On the Performance of Graph-based Scheduling Algorithms for Packet Radio Networks

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Abstract—**Many published algorithms used for scheduling transmissions in packet radio networks are based on finding maximal independent sets in an underlying graph. Such algorithms are developed under the assumptions of variations of *the Protocol Interference Model*, which does not take the aggregated effect of interference into consideration. We provide a probabilistic analysis for the throughput performance of such graph based scheduling algorithms under *the Physical Interference Model*. We show that in many scenarios a significant portion of transmissions scheduled based on the Protocol Interference Model result in unacceptable signal-to-interference and noise ratio (SINR) at intended receivers. Our analytical as well as simulation results indicate that, counter intuitively, maximization of the cardinality of independent sets does not necessarily increase the throughput of a network. We introduce the *truncated graph based scheduling algorithm* (TGSA) that provides probabilistic guarantees for the throughput performance of the network.**

I. INTRODUCTION

Packet radio networks are composed of a number of geographically scattered users that communicate via wireless links using radio signals. Examples include ad hoc wireless networks, IEEE 802.11 network systems, as well as a wide range of wireless military networks. One major common feature of packet radio networks is the scarcity of spectrum. An important issue is therefore to design multiple access mechanisms to control channel utilization efficiently. This has motivated the need for channel spatial reuse, i.e. having users sufficiently apart use the same time slot, frequency band, or code.

A prevalent medium access scheme for channel spatial reuse is *spatial time division multiple access* (STDMA), in which time is divided into fixed length slots that are organized cyclically. In each cycle, or timeframe, every slot is allocated to different users such that all transmissions are received successfully at their intended receivers. A number of STDMA algorithms have been proposed in the literature that can be categorized into *link scheduling* [1,3,4,5,6,8,9,15,17] and *broadcast scheduling* [1,9,12,13,14,16]. In link scheduling the transmission right is assigned to links and both transmitters and receivers should be determined a priori. In broadcast scheduling, the transmission right is allocated to nodes, which allows them to transmit to any of their neighbors. In this paper we concentrate on the STDMA link scheduling, though the same analysis can be readily adapted to STDMA broadcast scheduling, *spatial frequency division multiple access* (SFDMA) broadcast scheduling, and SFDMA link scheduling.

Depending on the signaling mechanism, transmissions may collide in two different ways: *primary interferences* and *secondary interferences* [1]. Primary interference is related to a scheduling, whereby a single node performs more than one operation in the underlying timeslot, such as receiving from different transmitters in the same timeslot. Secondary interference occurs under a scheduling, in which a receiver R tuned to a particular transmitter is in the interference range of another transmitter, whose intended destination is not R. This interference scheme, in its most generic form, is known as *the Protocol Interference Model* [2]. Clearly, the definition of the secondary interference implies that in the Protocol Interference Model the capture effect is disregarded.

Under most STDMA algorithms described in the literature, the reuse schedule is designed based on the Protocol Interference Model [1,4,5,9,12,14,16,17]. The algorithms introduced in these papers provide scheduling patterns that preclude primary and secondary interferences by utilizing different graph coloring methodologies. We refer to these algorithms as *the graph based scheduling algorithms.* (For details see section II). However, these algorithms do not take the aggregated interference into consideration. Thus, the graph based scheduling algorithms are not necessarily realistic due to the aggregative nature of interference in wireless communication networks. Simulation results show that reuse schedules obtained from these approaches may result in serious interferences in terms of *signal-to-interference and noise ratio* (SINR) and, hence, dramatic deterioration in network performance [7].

In this paper, we analyze the parameters that influence the throughput capacity of every graph based scheduling algorithm under *the Physical Interference Model*. We show that in many scenarios a significant portion of transmissions scheduled based on the Protocol Interference Model result in unacceptable signal-to-interference and noise ratio (SINR) at intended receivers. However, we provide a probabilistic

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guarantee (lower bound) for the performance of graph based scheduling algorithms and show the validity of this result through our simulations. Our results indicate that, counter intuitively, maximization of the cardinality of independent sets does not necessarily increase the throughput of a network.

The rest of the paper is organized as follows. In section II, we present the common features of graph based scheduling algorithms and their correspondence with the Physical Interference Model. The mathematical analysis and the Truncated Graph based Scheduling Algorithm (TGSA) are elaborated in section III and section IV, respectively. We describe our simulation results in section V. Conclusions are presented in section VI.

II. GRAPH-BASED SCHEDULING ALGORITHMS

In graph based link scheduling algorithms all transmissions are assumed to be performed under an identical transmission power level (P). Provided that node i_1 is the only node transmitting in the network, node i_2 is said to be in the *communication range* (r_c) of node i_1 if the signal-to-noise ratio (SNR) at node i_2 is not less than a communication threshold β_c. Similarly, assuming node i_1 is the only node transmitting in the network, node i_2 is said to be in the *interference range* (r_i) of node i_1 if the signal-to-noise ratio (SNR) at node i_2 is greater than or equal to an interference threshold β_i, but less than the communication threshold β_c.

A graph based link scheduling algorithm, in its most generic form, can be elaborated as follows: the underlying network is modeled by a graph $G(V,E_c,E_i)$, in which V is the set of nodes of graph G, and E_c and E_i are the set of communication arcs and the set of interference arcs, respectively. Every user in the network is represented by a node in graph G. Node i_1 is connected to node i_2 by a communication arc if and only if node i_2 is in the communication range of node i_1, i.e. node i_1 can communicate with node i_2 directly. Node i_1 is connected to node i_2 by an interference arc if and only if node i_2 is in the interference range of node i_1, but not in the communication range of node i_1. Thus, a packet transmitted by user i_1 is assumed to interfere with another packet arriving at node i_2 if and only if node i_1 is connected to node i_2 by either an interference arc or a communication arc. We note that some graph based scheduling algorithms in the literature do not differentiate between the communication range and the interference range, i.e. E_i is set to be equal to E_c.

Graph based scheduling algorithms utilize various graph coloring methodologies to color different communication arcs. All communication arcs colored by the same color are scheduled for transmission in the same time slot. For distinct nodes i_1, j_1, i_2, j_2, (communication) arcs (i_1, j_1) and (i_2, j_2) cannot be colored by the same color if there exits an arc either between i_1 and j_2 or between i_2 and j_1. The common characteristic of these algorithms is that they all attempt to maximize throughput based on maximizing the total number of arcs colored by a single color. This maximization is equivalent to finding a *maximum independent set of arcs* in graph $G'(V,E_c \cup E_i)$ [10]. In the remainder of the paper, we represent a transmission from node i_k to node j_k by $i_k \rightarrow j_k$.

In an alternative interference model the signal-to-interference and noise ratio (SINR) is used to describe aggregated interferences in the network as follows: let $\{i_k, i_k \in W\}$ be the subset of nodes simultaneously transmitting at some time instant (time slot) over a certain sub-channel. Then the transmission from a node $i_r, i_r \in W$, is successfully received by a node $j_s, j_s \notin W$, if and only if

$$\frac{P/d^{\alpha}(i_r, j_s)}{N + \sum_{\substack{k \neq r \\ i_k \in W}} P/d^{\alpha}(i_k, j_s)} \geq \beta_c, \qquad (1)$$

whereby $d(i_k, j_s)$ is the distance between nodes i_k and j_s, and N is the ambient noise power level. α is the path loss exponent, in which for outdoor environments we have $2 < \alpha \leq 5$ [18]. Such an interference model is commonly known as the Physical Interference Model [2]. We define the average number of simultaneous transmissions in a slot that do not interfere with each other based on the Protocol Interference Model (Physical Interference Model) as the *Protocol Throughput (Physical Throughput)*.

III. MATHEMATICAL ANALYSIS

Let nodes in the network be distributed independently and uniformly in a disk C with radius R.

Theorem 1. For a transmission scenario $\{(i_1 \rightarrow j_1), (i_2 \rightarrow j_2), \cdots, (i_M \rightarrow j_M)\}$ resulting from any graph based scheduling algorithm, the probability that transmission from node i_1 is successfully received at node j_1 is always bounded as

$$\Pr\{(i_1 \rightarrow j_1) \textit{ is successful}\} \geq 1 - (M-1) . E\left[\frac{1}{1/(\beta_c d^{\alpha}(i_1, j_1)) - N/P + \xi}\right] E\left[\frac{1}{d^{\alpha}(i_k, j_1)} + \xi \,\middle|\, k \neq 1\right], \qquad (2)$$

whereby ξ is a sufficiently small positive number and $E[A]$ is the expected value of A.

<u>Proof.</u> Based on the Physical Interference Model, transmission from node i_1 is successfully received at node j_1 if and only if

$$\frac{1}{d^{\alpha}(i_1, j_1)} - \beta_c \sum_{k=2}^{M} \frac{1}{d^{\alpha}(i_k, j_1)} \geq \frac{N\beta_c}{P}. \qquad (3)$$

Therefore,

$$\Pr\{(i_1 \rightarrow j_1) \textit{ is successful} \,|\, d(i_1, j_1) = x\} = 1 - \Pr\{\sum_{k=2}^{M} (\frac{1}{d^{\alpha}(i_k, j_1)}) + \xi \geq \frac{1}{\beta_c x^{\alpha}} - \frac{N}{P} + \xi\}, \qquad (4)$$

whereby ξ is a sufficiently small positive number.

Since $d(i_1, j_1) \le r_c$, we have

$$\frac{1}{\beta_c x^\alpha} - \frac{N}{P} + \xi > 0. \quad (5)$$

Based on Markov's inequality[1] and (5), we conclude

$$\Pr\{\sum_{k=2}^{M} (\frac{1}{d^\alpha(i_k, j_1)}) + \xi \ge \frac{1}{\beta_c x^\alpha} - \frac{N}{P} + \xi\}$$
$$\le E[\sum_{k=2}^{M} (\frac{1}{d^\alpha(i_k, j_1)}) + \xi] / (\frac{1}{\beta_c x^\alpha} - \frac{N}{P} + \xi). \quad (6)$$

Considering (4) and (6), we have

$$\Pr\{(i_1 \to j_1) \textit{ is successful}\}$$
$$\ge \int_0^{r_c} [1 - (M-1) E[\frac{1}{d^\alpha(i_k, j_1)} + \xi | k \ne 1] / (\frac{1}{\beta_c . x^\alpha} - \frac{N}{P} + \xi)] . f_{d(i_1, j_1)}(x) dx \quad (7)$$
$$= 1 - E[\frac{1}{1/(\beta_c d^\alpha(i_1, j_1)) - N/P + \xi}] E[\frac{1}{d^\alpha(i_k, j_1)} + \xi | k \ne 1](M-1). \quad (8)$$

■

Lemma 2. For a transmission scenario $\{(i_1 \to j_1), (i_2 \to j_2), \cdots, (i_M \to j_M)\}$ resulting from any graph based scheduling algorithm we have

$$E[\frac{1}{1/(\beta_c d^\alpha(i_1, j_1)) - N/P + \xi}] \le \frac{2/3}{1/(\beta_c r_c^\alpha) - N/P + \xi}, \quad (9)$$

in which ξ is a sufficiently small positive number.

Proof. Let $g(d(i_1, j_1))$ represent $[1/\beta_c d^\alpha(i_1, j_1) - N/P + \xi]^{-1}$. Assuming $0 < \xi < N/P$, $g(d(i_1, j_1))$ is a convex function of $d(i_1, j_1)$ in the interval [0, r_c]. By applying the Edmundson-Mandansky upper bound for the expected value of convex functions [19] (, and considering a single partition), it can be shown that

$$E[\frac{1}{1/(\beta_c d^\alpha(i_1, j_1)) - N/P + \xi}] \le \frac{2/3}{1/(\beta_c r_c^\alpha) - N/P + \xi}. \quad (10)$$

■

Remark: In Lemma 2 equality can be achieved as the number of partitions of interval [0, r_c] in the Edmundson-Mandansky bound is sufficiently large. We represent such an upper bound by C_1.

Lemma 3. For a transmission scenario $\{(i_1 \to j_1), (i_2 \to j_2), \cdots, (i_M \to j_M)\}$ resulting from any graph based scheduling algorithm we have

$$E[\frac{1}{d^\alpha(i_k, j_1)} | k \ne 1] \le \begin{cases} (\frac{2}{(\alpha-2)(R^2 - r_i^2)})[\frac{1}{r_i^{\alpha-2}} - (\frac{1-2^{\alpha-4}}{4-\alpha} - \frac{1-2^{\alpha-3}}{2(3-\alpha)})(\frac{1}{2^{\alpha-5} R^{\alpha-2}})], & \alpha \ne 3, \alpha \ne 4 \\ (\frac{2}{R^2 - r_i^2})[\frac{1}{r_i} - \frac{2(1-\ln(2))}{R}], & \alpha = 3 \\ (\frac{1}{R^2 - r_i^2})[\frac{1}{r_i^2} - \frac{(2\ln(2)-1)}{R^2}], & \alpha = 4 \end{cases}. \quad (11)$$

Proof. Proof is based on conditioning on the polar coordinates of node j_1. Details are omitted here due to the space limitation.

■

We represent the derived upper bound in Lemma 3 by C_2. Clearly, by applying Lemma 2 and Lemma 3 to Theorem 1, a lower bound for the probability of a successful transmission in graph based scheduling algorithms can be easily calculated. In Fig. 1 we illustrate the sensitivity of probability of successful reception with respect to variations in number of simultaneous transmissions (M). In this analysis, we assume R=700m, N=-80dBm, P=10mW, β_c=15dB, α=5, r_c=31.6 m, and r_i=50 m. We note that the lower bound for the probability of a successful transmission is a linear function of M, the number of parallel transmissions in a single time slot. This observation is intuitively correct, since as M increases the aggregated interference in each receiver increases. Clearly, the probability of successful reception is equal to 1 at M=1. The sensitivity of the lower bound for probability of successful reception can be similarly depicted with respect to the other parameters influencing the Physical Interference Model.

IV. The Truncated Graph-based Scheduling Algorithms (TGSA)

We observe that as the number of parallel transmissions in a time slot (M) based on a graph based scheduling algorithm increases the throughout of the network in the underlying time slot may not necessarily increase. This is due to our recent result (Theorem 1), which shows that the probability of successful reception for every transmission linearly decreases with the increase in M. So, the question is: What is the optimum value of M?

Theorem 4. The lower bound for the throughput of any graph based scheduling algorithm is maximized if the cardinality of the set of simultaneous transmissions is set to be equal to either

$$M^* = \lfloor (1 + C_1 C_2) / 2 C_1 C_2 \rfloor \quad (12)$$

or

$$M^* = \lceil (1 + C_1 C_2) / 2 C_1 C_2 \rceil \quad (13)$$

Proof. For an arbitrary transmission scenario

[1] Markov's inequality [11]: Let h(X) be a nonnegative Borel-measurable function of a random variable X. If E(X) exists, then, for every $\varepsilon > 0$, $\Pr\{h(X) \ge \varepsilon\} \le E[h(X)]/\varepsilon$.

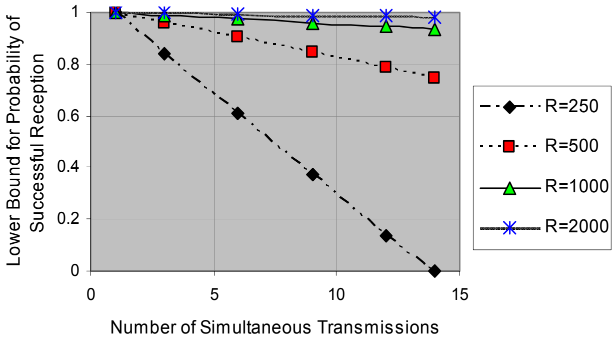

Figure. 1. Sensitivity of probability of successful reception to variations in number of simultaneous transmissions.

$\{(i_1 \to j_1),(i_2 \to j_2),\cdots,(i_M \to j_M)\}$ resulting from any graph based algorithm, the throughput in the underlying time slot can be calculated as

$$TH = E[\sum_{k=1}^{M} X_k], \tag{14}$$

whereby $X_k, k = 1,2,\cdots,M$, are dependent Bernoulli random variables and are defined as

$$X_k = \begin{cases} 1, \textit{if transmission from } i_k \textit{ is successfully received at } j_k \\ 0, \quad \textit{otherwise} \end{cases}. \tag{15}$$

From (14) we have

$$TH = \sum_{k=1}^{M} \Pr[X_k = 1]. \tag{16}$$

Based on Theorem 1, Lemma 2, and Lemma 3, we have

$$\Pr[X_k = 1] \geq 1 - C_1 C_2 (M-1). \tag{17}$$

By substituting (17) in (16), we obtain

$$LTH = (1 + C_1 C_2) M - C_1 C_2 M^2, \tag{18}$$

which LTH is the lower bound for the throughput. Based on the fact that C_1 and C_2 are non-negative constants, it can be readily seen that LTH is a concave quadratic function of M and its global maximum is achieved if

$$M = (1 + C_1 C_2) / 2C_1 C_2. \tag{19}$$

Since, by definition, M is a non-negative integer the global optimum value of M is equal to either $\lfloor (1 + C_1 C_2)/2C_1 C_2 \rfloor$ or $\lceil (1 + C_1 C_2)/2C_1 C_2 \rceil$, depending on which one maximizes equation (18). ■

TGSA uses the same principles as other graph based scheduling algorithms (see section II), with the difference that it does not strive to maximize the number of simultaneous transmissions. Instead, it calculates the optimal number of simultaneous transmissions and prohibits the number of simultaneous transmissions from exceeding this optimal value.

Let M^* represent the optimal cardinality of set of simultaneous transmissions calculated based on Theorem 4. The TGSA algorithm can be represented as follows:

Step 1. Find a maximal independent set of arcs in graph $G'(V, E_c \cup E_i)$. Assume $M = kM^* + a$, where M is the cardinality of the independent set, k and a are nonnegative integers, and $a < M^*$.

Step 2. Partition the set of M arcs into k+1 arbitrary subsets (or k subsets if a=0) such that the size of k of the subsets is equal to M^*.

Step 3. If k=0, assign all the arcs in the independent set to a single new slot. Otherwise, assign each subset with cardinality M^* to a distinct new time slot.

Step 4. Remove all the allocated arcs from graph $G'(V, E_c \cup E_i)$. If $E_c \cup E_i$ is null, stop. Otherwise, go back to Step 1.

V. Simulation Results

In our simulation environment nodes are randomly located in a disk with radius R based on a uniform distribution. Every node generates packets based on a Poisson arrival process with intensity λ packets per slot for each of its neighbors. Throughout the simulation we assume R=450 m, N=-90 dBm, P=10 mW, β_c=10 dB, α=4, and r_i=r_c=177.8 m, unless otherwise is specified.

In Fig. 2 we compare the Physical Throughput and the Protocol Throughput of a randomly generated topology with 30 nodes (n=30) as a function of packet generation rate. We observe that as the packet generation rate increases the Protocol Throughput increases, since the graph algorithm obtains higher degree of freedom for spatial reuse provision. However, the Physical Throughput has a global maximum with respect to variations in packet generation rate. This is due to the fact that the increase in number of simultaneous transmissions decreases the probability of successful reception. We observe that for high values of λ, less than 40 percent of all transmissions based on the graph based scheduling algorithm are successfully received at destinations.

In Fig. 3, we compare the percentage of successful transmissions in a graph based scheduling algorithm for a randomly generated topology (n=30) with the derived lower bound for probability of successful reception. We note that within the horizon of our simulation the graph based scheduling algorithm did not result in a time slot with a single transmission (Fig. 3).

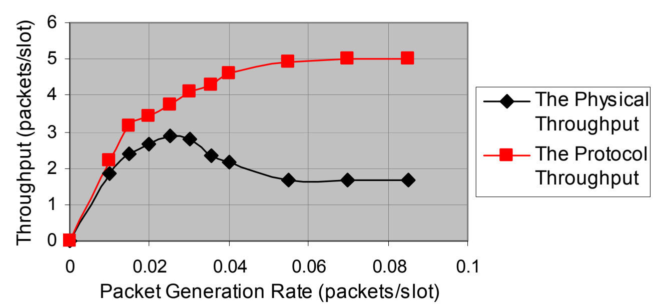

Figure. 2. Comparison between the Physical Throughput and the Protocol Throughput in a graph based scheduling algorithm.

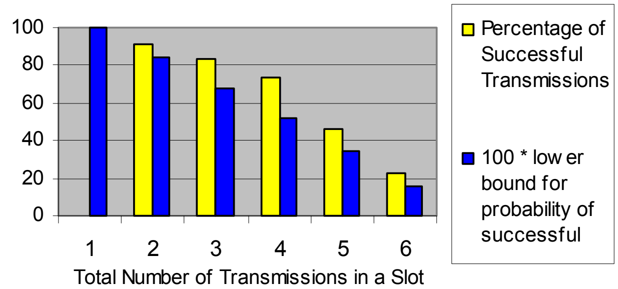

Figure 3. Comparison between the percentage of successful transmissions and the theoretical lower bound for the probability of successful transmission.

In Fig. 4 we represent the Physical Throughput of a graph based scheduling algorithm as a function of an upper bound on maximum number of simultaneous transmissions. We observe that in the random topologies with n=25 and n=30, by limiting the number of simultaneous transmissions to 4, we achieve the highest Physical Throughput, while for n=20 the maximum Physical Throughput is obtained when the upper bound is set to be equal to 3. We note that as the upper bound gets larger than the maximum possible number of simultaneous transmissions in a slot for the underlying topology, it becomes redundant and, hence, the Physical Throughput converges. Clearly, this value of the Physical Throughput is associated with the conventional graph based scheduling algorithms that do not consider any bounds on the maximum number of simultaneous transmissions.

TGSA sets the upper bound for the latter scenario to be equal to the optimum number of simultaneous transmissions, which based on Theorem 4 is equal to 3. As a result, TGSA provides the highest Physical Throughput for the topology with n=20 (Fig. 4). For the topologies with n=25 and n=30, TGSA results in a suboptimal Physical Throughput. However, the Physical Throughput levels derived based on TGSA for the latter topologies are still higher than those of a conventional graph based scheduling algorithm, which strives to maximum the number of simultaneous transmissions (Fig. 4).

We note that the optimal number of simultaneous transmissions (M*) for some scenarios may be larger than the maximum possible number of simultaneous transmissions in a slot for the underlying topology. In this case, TGSA behaves similar to the conventional graph based scheduling algorithms and strives to maximize the number of simultaneous transmissions in every slot.

VI. CONCLUSIONS

In this paper, we show that in many scenarios a significant portion of transmissions scheduled according to the graph based scheduling algorithms result in unacceptable signal-to-interference and noise ratio (SINR) at intended receivers. Our analytical as well as simulation results indicate that, counter intuitively, maximization of the cardinality of independent sets does not necessarily increase the throughput of a network. The analysis of the effect of the interference range on the Physical Throughput of graph based scheduling algorithms is an interesting open problem and is part of our ongoing research.

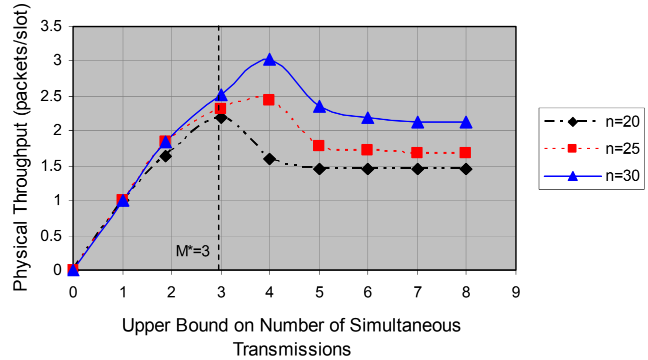

Figure. 4. The physical throughput of a graph based scheduling algorithm with an upper bound on number of simultaneous transmissions.

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