Integrated Power Controlled Rate Adaptive Medium Access Control in Wireless Mesh Networks

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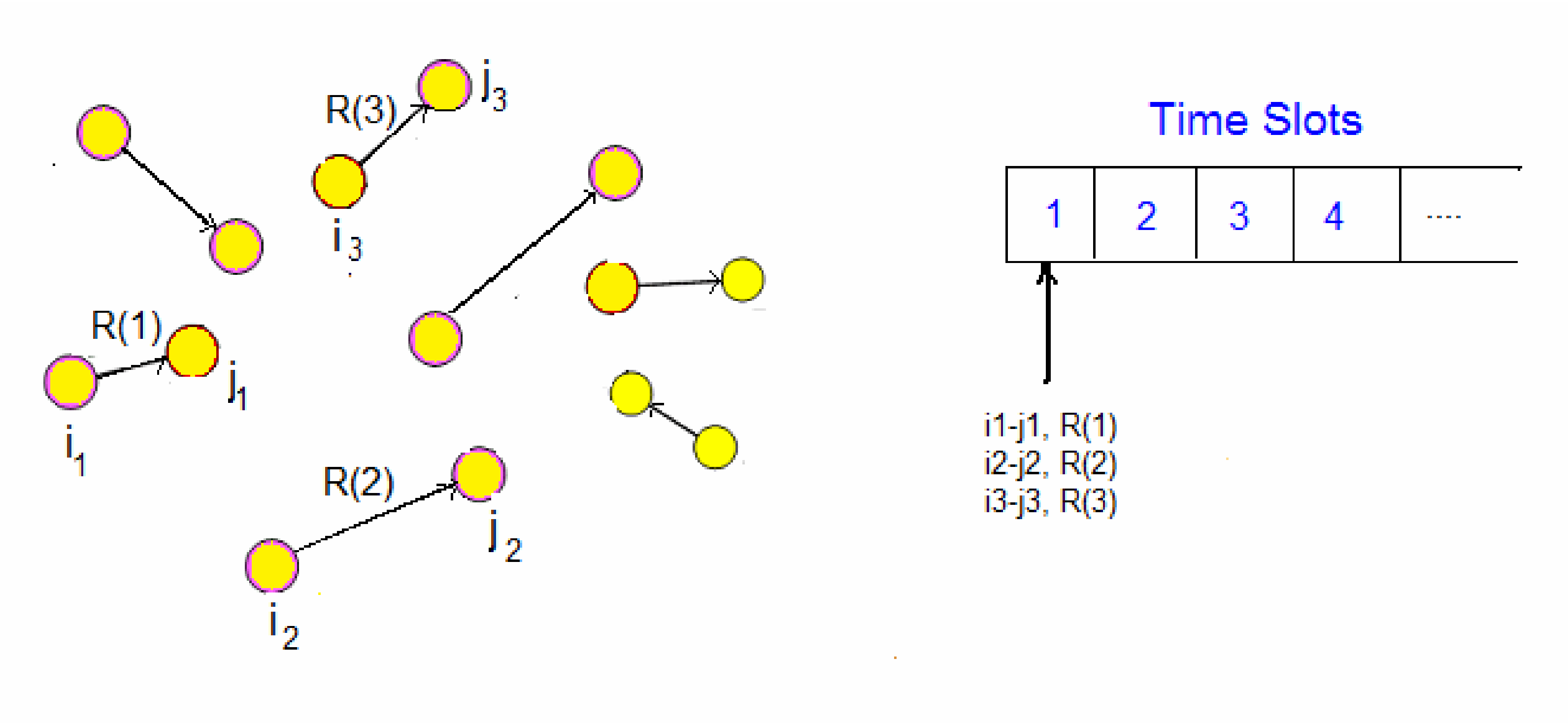
Agenda:

- 1. Problem definition
- 2. Mathematical modeling of the problem
- 3. Heuristic approach to the problem
- 4. Conclusions

Problem Definition

- In a wireless mesh network a scheduling based medium access control (MAC) is used, to allocate time slots to nodal stations to transmit their messages.
- Modern stations use adaptable software defined radios (SDR) which employs a selected modulation/coding scheme (MCS), to operate at a corresponding transmission rate (R). In addition, it is desirable, at the same time to adjust the level of station's transmit power across the allocated time slot.
- It is desirable to set a schedule that will achieve a high spatial reuse factor, so that several simultaneous transmissions can be successfully executed while meeting the required signal-to-interference-plus-noise ratio levels at intended receivers.

Spatial TDMA



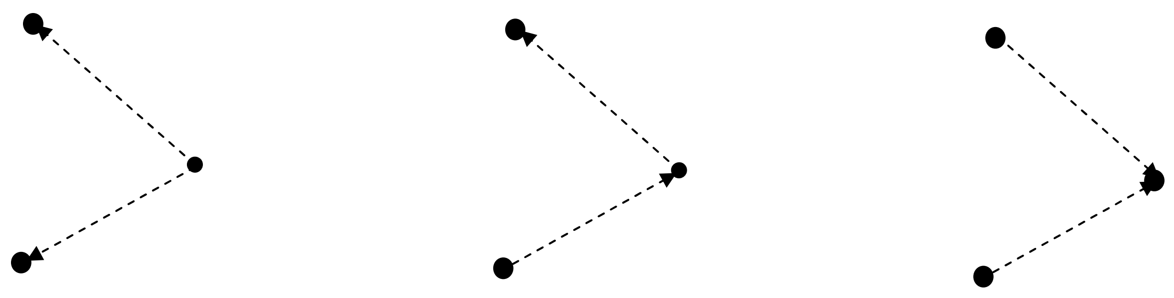
Objective

- Assume the routing has already taken place and we have the load of *Kij* packets to be transmitted across each designated link $l_{ij} \in L$.
- We have developed a scheme to design a time frame with minimum schedule length in spatial TDMA wireless networks based on the optimal joint scheduling of link transmissions and allocation of transmit power levels and data rates, while meeting required signal-to-interference-plus-noise ratio levels at intended receivers.
- In addition, we have minimized the total power consumption of each node for the prescribed optimum schedule and the corresponding transmit rates.

Mathematical modeling of the problem

- *Gij* is the propagation gain from node *i* to node *j.*
- η_j is the thermal noise power at receiver *j.*
- $\gamma(R(k))$ is the SINR threshold at rate *R(k).*
- Constarints:

unicasting half-duplexing receptivity



Feasible Transmission

A transmission scenario

$$S(t) = \{ i_1 \xrightarrow{R(1)} j_1, i_2 \xrightarrow{R(2)} j_2, \Lambda\ , i_M \xrightarrow{R(M)} j_M \},$$

is feasible if there is a power vector

$$P(t) = (P_{i_1 j_1}^{(t)}, P_{i_2 j_2}^{(t)}, \ldots, P_{i_M j_M}^{(t)})$$

under which all the transmissions of *S(t)* at the designated data rates are successful (SINR requirement is met).

$$G_{i_k j_k} P_{i_k j_k}^{(t)} / (\eta_j + \sum_{\substack{z=1 \\ z \neq k}}^{M} G_{i_z j_k} P_{i_z j_z}^{(t)}) \geq \gamma(R(k)), \quad k = 1, 2, \mathrm{L}\ , M,$$

Mixed Linear Integer Programming Formulation

The decision variables are $X_{ijh}^{(t)}$ and $P_{ij}^{(t)}$

$$X_{ijh}^{(t)} = \begin{cases} 1, & if\ \ packet\ is\ transmitted\ in\ slot\ t\ over\ l_{ij}\ under\ rate\ r_h \\ 0, & otherwise \end{cases}$$

$$(i,j) \in L, h = 1,2,\ldots,m, t = 1,2,\ldots,T_{\max}.$$

at rate *rk*, we can transmit 2^{k-1} packets/time slot

$$T_{\max} = \sum_{\{(i,j):l_{ij} \in L\}} K_{ij}$$ (1 transmission/slot at lowest rate)

Mixed Linear Integer Programming Formulation

minimize $$Z(X,P)=\sum_{t=1}^{T_{\max}}\sum_{(i,j)\in L}(c_t\sum_{h=1}^{m}X_{ijh}^{(t)}+\varepsilon P_{ij}^{(t)})$$

s.t.

$$\sum_{t=1}^{T_{\max}}\sum_{h=1}^{m}2^{k-1}X_{ijh}^{(t)}\geq K_{ij},\quad (i,j)\in L \qquad (1)$$

$$\sum_{h=1}^{m}(\sum_{(i,j)\in L}X_{ijh}^{(t)}+\sum_{(j.f)\in L}X_{jfh}^{(t)})\leq 1,\quad j\in N^{Tx}\cup N^{Rx} \qquad (2)$$

$$G_{ij}P_{ij}^{(t)}-\gamma(r_h)\sum_{(f,s)\in L-(i,j)}G_{fj}P_{fs}^{(t)}-\gamma(r_h)\eta_j\geq\theta(X_{ijh}^{(t)}-1) \qquad (3)$$

$$0\leq P_{ij}^{(t)}\leq P_{\max}, \qquad (4)$$

$$X_{ijh}^{(t)}=0\ or\ 1, \qquad (5)$$

ε is a sufficiently small positive number,

θ is a sufficiently large positive number, and

$$c_t = t.|L|.c_{t-1},\quad t=2,\ldots,T_{\max}$$ $$c_1=1$$

Theorem 1: Every optimum solution of the MILP formulation is a strongly Pareto optimal power vector with respect to the transmission scenario at each time slot.

- Let the optimum solution be:

$$S^*(t) = \{ i_1 \xrightarrow{R(1)} j_1, i_2 \xrightarrow{R(2)} j_2, \Lambda\ , i_M \xrightarrow{R(M)} j_M \}$$

$$P^*(t) = (P_{i_1 j_1}^{*(t)}, P_{i_2 j_2}^{*(t)}, \ldots, P_{i_M j_M}^{*(t)})$$

- M constraints from (3) are non-redundant, so we write (3) as MxM linear inequalities:

$$G_{i_k j_k} P_{i_k j_k}^{*(t)} - \gamma(R(k)) \sum_{\substack{z=1 \\ z \neq k}}^{M} G_{i_z j_k} P_{i_z j_z}^{*(t)} \geq \gamma(R(k)) \eta_{j_k}, \ k = 1, \mathrm{L}\ , M.$$

- We assume G is nonsingular so $P^*(t)$ meets the M inequality with equality. Therefore $P^*(t)$ is strongly Pareto optimal.

Theorem 2: The integrated power controlled rate adaptive scheduling problem (IPRS) is NP-complete.

- It is known that the edge coloring (EC) problem is NP-complete. We can reduce the IPRS problem to an instance of the EC problem.
- The edge coloring problem is to determine, whether all edges of $G = (V, E)$ can be colored by less than or equal to λ colors.
- Assume every link in *L* requires one time slot per timeframe. There is a one-to-one correspondence between every edge in *G* and every communication link in the IPRS problem,.
- All edges of graph *G* can be colored by λ colors iff there is a (feasible) frame for the associated IPRS problem whose length is equal to λ (same color means same time slot).

Heuristic Algorithm

First we create the Extended Interference Graph (EIG) based on pair wise transmission feasibility.

- The *Extended Interference Graph* is an undirected weighted graph *G(V,E,w)*. Every vertex in *V* is represented by an ordered triplet (i, j, r_h) .
- The set of vertices $\{(i, j, r_h) \mid h = 1,2,\Lambda\ , m'\}$ are in *V* if and only if the communication link l_{ij} belongs to *L*, where the value of m' is derived based on the following relation:

$$m' = \begin{cases} m, & k_{ij} > r_m / r_1 \text{ and } G_{ij} P_{\max} / \eta_j \geq \gamma(r_m) \\ h+1, & r_h / r_1 < K_{ij} \leq r_{h+1} / r_1 \text{ and } G_{ij} P_{\max} / \eta_j \geq \gamma(r_{h+1}), h < m \\ 0, & \text{otherwise} \end{cases}$$

- The weight of vertex (i, j, r_h) is set to 2^{h-1} which is the number of packets that can be transmitted over link l_{ij} in a time slot.

Extended Interference Graph (EIG)

Vertices *(i_1,j_1,R(1))* and *(i_2, j_2, R(2))* are connected to each other by an edge in graph *G* iff:

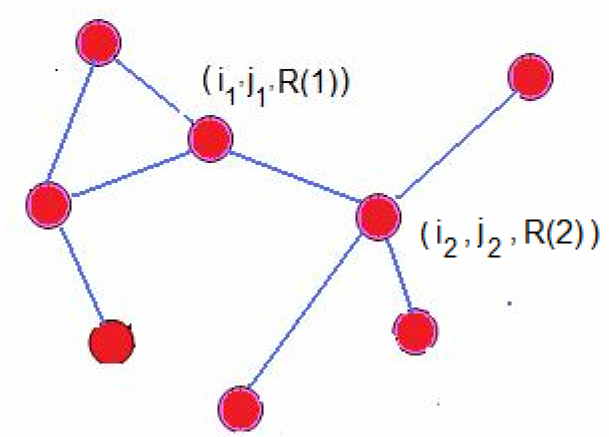
1.Nodes i_1,j_1,i_2, and j_2 are not mutually distinct.

2. Transmission scenario $S(t) = \{i_1 \xrightarrow{R(1)} j_1, i_2 \xrightarrow{R(2)} j_2\}$ is not feasible.

Iff:

$$\begin{cases} G_{i_1 j_1} P^{(t)}_{i_1 j_1} - \gamma(R(1)) G_{i_2 j_1} P^{(t)}_{i_2 j_2} \geq \gamma(R(1)) \eta_{j_1} \\ -\gamma(R(2)) G_{i_1 j_2} P^{(t)}_{i_1 j_1} + G_{i_2 j_2} P^{(t)}_{i_2 j_2} \geq \gamma(R(2)) \eta_{j_2} \end{cases}$$

Doesn't have a solution



The Integrated Power Controlled Rate adaptive Scheduling Algorithm

Step 1. We use a Greedy algorithm to find a maximal weighted independent set in the EIG. At each iteration of the algorithm one vertex is selected for inclusion into the weighted independent set, then the selected vertex and all its neighbors are removed from the graph. This process is repeated until the set of vertices of the residual graph is null.

The Greedy algorithm has two variations:

- 1. at the *i*-th iteration, selects vertex *v* such that

$$w(v)/(d_{Gi}(v)+1) = \max_{u \in V(Gi)} \{w(u)/(d_{Gi}(u)+1)\}$$

$d_{Gi}(u)$ is the degree of vertex u.

- 2. at the *i*-th iteration, selects vertex *v* such that

$$w(v)/\sum_{y \in N(Gi,v)} w(y) = \max_{u \in V(Gi)} \{w(u)/\sum_{y \in N(Gi,u)} w(y)\}$$

Step 2 (Feasibility stage)

- In Step 1 we found a maximal weighted independent set:

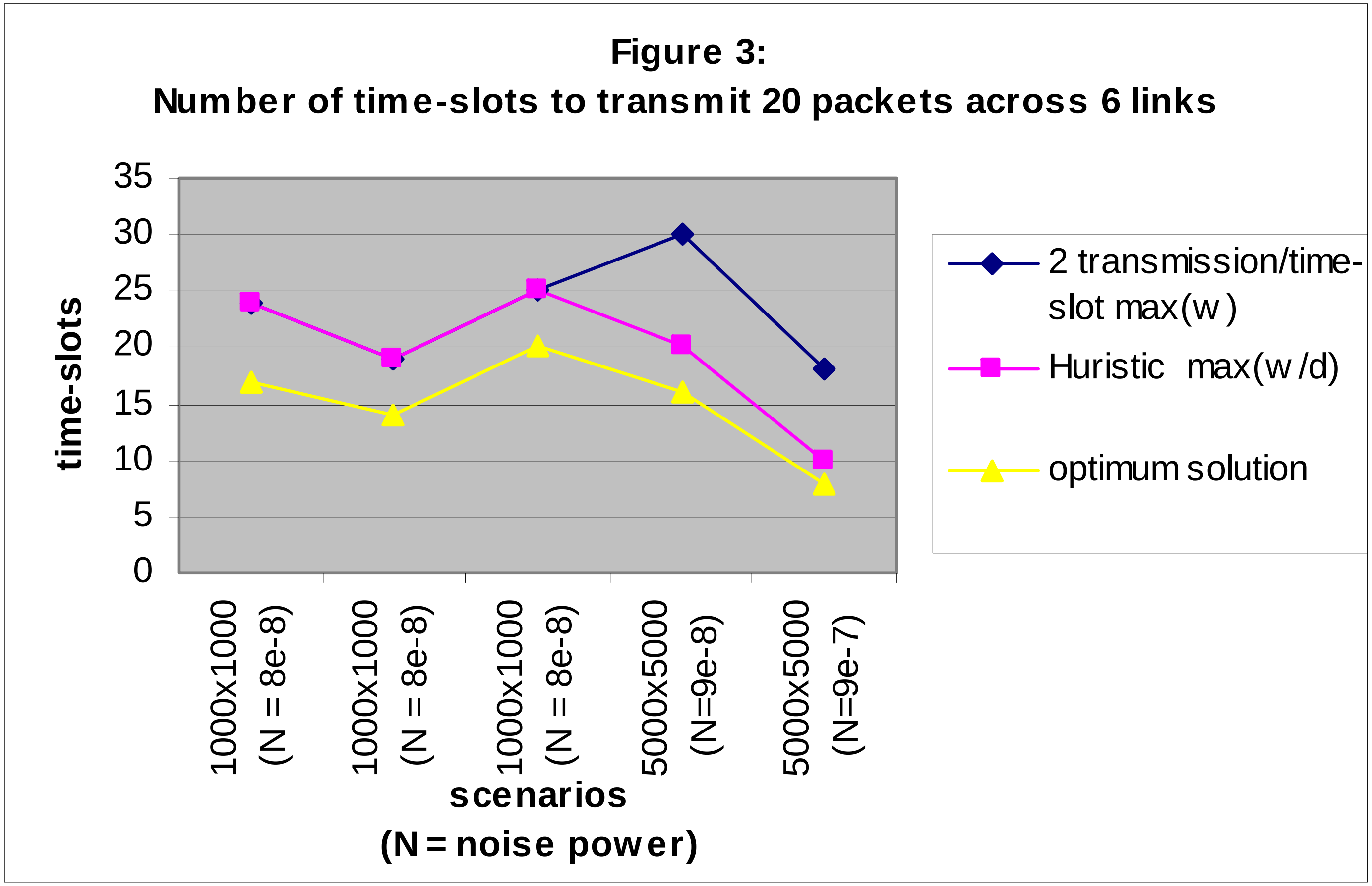
$$S(1) = \{i_1 \xrightarrow{R(1)} j_1, i_2 \xrightarrow{R(2)} j_2, \Lambda\ , i_M \xrightarrow{R(M)} j_M\}$$

- This set is not necessarily a feasible transmission scenario. We need to check if the set is feasible and if not, remove the node that causes the most interference to others and repeat until we get a feasible transmission scenario.
- We modify the *Stepwise Maximum Interference Removal Algorithm* (SMIRA) that was originally introduced for the downlink connection removal of the cellular radio systems. In SMIRA, at every step a transmission is removed that causes most interference to other receivers. This process iterates until the resulting transmission scenario is feasible.

Step 3 (Super Maximality stage)

- The feasible transmission scenario induced by the Feasibility Stage is not necessarily maximal with respect to the Extended Interference Graph. The maximality is ensured by iteratively considering the other remaining transmissions for possible inclusion in the transmission scenario.
- The second objective of this stage is to guarantee that the resulting feasible transmission scenario is super-maximal. This is done iteratively by assigning the maximum possible rate to each transmission while maintaining the feasibility of the transmission scenario.

Numerical Illustration



Conclusions

- We model the optimum cross-layer scheduling problem as a Mixed Integer Linear Program (MILP) and solve it for a system with small number of links and data rate levels.
- Showed that the optimum method is NP hard. Hence, we present a heuristic algorithms for solving the problem.
- For the cases that we analyzed, the performance of the heuristic method is in the 75 percentile of the optimum solution. We note that for systems that offer a high special reuse factor, our heuristic algorithm results in a schedule length that is close to the optimum solution.

References

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