Spectrum Sensing by Cognitive Radios at Very Low SNR

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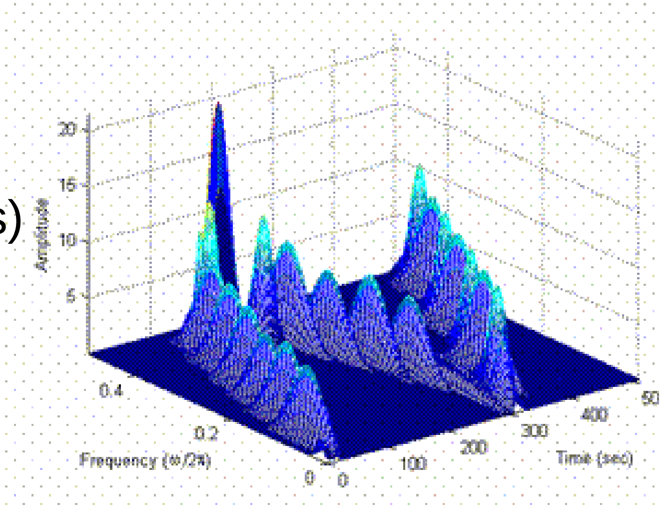
Introduction and Motivation

- FCC is developing rules for dynamic use of radio spectrum
  - Radio spectrum is currently under-utilized
  - Empty TV bands
- Cognitive Radios (IEEE 802.22)
  - Sense spectrum and identify **White Space** for dynamic use
  - Not allowed to interfere with licensed transmissions (TV & Wireless Mic.)
- Market Opportunities
  - Google, Microsoft, and Motorola (Wi-Fi)
  - Qualcomm (improve capacity of cellular networks)
  - Philips, Dell, et. al. (new wireless devices)
- Technical Challenges: **Spectrum Sensing**
  - Detect signals at very low SNR **(-10 to -20 dB)**
  - Quick sensing (less than 2 seconds)
  - Multipath fading and shadowing: hidden node problem



Source: Internet

Outline

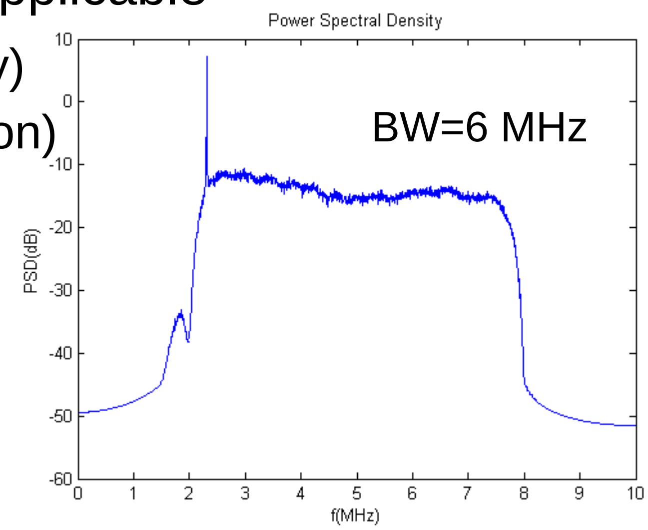
- Overview of Spectrum Sensing Techniques
- Spectral Feature Detection
  - An asymptotically optimal detector
  - Equivalent to the likelihood ratio test (LRT) at very low SNR
- Spectral Feature Selection
  - Optimization Analysis (non-convex problem)
- Threshold Estimation
  - Moment Method
- Simulation
  - Detects primary TV signals (NTSC and ATSC) at SNR around -20 dB

Spectrum Sensing Techniques

- Energy Detection (Radio-Meter)
  - Non-coherent detection (synchronization not needed)
  - Optimal if only noise power is known
- Matched Filtering
  - Coherent detection
  - Synchronization needed (if a deterministic sequence is known)
  - Performance better than energy detection
- Feature Detection
  - Non-coherent method (synchronization not needed)
  - Cyclostationarity (exploits inherent periodicities in first- and second-order statistics, assuming that the modulation scheme is known)

Spectrum Sensing at very low SNR

- Traditional detection techniques are no longer applicable
  - Energy detection (failure due to noise uncertainty)
  - Matched filtering (difficult to obtain synchronization)
  - Cyclo-stationary detector (too complex)



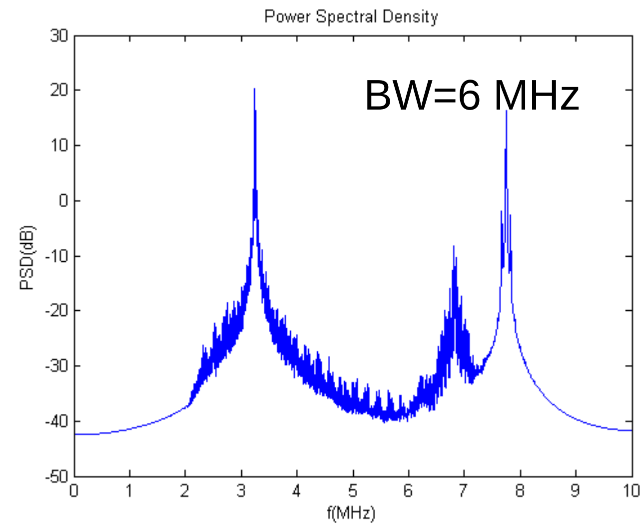
- Our strategy: Spectral Feature Detection
  - Exploits spectral features of primary signals

ATSC (Digital TV standard)

- Pilot tone at 309 KHz above band edge
- 8VSB Modulation

NTSC (Analog TV standard)

- Video carrier (AM), 1.25 MHz above band edge
- Color carrier (QAM), 3.58 MHz above video carrier
- Audio Carrier (FM), 4.5 MHz above video carrier



Spectral Feature Detector

- Extract the unique spectral features of a specific TV signal
- Use the *a priori* known feature to match the signal under detection
- Spectrum sensing is modeled as a binary hypothesis test:

$$\begin{aligned} \mathcal{H}_0 :& \quad y(l) = v(l) \\ \mathcal{H}_1 :& \quad y(l) = x(l) + v(l) \qquad v(l) \sim \mathcal{CN}(0, \sigma_v^2) \end{aligned}$$

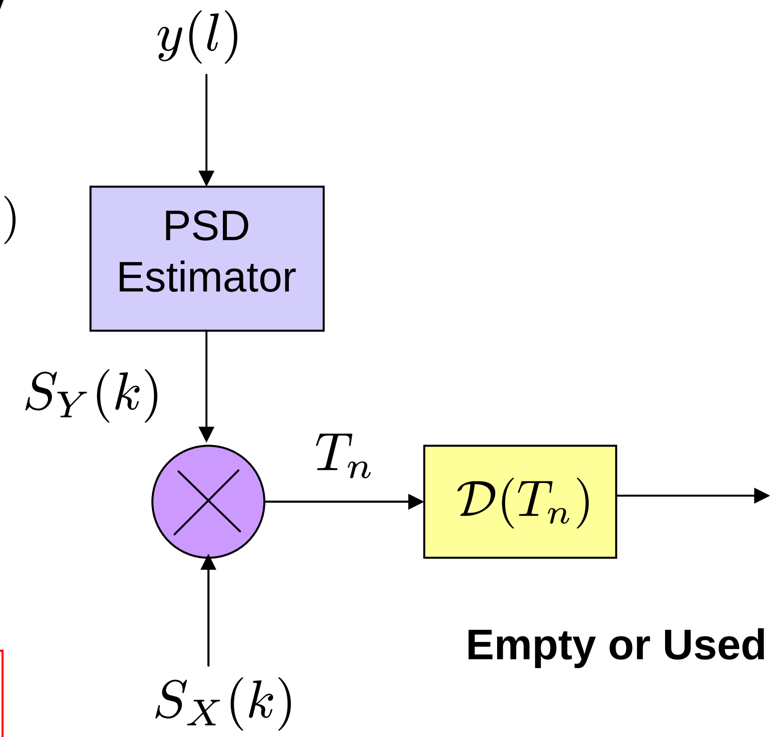
- PSD is estimated via *n*-point DFT (periodogram)

$$\begin{aligned} \mathcal{H}_0 :& \quad S_Y^{(n)}(k) = \hat{\sigma}_v^2 \\ \mathcal{H}_1 :& \quad S_Y^{(n)}(k) = S_X^{(n)}(k) + \hat{\sigma}_v^2 \end{aligned}$$

- Decision rule

$$T_n = \frac{1}{n} \sum_{k=0}^{n-1} S_Y^{(n)}(k) S_X^{(n)}(k) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} t_n$$

where $S_X^{(n)}(k)$ is the pre-stored spectral feature (e.g., ATSC or NTSC).



Asymptotical Optimality

- Assumption: TV signals $\mathbf{x} = [x(0), x(1), \ldots, x(n-1)]^T$ are a second-order stationary stochastic (Gaussian) process

$$\begin{aligned} \mathcal{H}_0: \quad & \mathbf{y} \sim \mathcal{CN}\left(\mathbf{0}, \sigma_v^2 \mathbf{I}\right) \\ \mathcal{H}_1: \quad & \mathbf{y} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{\Sigma}_n + \sigma_v^2 \mathbf{I}\right) \end{aligned}$$

  where $\mathbf{\Sigma}_n = \mathbb{E}\left(\mathbf{x}\mathbf{x}^T\right)$ is the covariance matrix.

- Optimal likelihood ratio test (LRT) at very low SNR is given by

$$T_{\mathrm{LRT},n} = \frac{1}{n} \mathbf{y}^T \mathbf{\Sigma}_n \mathbf{y} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} t_{\mathrm{LRT}}$$

(Proof in backup slides)

- T_n is asymptotically equivalent to $T_{\mathrm{LRT},n}$ for large block size *n, i.e.,*

$$\lim_{n \to \infty} \left| T_n - T_{\mathrm{LRT},n} \right| = 0$$

(Proof in backup slides)

The spectral feature detector is asymptotically optimal at very low SNR and for large block size *n*.

Feature Selection

- What's a good spectral feature for detection? (useful insights for signal design)
- Detector (mean) under hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$

$$T_{n,0} = \frac{1}{n}\sigma_v^2 \sum_{k=0}^{n-1} S_X(k) = \sigma_v^2 P_x$$

$$T_{n,1} = \sigma_v^2 P_x + \frac{1}{n}\sum_{k=0}^{n-1} S_X^2(k)$$

Primary transmit power (constant)

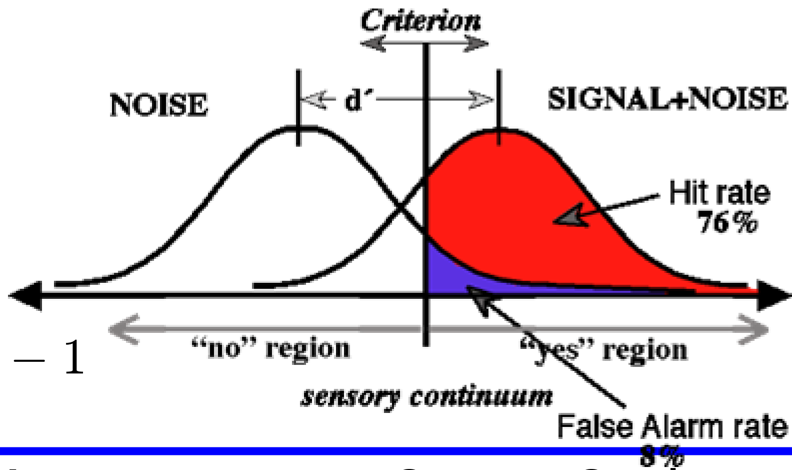
$$P_x = \frac{1}{n}\sum_{k=0}^{n-1} S_X(k)$$

- The difference determines the detection performance
- How to choose $\{S_X(k)\}$ to maximize the difference?

$$\text{maximize} \quad T_{n,1} - T_{n,0}$$

$$\text{s.t.} \quad \frac{1}{n}\sum_{k=0}^{n-1} S_X(k) = P_x$$

$$S_X(k) \geq 0, \quad k = 0, 1, \ldots, n-1$$



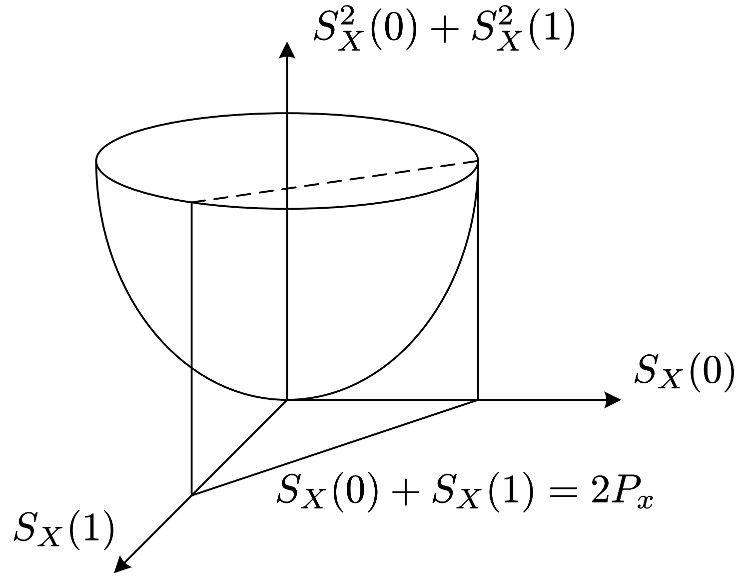
What Feature is Best (or Worst)?

- Maximizes a convex function over a hyper-plane (non-convex)

$$\max \sum_{k=0}^{n-1} S_X^2(k)$$

$$\text{s.t.} \sum_{k=0}^{n-1} S_X(k) = nP_x$$

$$S_X(k) \geq 0, \quad k = 0, 1, \ldots, n-1$$



- Optimal: all transmit power concentrated in a frequency bin (sinusoid)

$$\begin{cases} S_X(j) = nP_x, & j \in \{0, 1, \ldots, n-1\} \\ S_X(k) = 0, & 0 \leq k \leq n-1, \text{ and } k \neq j \end{cases}$$

$$T_{n,1} - T_{n,0} = n^2 P_x^2$$

- Worst: transmit power evenly distributed (white Gaussian like)

$$S_X(k) = P_x, \qquad k = 0, 1, \ldots, n-1$$

$$T_{n,1} - T_{n,0} = nP_x^2$$

Sharp spectral feature provides better detection performance.

Threshold Estimation

- Suppose $\mathcal{H}_0$ is true (absence of primary signals)
- Periodogram is *chi-squared* distributed, i.e., $\hat{S}_Y(k) \sim \chi_2^2$
- The detector is a linear combination of chi-squared R.V.

$$T_n = \frac{1}{n}\sum_{k=0}^{n-1} \hat{S}_Y(k) S_X(k) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} t_n$$

What is its distribution?

- Moment Method: calculate the logarithm of its moment generating function

$$g(\tau) = \log \mathbb{E}\left(e^{\tau T_L}\right) = \sum_{k=0}^{n-1} \log \mathbb{E}\left(e^{\tau \hat{S}_Y(k) S_X(k)/n}\right) = -\sum_{k=0}^{n-1} \log\left(1 - \tau\theta_k\right)$$

- Cumulants

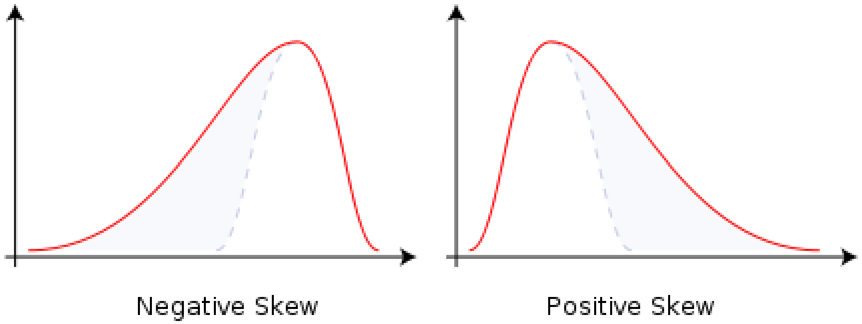
$$c_n = g^{(n)}(0) = (n-1)! \sum_{k=0}^{n-1} \theta_k^n \qquad \theta_k = \frac{\sigma_v^2 S_X(k)}{n}$$

Chi-Square Approximation

- High-order statistics

c_1 *Mean*

c_2 *Variance*



$$S_T = \frac{c_3}{\sqrt{c_2^3}}$$ *Skewness* (measure of asymmetry)

$$K_T = \frac{c_4}{c_2^2}$$ *Kurtosis* (measure of peakedness)

- Non-central *chi-squared* distribution $\chi_\eta^2(\delta)$

$$S_{\chi^2} = \frac{2\sqrt{2}(\eta + 3\delta)}{(\eta + 2\delta)^{3/2}} \qquad K_{\chi^2} = \frac{12(\eta + 4\delta)}{(\eta + 2\delta)^2}$$

- Approximation (solved for η and δ)

$$\begin{aligned} \text{minimize} \quad & \Delta_K = \left| K_T - K_{\chi^2} \right| \\ \text{s.t.} \quad & S_T = S_{\chi^2} \end{aligned}$$

Prob. False Alarm

- Prob. false alarm is the tail prob.:

$$\begin{aligned}\Pr(T_L > t) &= \Pr\left(\frac{T_L - c_1}{\sqrt{c_2}} > t^*\right)\\ &\approx \Pr\left(\frac{\chi^2_\eta(\delta) - \mu_{\chi^2}}{\sigma_{\chi^2}} > t^*\right)\\ &= \Pr\left(\chi^2_\eta(\delta) > t^*\sigma_{\chi^2} + \mu_{\chi^2}\right)\end{aligned}$$

$$t^* = \frac{t - c_1}{\sqrt{c_2}}$$

$$\mu_{\chi^2} = \eta + \delta$$

$$\sigma_{\chi^2} = \sqrt{2(\eta + 2\delta)}$$

- Cumulative distribution function (CDF) of non-central chi-square:

$$P(x;\eta,\delta) = e^{-\delta/2}\sum_{k=0}^{\infty}\frac{(\delta/2)^k}{k!}F(x;\eta+2k)$$

- CDF of central chi-square:

$$F(x;\eta) = \frac{\gamma(\eta/2, x/2)}{\Gamma(\eta/2)}$$

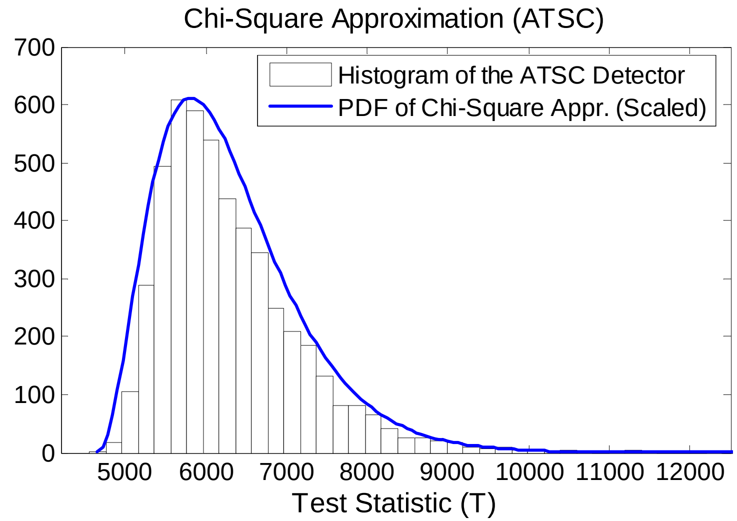
Incomplete gamma function

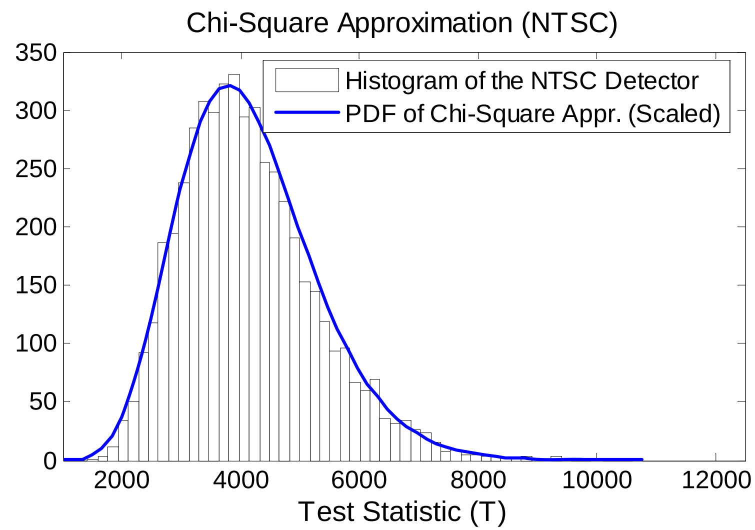
$$\gamma(z,x) := \int_0^x \tau^{z-1}e^{-\tau}\,d\tau$$

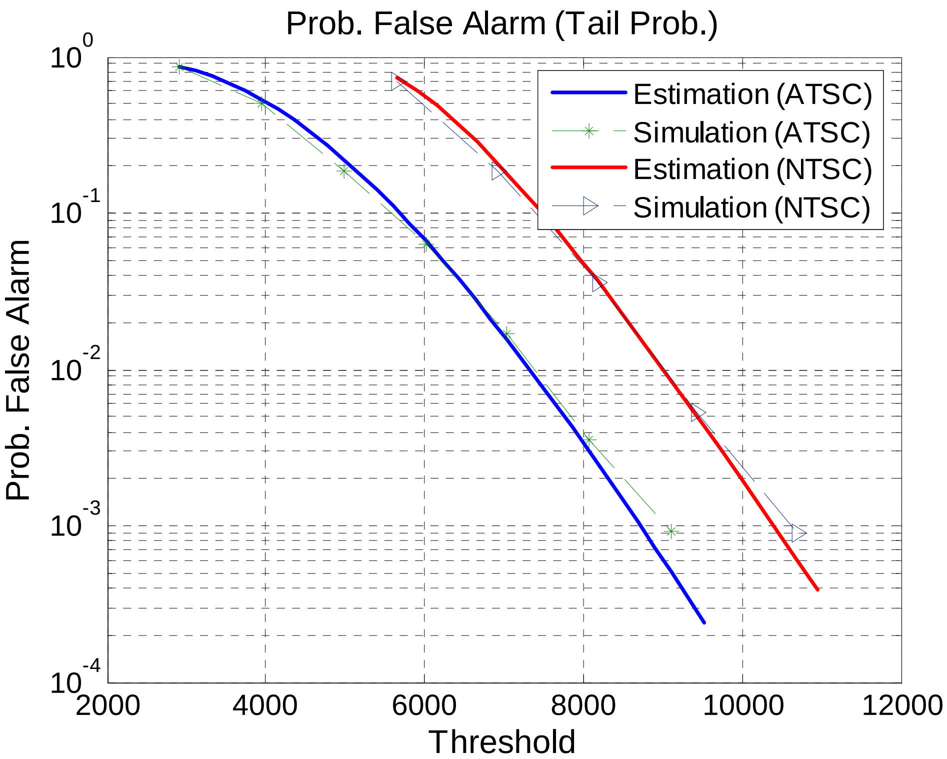
Gamma function

$$\Gamma(z) := \int_0^\infty \tau^{z-1}e^{-\tau}\,d\tau$$

Numerical Results







Threshold of spectral feature detector can be estimated numerically.

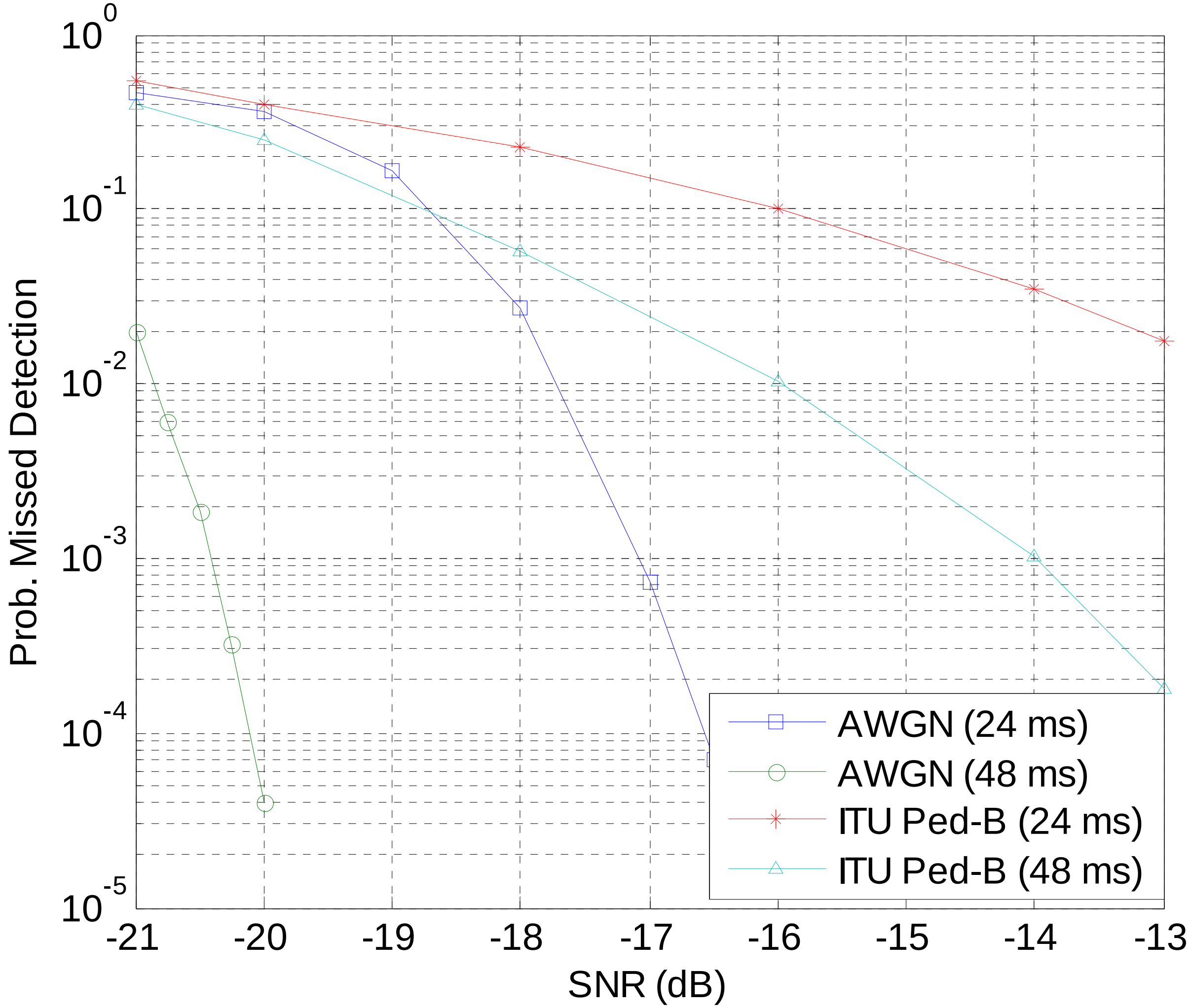
Computational Complexity

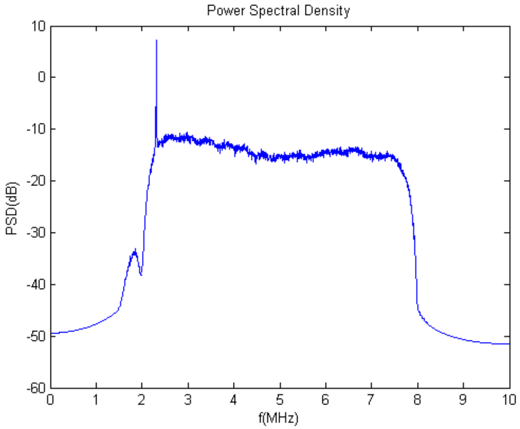
- Likelihood Ratio Test (LRT)
  - Computing covariance matrix (off-line): $O(n^2)$
  - Quadratic detector (on-line): $O(n^2)$
- Eigenvalue Based Spectrum Sensing (Zeng, etc.)
  - Computing covariance matrix (on-line): $O(n^2)$
  - Eigen-decomposition (on-line): $O(n^3)$
- Cyclo-stationary Detection
  - Computing the auto-correlation function (on-line): $O(n^2)$
- Spectral Feature Detector
  - Calculating periodogram (using FFT) : $O(n \log_2 n)$
  - Correlation (on-line): $O(n)$

The spectral feature detector has less computational complexity.

Simulation (Digital TV - ATSC)

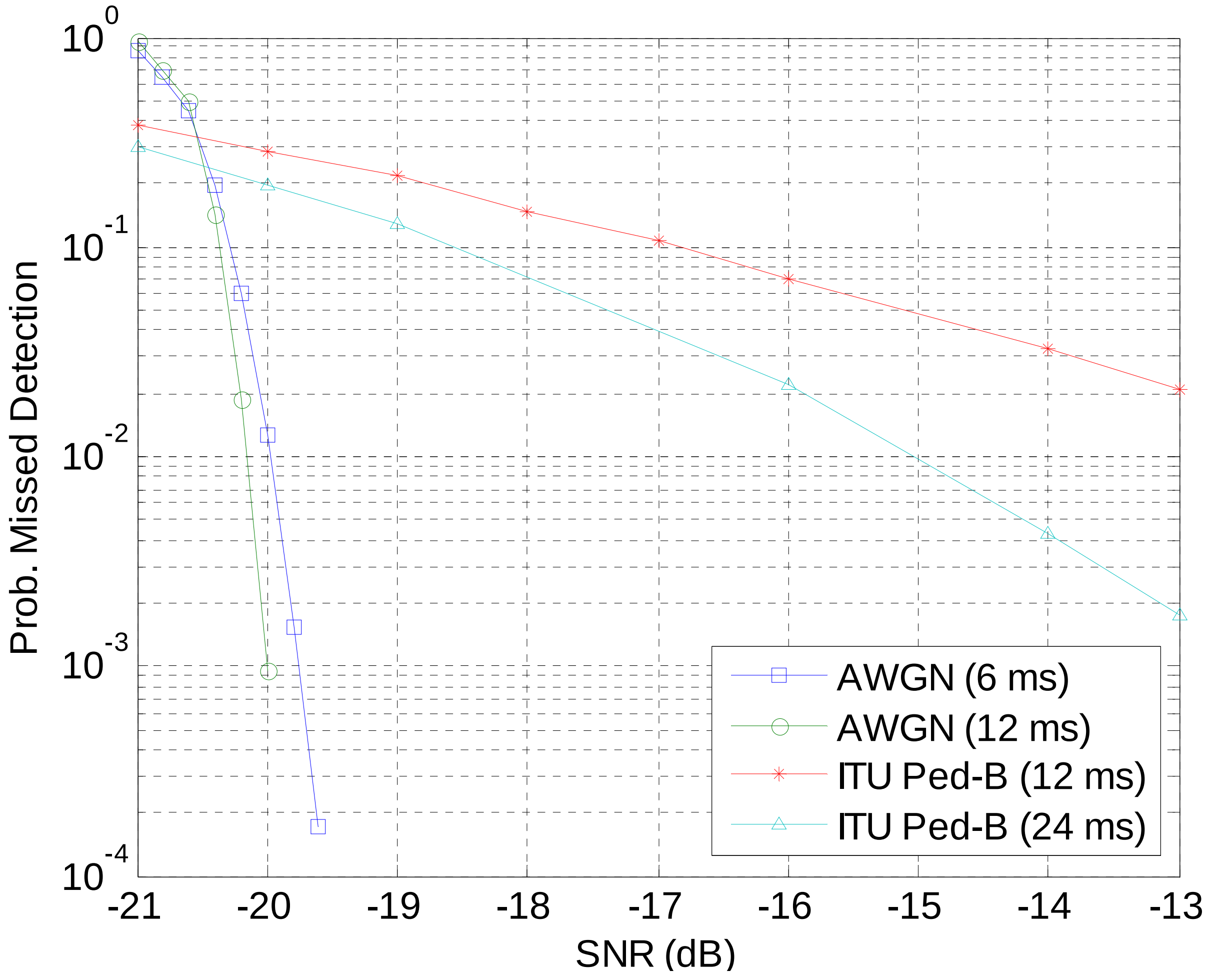
- Prob. false alarm less than 0.001

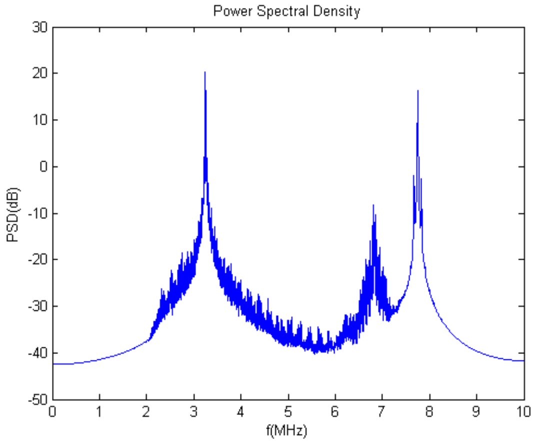




Simulation (Analog TV - NTSC)

- Prob. false alarm less than 0.001





Summary

- Spectral feature detector is asymptotically optimal
  - At very low SNR
  - With a large block size
- Reliably detects TV signals at SNR levels around -20 dB or so.
- Non-coherent detection (synchronization is not needed)
- Low computational complexity compared with the LRT detector.
- Threshold can be estimated numerically.
- Feature selection: prefer sharp spectral features

Thank you!