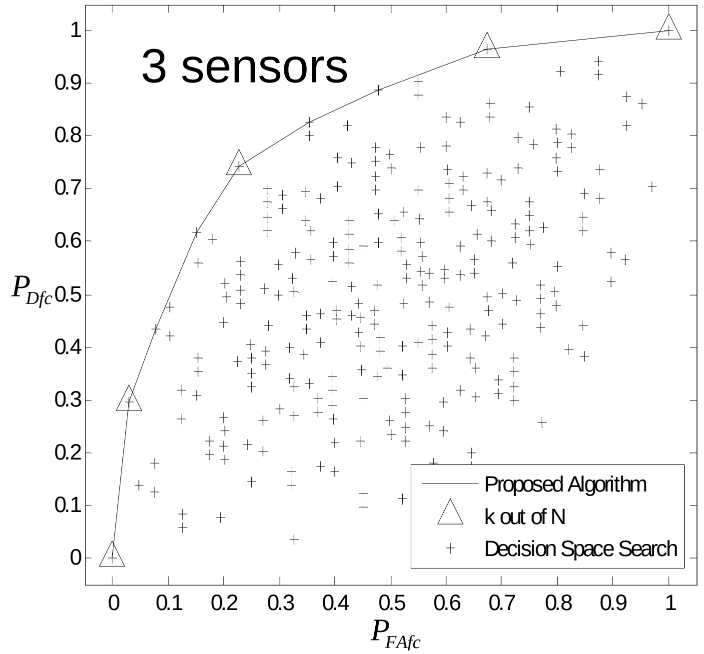


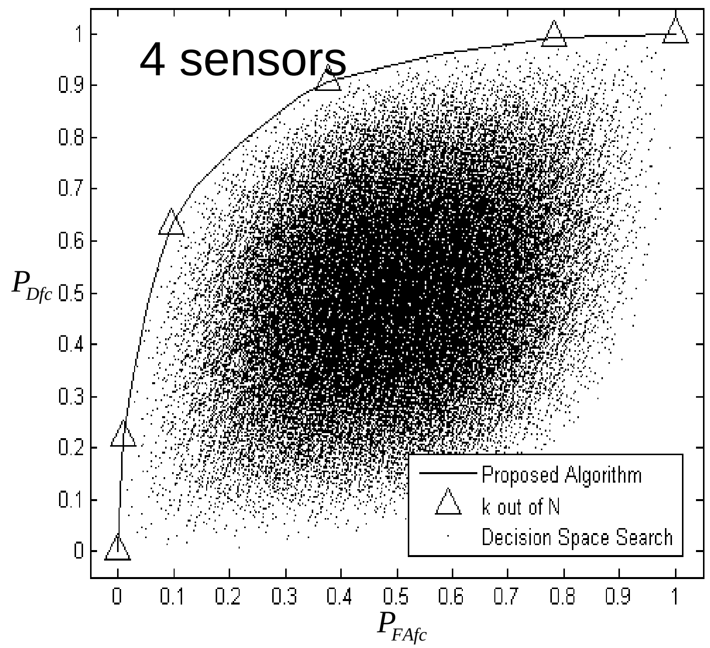
Fusion of Two Sensors



Examples

- Proposed algorithm
- K out of N
  - FC declares H_1 if k or more than k sensors declare H_1. Otherwise, FC declares H_0.
- Decision Space search $f_0 : \{-1,1\}^N \rightarrow \{-1,1\}$
  - All possible combinations of decision fusion rule





Detection under Fading---
Likelihood Ratio Test with Fading Statistics (LRFS)

- Signal Model $\underline{X} = \begin{cases} \underline{N}, H_0 \\ \kappa \underline{S} + \underline{N}, H_1 \end{cases}$
- Fading Gains
  - Rayleigh $p_{Ray}(\kappa;\sigma_R) = \kappa \exp(\frac{-\kappa^2}{2\sigma_R^2}) \Big/ \sigma_R^2$
  - Rician $p_{Rician}(\kappa;\sigma_{Ri},\nu) = \frac{\kappa}{\sigma_{Ri}^2} \exp(\frac{-(\kappa^2+\nu^2)}{2\sigma_{Ri}^2}) I_0[\frac{\kappa\nu}{\sigma_{Ri}^2}]$
- Test Statistic

$$\Lambda_{LRFS}(\underline{X}) = \frac{p(\underline{X}\,|\,H_1)}{p(\underline{X}\,|\,H_0)} = \frac{\int_0^\infty \frac{1}{(2\pi\sigma_n^2)^{m/2}} e^{-(\underline{X}-\kappa\underline{S})^T(\underline{X}-\kappa\underline{S})/(2\sigma_n^2)} p(\kappa)d\kappa}{\frac{1}{(2\pi\sigma_n^2)^{m/2}} e^{-\underline{X}^T\underline{X}/(2\sigma_n^2)}}$$

$$= \int_0^\infty e^{(2\kappa\underline{X}^T\underline{S}-\kappa^2\underline{S}^T\underline{S})/(2\sigma_n^2)} p(\kappa)d\kappa$$

LRFS

- Explicit Expressions of the Test Statistics
  - Rayleigh: $$\Lambda_{LRFS}(\underline{X}) = \left[e^{\frac{\sigma_R^2(\underline{X}^T\underline{S})^2}{2(\sigma_R^2\sigma_n^2\underline{S}^T\underline{S}+\sigma_n^4)}} \sqrt{2\pi}\,\underline{X}^T\underline{S} + 2\sigma_n^2\sqrt{\frac{1}{\sigma_R^2}+\frac{\underline{S}^T\underline{S}}{\sigma_n^2}} + \right.$$
    $$\left. \sqrt{2\pi}\,\underline{X}^T\underline{S}\; Erf\left[\frac{\underline{X}^T\underline{S}}{\sigma_n^2\sqrt{\frac{2}{\sigma_R^2}+\frac{2\underline{S}^T\underline{S}}{\sigma_n^2}}}\right] e^{\frac{\sigma_R^2(\underline{X}^T\underline{S})^2}{2(\sigma_R^2\sigma_n^2\underline{S}^T\underline{S}+\sigma_n^4)}} \right] \Bigg/ 2\left(\sigma_R^2\underline{S}^T\underline{S}+\sigma_n^2\right)\sqrt{\frac{1}{\sigma_R^2}+\frac{\underline{S}^T\underline{S}}{\sigma_n^2}}$$
  - Rician: $$\Lambda_{LRFS}(\underline{X}) = \frac{1}{\sigma_{Ri}^2}\int_0^\infty \kappa e^{\frac{-\nu^2-\kappa^2}{2\sigma_{Ri}^2}+\frac{2\kappa\underline{X}^T\underline{S}-\kappa^2\underline{S}^T\underline{S}}{\sigma_n^2}} I_0[\frac{\kappa\nu}{\sigma_{Ri}^2}]d\kappa$$

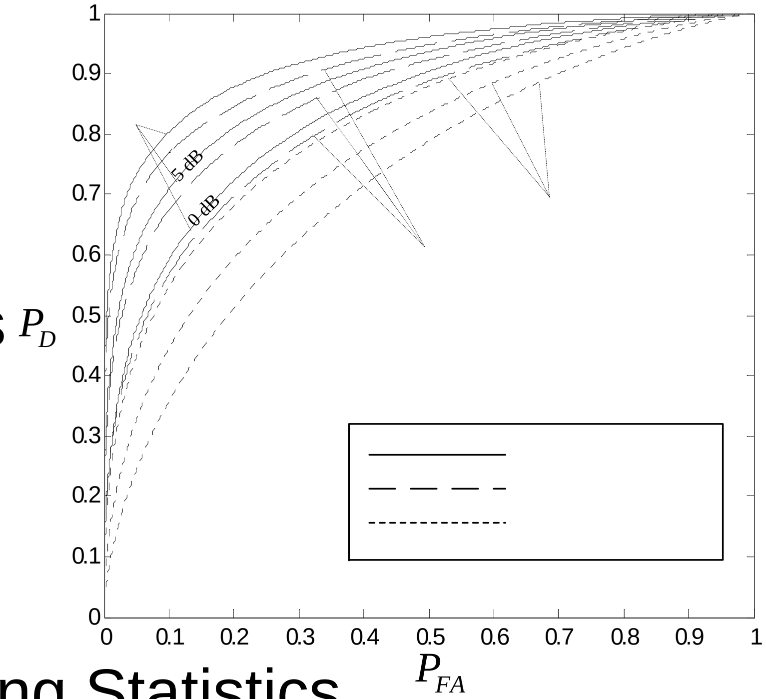
Multi-Sensor Decision Fusion under Fading

- Signal Model: $y_i = \kappa_i u_i + n_i$
- Likelihood Ratio: $\Lambda(\underline{Y}) = p(\underline{Y} \mid H_1) / p(\underline{Y} \mid H_0)$
- Reformulate by fading statistics
  - Under H_1: $p(\underline{Y} \mid H_1) = \sum_{\forall \{u_i\} \in \{-1,+1\}^N} \int p(\underline{Y} \mid \{\kappa_i\}, \{u_i\}, H_1) p(\{u_i\} \mid H_1) p(\{\kappa_i\}) d\kappa_1 d\kappa_2 ... d\kappa_N$
  - Under H_0: $p(\underline{Y} \mid H_0) = \sum_{\forall \{u_i\} \in \{-1,+1\}^N} \int p(\underline{Y} \mid \{\kappa_i\}, \{u_i\}, H_0) p(\{u_i\} \mid H_0) p(\{\kappa_i\}) d\kappa_1 d\kappa_2 ... d\kappa_N$
- Test Statistics

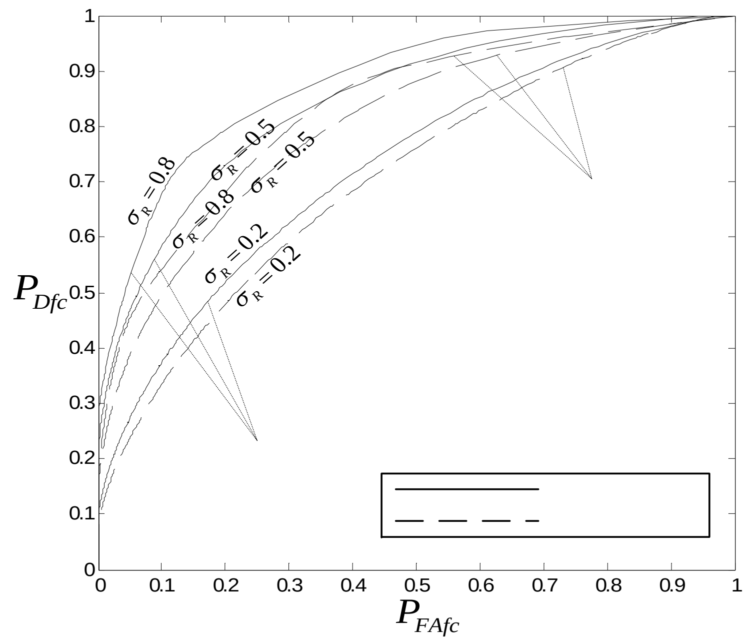
$$\Lambda(\underline{Y}) = \frac{\sum_{\forall \{u_i\} \in \{-1,+1\}^N} \int p(\underline{Y} \mid \{\kappa_i\}, \{u_i\}, H_1) p(\{u_i\} \mid H_1) p(\{\kappa_i\}) d\kappa_1 d\kappa_2 ... d\kappa_N}{\sum_{\forall \{u_i\} \in \{-1,+1\}^N} \int p(\underline{Y} \mid \{\kappa_i\}, \{u_i\}, H_0) p(\{u_i\} \mid H_0) p(\{\kappa_i\}) d\kappa_1 d\kappa_2 ... d\kappa_N} \underset{u_{fc}=-1}{\overset{u_{fc}=+1}{\gtrless}} \lambda_{fc}$$

Numerical Examples

- LRT with Fading Statistics
  - LRFS under Rayleigh
  - LRFS under Rician
  - Matched Filter



- Decision Fusion with Fading Statistics
  - 3 Sensors
  - 2 Sensors



Sequential Likelihood Ratio Test for Spectrum Sensing

- Problem: Spectrum Sensing under Fading
- Goal
  - Faster decision
  - Allow setting both P_{FA} and P_D
- Conventional Approaches
  - Collect fixed amount of data
  - Uncertain signal strength in fading
  - Can we reach faster decision when signal is strong?
- Sequential Likelihood Ratio Test

[W. Chung and K. Yao, “Sequential Likelihood Ratio Test under Incomplete Signal Model for Spectrum Sensing,”]

Formulation

- Signal Model
  - Received $x_t = \begin{cases} n_t, H_0 \\ \alpha_t + n_t, H_1 \end{cases}$
  - Signal (primary user) α_t
  - Signal follows AR model

$$\alpha_t = a_1\alpha_{t-1} + a_2\alpha_{t-2} + a_3\alpha_{t-3} + ... + a_q\alpha_{t-q} + w_t$$

- Received signal follows ARMA

$$x_t = a_1x_{t-1} + a_2x_{t-2} + a_3x_{t-3} + ... + a_qx_{t-q} + w_t + n_t - a_1n_{t-1} - a_2n_{t-2} - ... - a_qn_{t-q}$$

Sequential Decision

- Decision at time t

$$D_t = \begin{cases} H_1 \\ H_0 \\ continue \end{cases}$$

- Log LR

$$\Lambda_t(x_1, x_2, ..., x_t) = \log \frac{P(x_1, x_2, ..., x_t \mid H_1)}{P(x_1, x_2, ..., x_t \mid H_0)}$$

- Sequential decision

$$\lambda_t(x_t \mid x_{t-1}, x_{t-2}, ..., x_{t-p-1}) = \log \frac{P(x_t \mid x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_1)}{P(x_t \mid x_{t-1}, x_{t-2}, ..., x_{t-p-1}, H_0)}$$

$$\Lambda_t = \Lambda_{t-1} + \lambda_t$$

Decision and Thresholds

- Decision

$$D_t = \begin{cases} H_1 & \text{if } \Lambda_t > \log A \\ H_0 & \text{if } \Lambda_t < \log B \\ continue & \text{if } \log A \le \Lambda_t \le \log B \end{cases}$$

- Thresholds

$$A = \frac{1 - P_M}{P_{FA}}$$

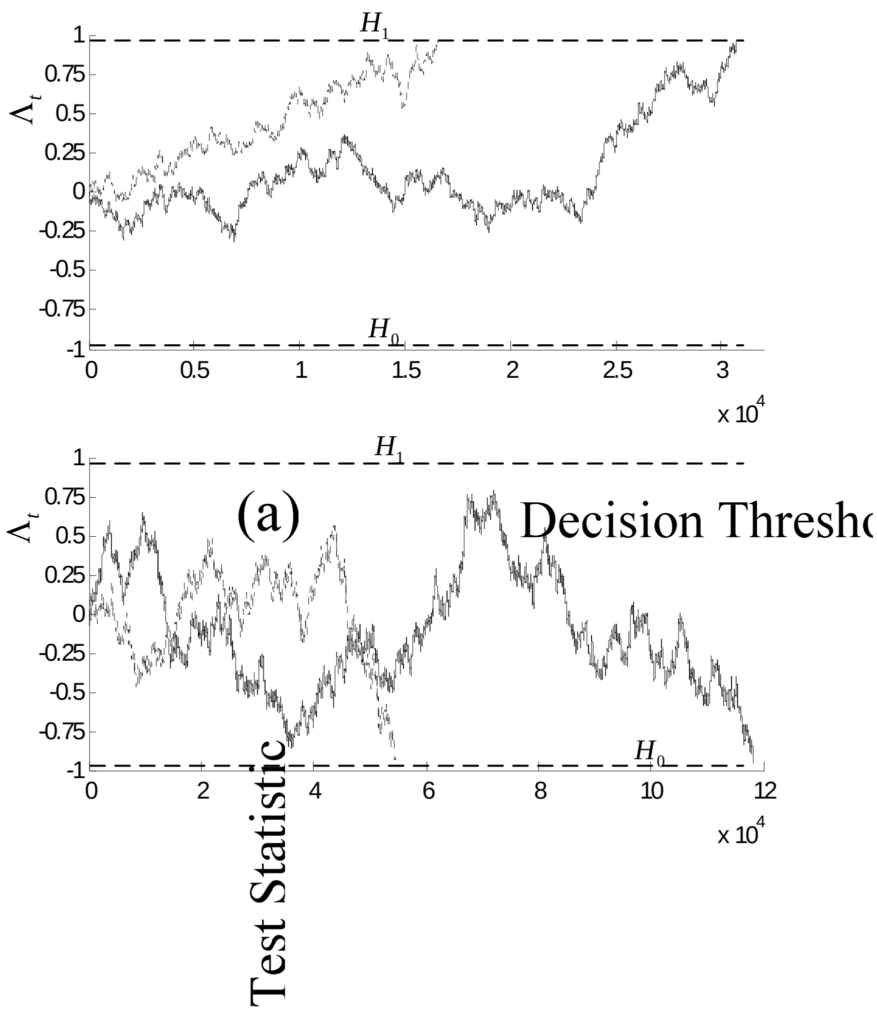
$$B = \frac{P_M}{1 - P_{FA}}$$

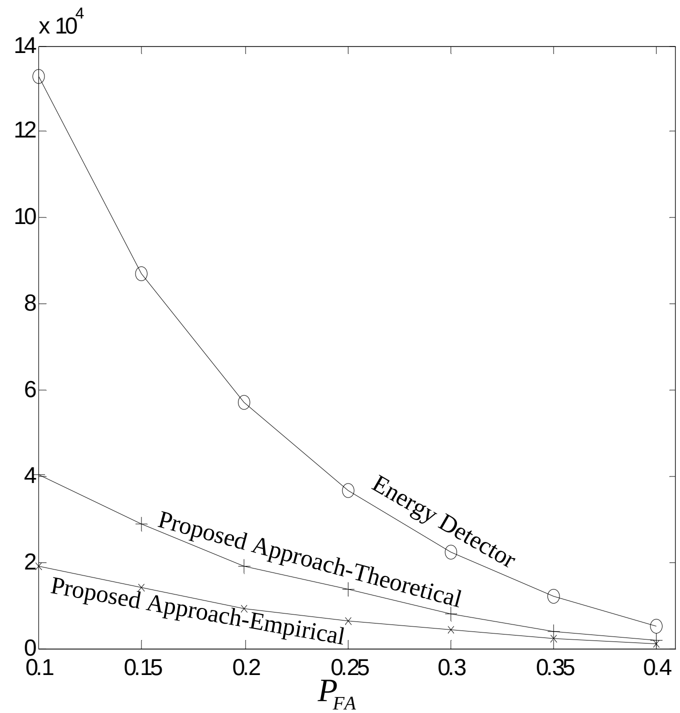
- Expected termination time can be derived as a function of accuracy and SNR

Example 1-Scenario

- SNR uniformly distributed between -20 dB to 10 dB
- Prior 0.5 for H1 and H0
- Jake's ACF

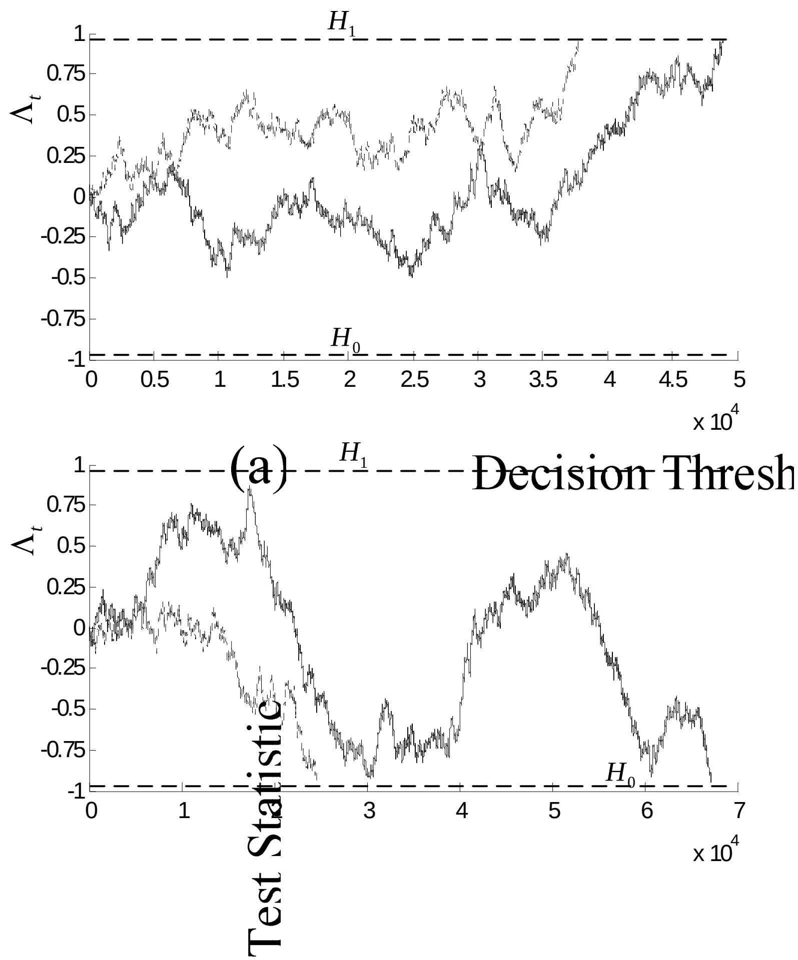
Example 1-results

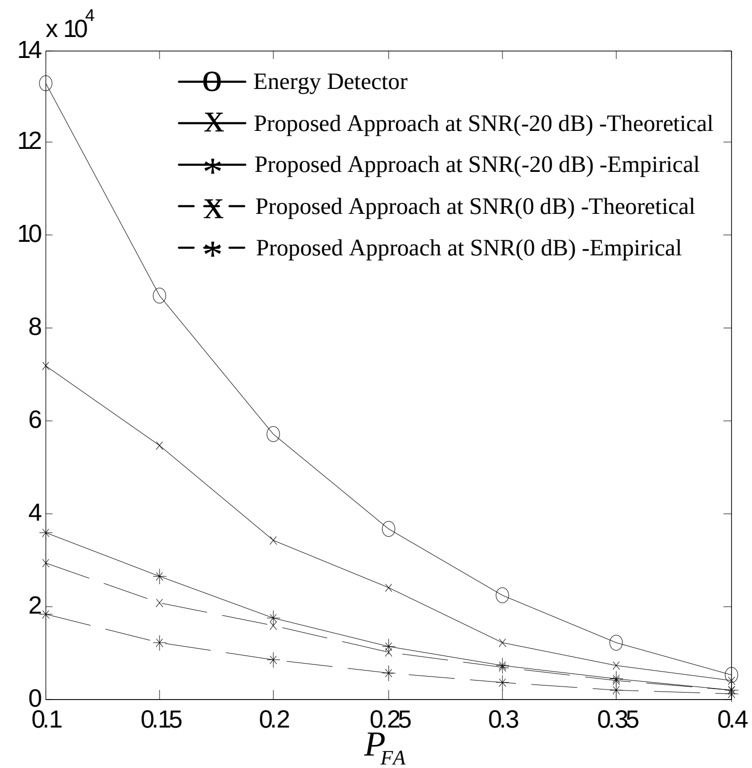




Decision Threshold

Example 2—fixed SNR at -20 dB





Decision Threshold

Summary

- Explicit Algorithms
  - Neyman-Pearson
  - Lowered-Bounded Probability of Detection
- Test Statistics under Rayleigh and Rician Fading.
- Performance Improvements by Incorporating Fading Statistics
  - Single-Sensor Detection under Fading
  - Decision Fusion under Fading
- Sequential LRT
  - Faster decision
  - Allows explicit settings of P_{FA} and P_D

Thank you