High Transmission Power Increases the Capacity of Ad Hoc Wireless Networks

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***Abstract*—In this paper, we analyze and investigate the effect of transmission power on the throughput capacity of finite ad hoc wireless networks. We prove that, independent of nodal distribution and traffic pattern, the capacity of an ad hoc wireless network is maximized by properly increasing the nodal transmission power. Under the special case of our analysis that the maximum transmission power can be arbitrary large, we prove that the fully connected topology (i.e. the topology under which every node can directly communicate with every other node in the network) is always an optimum topology, independent of nodal distribution and traffic pattern. Our result stands in sharp contrast with previous results that appeared in the literature for networks with random nodal distribution and traffic pattern, which suggest the use of minimal common transmission power that maintains connectivity in the network, showing it to asymptotically achieve a throughput level that is in the order of the throughput capacity. We also derive a linear programming (LP) formulation for calculating the capacity of finite ad hoc wireless networks. Our LP based performance evaluation results confirm the capacity improvement attained under our recommended approach, as well as identify the magnitude of capacity upgrade that can be realized for networks with random topologies and traffic patterns.**

I. INTRODUCTION

Ad hoc wireless networks are infrastructure-free wireless networks consisting of nodes that communicate with each other across wireless links directly or through possibly intermediate nodes. Capacity (throughput capacity) of an ad hoc wireless network is defined in the usual manner as the maximum data rate that is achievable by all source-destination pairs of nodes ([5], [7], [11], [9], [3], [4]). This value is one of the fundamental characteristics of the network. In an ad hoc wireless network, capacity is a function of various factors, including nodal density and distribution, mobility, traffic pattern, size of the network, transmission power and bandwidth constraints, and antenna directionality.

In a recent landmark paper [5], Gupta and Kumar studied the capacity of ad hoc wireless networks in the limit as the number of nodes grows to an arbitrarily large level. Under this model, stationary nodes are randomly and uniformly located (over a disk area) and each node sends data to a randomly and uniformly selected destination. Their main result indicates that as the number of nodes per unit area (n) increases, the throughput capacity decreases approximately as $1/\sqrt{n}$. In particular, the authors prove that under the *Physical Interference Model* the upper bound for the throughput capacity is $\theta(1/\sqrt{n})$, while their constructive spatial and temporal joint scheduling scheme asymptotically (i.e., as the number of nodes becomes very large) attains a throughput of $\theta(1/\sqrt{n \log n})$. Under the latter scheme, each node sets its transmit power level to a common value that is equal to the minimum common transmission power that maintains connectivity in the network.

Grossglauser and Tse [11] exploit nodal mobility to attain *multiuser diversity*. Allowing for unbounded delay and using only one-hop relaying, they show that mobility increases the capacity considerably. Yi et al. [4] investigate the capacity of ad hoc wireless networks using directional antennas. They show that with the use of directional antennas, the throughput capacity can be upgraded by realizing a gain as large as $4\pi^2/\alpha\beta$, where α and β are the antenna transmission angle and reception angle, respectively. Liu et al. [10] study the capacity of a hybrid wireless network that is formed by placing a sparse network of base stations in an ad hoc network. The base stations are assumed to be connected by a high-bandwidth wired network. Their results show that if the number of base stations grows faster than $\sqrt{n}$, the capacity increases linearly with the number of base stations, providing an effective improvement over a pure ad hoc network. Toumpis and Goldsmith [7] investigate the capacity regions for finite ad hoc wireless networks. A capacity region characterizes the set of achievable rate combinations involving all source-destination pairs in the network. Comments are made as to the impact of some simple power level variations on the capacity region.

Based on the *Protocol Interference Model*, a transmission is successfully received if the intended destination is sufficiently apart from the source of any other simultaneous transmission. In [1] and [5] the authors also show that based on the Protocol Interference Model there is a trade-off between the spatial reuse factor and the average route length (in hops) as a function of the common transmission range. Specifically, through a quantitative argument based on the Protocol Interference Model, the authors prove that the upper bound for the throughput capacity is inversely proportional to the common transmission range r.

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Let P_i denote the transmit power level of the *ith* node, $i = 1, \cdots, n$. Given a selected set of nodal transmit power levels $P = (P_1, \cdots, P_n)$ (that we assume can be different from node to node but are fixed in time), the throughput capacity $\lambda(P)$ is achieved (in finite time or asymptotically in time) as the system designer selects an *optimal spatial* (based on the routing mechanism) *and temporal* (based on the channel sharing MAC protocol) *joint scheduling scheme* (or simply, *optimal joint scheduling scheme*) over the underlying (finite or infinite) time period *T*.

Our aim in this paper is to characterize the key features of a power vector solution that achieves the supreme throughput capacity level λ^* over the set of power vectors $P = (P_1, \cdots, P_n), 0 \le P_i \le P_{max}, i = 1, \cdots, n$, where P_{max} denotes the maximum allowable transmission power. That is,

$$\lambda^* = \underset{P}{Sup}\{\lambda(P) : P = (P_1, \cdots, P_n), 0 \le P_i \le P_{max}, i = 1, \cdots, n\}. \quad (1)$$

We call such a power vector an *optimum power vector*, identify an associated optimal joint scheduling scheme as an *optimum joint scheduling scheme*, and denote the resulting throughput capacity level as the *optimum (or maximum) throughput capacity.* We note that the definition of the network topology (i.e. the connectivity graph layout of the network) for a given nodal transmit power vector is as usual based on a link connecting two nodes if they can directly communicate with each other successfully (under a specified minimum required signal-to-noise ratio level) when no other transmissions are invoked in the network. We refer to the topology associated with an optimum power vector as an *optimum topology*.

In this paper, we analyze and investigate the effect of transmission power on the throughput capacity of finite ad hoc wireless networks. We prove that, independent of nodal distribution and traffic pattern, the capacity of an ad hoc wireless network is maximized by properly increasing the nodal transmission power. In particular, we prove that, independent of nodal distribution and traffic pattern, there exists an optimum power vector that at least one of its components is equal to P_{max}. Under the assumption that P_{max} can be arbitrarily large, we prove that the fully connected topology (i.e. the topology under which every node can directly communicate with every other node in the network in the absence of interference) is always an optimum topology, independent of nodal distribution and traffic pattern. Our result is valid for any interference model that uses the received SINR as the measure of successful reception.

Under the special case that the transmission power levels of all nodes are assumed to be identical (yet programmable), we prove that the power vector $P = (P_1 = P_{max}, \cdots, P_n = P_{max})$ always (i.e. independent of nodal distribution and traffic pattern) maximizes the capacity of the ad hoc wireless network. This result is in sharp contrast to the results in [1] and [5] which are based on the Protocol Interference Model and are valid for any number of nodes. The latter results state that the upper bound for the throughput capacity is inversely proportional to the common transmission range *r*. The authors then conclude that the common nodal transmit power level should be reduced to the lowest value at which the network is connected. We note that the Protocol Interference Model does not generally provide a comprehensive scrutiny of reality due to the relativity of transmission power and the aggregate effect of interference in wireless networks, among other reasons ([8], [6]).

As another special case of our analysis, we assume the following: i) transmission power of all nodes are identical, ii) nodes are randomly and uniformly located, iii) every node is a source of a transmission whose destination is uniformly and independently distributed,[2] and iv) the Physical Interference Model [5] is used as the measure for successful reception of transmissions. Under such a model, our result regarding the optimality of $P = (P_1 = P_{max}, \cdots, P_n = P_{max})$ stands in sharp contrast with the asymptotic behavior result presented in [5]. The latter result, which is proven to hold for the Protocol Interference Model as well as for the Physical Interference Model, suggests the use of minimal common transmission power that maintains connectivity in the network, showing it to asymptotically achieve a (per source-destination) throughput level that is in the order of the throughput capacity. We note that the asymptotic result in [5] regarding the optimality of the minimum common transmission power, inherently disregards the constant factors, which may lead to inaccurate interpretations of the effect of transmission power on the capacity of finite ad hoc wireless networks.

We also derive a linear programming (LP) formulation for calculating the capacity of finite ad hoc wireless networks. We compare our results versus previous results in the literature by solving about 2000 linear programming problems (corresponding to distinct *Random Networks*) in ILOG CPLEX 7.0 software.

The rest of the paper is organized as follows: in section II, the system model is presented. Mathematical analysis and numerical results are discussed in section III and section IV, respectively. Conclusions are presented in section V.

II. System Model

We consider an ad hoc wireless network that consists of *n* stationary nodes, which are located based upon any arbitrary distribution in a given area. Every node transmits at a fixed data rate of *W* bits per second, and variations in transmission power merely affect the transmission range. Every transmission is intended for a single receiver. All nodes are equipped with identical half-duplex radios and with omni-directional antennas. A node can receive from at most one other node in the same time instant. We assume node *i* to transmit at a fixed (yet programmable) transmission power P_i, $0 \le P_i \le P_{max}$, *i=1,...,n*; assume a transmission to occupy the entire bandwidth of the system under consideration. Channel time is slotted into identical synchronized time slots. Slot duration is assumed to be equal

[2] Reference [5] refers to a network holding properties (i), (ii), and (iii) as a *Random Network.*

to the transmission time of a packet plus some overhead duration that includes the maximum propagation delay. Nodes are continuously active so that source nodes have infinite reservoirs of packets to send to their destinations. Without loss of generality and for the sake of presentation simplicity, we assume every source node to be associated with a single destination. The source-destination association can be selected based on an arbitrary traffic pattern. Consequently, some nodes may not necessarily function as source or destination nodes.

There is a communication link from node i to node j if node i can directly communicate with node j under power level P_i in the absence of interference. Let us represent a direct transmission from node i to node j whose source is node s by $i \xrightarrow{s} j$. A *transmission scenario* $S_{(M)} = \{i_1 \xrightarrow{s_1} j_1, \ldots i_M \xrightarrow{s_M} j_M\}$ is defined as a candidate set of direct transmissions that are considered to all take place at the same time slot, where all transmitting and receiving nodes are distinct.

For such a transmission scenario $S_{(M)}$ under nodal transmit power vector $P_{(M)} = (P_{i_1}, \ldots, P_{i_M}), 0 \le P_{i_k} \le P_{\max}, k = 1,2,\ldots,M$, we say that the transmission from i_k is *successful* if the received SINR at the intended receiver j_k is not less than the minimum required threshold γ [2], i.e.

$$\frac{G_{i_k j_k} P_{i_k}}{N_{j_k} + \sum_{\substack{r=1 \\ r \ne k}}^{M} G_{i_r j_k} P_{i_r}} \ge \gamma, \quad k = 1,2,\ldots,M, \tag{2}$$

in which G_{ij} is the propagation gain (incorporating the effects of link loss phenomena such as fading and shadowing) for direct transmission from node i to node j, and N_{j_k} is the thermal noise power at receiver j_k. We refer to such a generic model for successful reception of a packet as the *SINR-based Interference Model* (Clearly, the Physical Interference Model [5] is a special case of the SINR-based Interference Model).

Based on the definition of transmission scenario, for an ad hoc wireless network with n half-duplex nodes there can be at most

$$N_S = \sum_{i=1}^{\lfloor n/2 \rfloor} \binom{n}{2i} \binom{2i}{i} (i!)(n-1)^i \tag{3}$$

distinct transmission scenarios, noting that the maximum number of simultaneous transmissions in a time slot is equal to $\lfloor n/2 \rfloor$ [7], and the maximum number of transmitter-receiver pairs is given by $\binom{n}{2i}\binom{2i}{i}(i!)$ when there are i simultaneous transmissions, $i = 1, \cdots, \lfloor n/2 \rfloor$.

We define the cardinality of the set of successful transmissions in a transmission scenario $S_{(M)} = \{i_1 \xrightarrow{s_1} j_1, \ldots, i_M \xrightarrow{s_M} j_M\}$ employing power vector $P_{(M)} = (P_{i_1}, \ldots, P_{i_M})$, $0 \le P_{i_k} \le P_{\max}, k = 1,2,\ldots,M$, as the *spatial reuse factor of the transmission scenario $S_{(M)}$* with respect to $P_{(M)}$. We define a transmission scenario $S_{(M)}$ to be *feasible* under power vector $P_{(M)}$ (or equivalently, under power vector $P = (P_1, \cdots, P_n), 0 \le P_i \le P_{\max}, i = 1, \cdots, n$) if all its transmissions are successful. Consequently, the spatial reuse factor of a feasible transmission scenario $S_{(M)}$ under power vector $P_{(M)}$ is equal to M. Clearly, every admissible spatial and temporal joint scheduling scheme under power vector $P = (P_1, \cdots, P_n), 0 \le P_i \le P_{\max}, i = 1, \cdots, n$, over the underlying (finite or infinite) time period can be represented by a sequence of feasible transmission scenarios under power vector P allocated to (finite or infinite) consecutive time slots. We refer to such a sequence as a *scenario sequence* with respect to power vector P. The *ith* scenario sequence with respect to power vector P, and the associated (per source-destination) throughput are denoted as $SQ_i(P)$ and $\lambda_{SQ_i(P)}$, respectively. Furthermore, the set of all possible distinct scenario sequences, each operating under the same power vector P, is denoted as $X(P)$. Then, based on the definition of the scenario sequence, we can express the throughput capacity under power vector P also as follows:

$$\lambda(P) = \operatorname*{Sup}_i \{\lambda_{SQ_i(P)} : SQ_i(P) \in X(P)\}. \tag{4}$$

III. Mathematical Analysis

A. Some Theoretical Results

Definition: Relative Maximality. Let $P_{(M)} = (P_{i_1}, \ldots, P_{i_M})$ be an arbitrary power vector, whereby $0 < P_{i_k} \le P_{\max}, k = 1,\ldots,M$. Power vector $P'_{(M)} = (P'_{i_1}, \ldots, P'_{i_M})$ is said to be *relatively maximized* with respect to power vector $P_{(M)}$ if

$$P'_{(M)} = \alpha(P_{(M)}) P_{(M)}, \tag{5}$$

where $\alpha(P_{(M)})$ is a real positive scalar defined as

$$\alpha(P_{(M)}) = \min_{k=1,\cdots,M} \{P_{\max} / P_{i_k}\}. \tag{6}$$

Furthermore, a power vector is said to be *relatively maximum* if at least one of its components is equal to P_{max}.

Lemma 1. Let $S_{(M)} = \{i_1 \xrightarrow{s_1} j_1, \ldots, i_M \xrightarrow{s_M} j_M\}$ be an arbitrary transmission scenario under power vector $P_{(M)} = (\beta P_{i_1}, \ldots, \beta P_{i_M})$, $0 < \beta P_{i_k} \le P_{\max}, k = 1,\ldots,M$, where β is a real positive number. The spatial reuse factor of transmission scenario $S_{(M)}$ with respect to this power vector $P_{(M)}$ is a monotonically non-decreasing function of β in interval $(0, \alpha(\beta^{-1} P_{(M)})]$, independent of nodal distribution and traffic pattern.

Proof. Let us consider an arbitrary transmission $i_k \xrightarrow{s_k} j_k, k = 1,\ldots,M$ in $S_{(M)}$. Based on (2), transmission $i_k \xrightarrow{s_k} j_k$ is successfully received at node j_k if

$$\frac{G_{i_k j_k}\beta P_{i_k}}{N_{j_k}+\sum_{\substack{r=1\\ r\neq k}}^{M} G_{i_r j_k}\beta P_{i_r}} \geq \gamma. \qquad (7)$$

The derivative of the left-hand-side of (7) with respect to β can be calculated as

$$\frac{\partial}{\partial\beta}\left(\frac{G_{i_k j_k}\beta P_{i_k}}{N_{j_k}+\sum_{\substack{r=1\\ r\neq k}}^{M} G_{i_r j_k}\beta P_{i_r}}\right) = \frac{G_{i_k j_k} P_{i_k} N_{j_k}}{(N_{j_k}+\sum_{\substack{r=1\\ r\neq k}}^{M} G_{i_r j_k}\beta P_{i_r})^2}, \qquad (8)$$

and is noted to be always nonnegative. Therefore, by increasing the value of β, the SINR at j_k remains constant (when $N_{j_k}=0$) or increases. In fact, in the limit as $\beta \to \infty$, the SINR at j_k converges to a constant; i.e.,

$$\lim_{\beta\to\infty}\left(\frac{G_{i_k j_k}\beta P_{i_k}}{N_{j_k}+\sum_{\substack{r=1\\ r\neq k}}^{M} G_{i_r j_k}\beta P_{i_r}}\right) = \frac{G_{i_k j_k} P_{i_k}}{\sum_{\substack{r=1\\ r\neq k}}^{M} G_{i_r j_k} P_{i_r}}. \qquad (9)$$

Similarly, the SINR at all other intended receivers increase as β increases. Therefore, the spatial reuse factor of transmission scenario $S_{(M)}$ under $P_{(M)}$ is a monotonically non-decreasing function of β, $\beta \in (0, \alpha(\beta^{-1}P_{(M)})]$. ■

We note that the result described by Lemma 1 contrasts similar results that have been derived by assuming the Protocol Interference Model, in which the "interference range" of every transmission increases with the increase in nodal transmit power. Specifically, based on the Protocol Interference Model, the spatial reuse factor of any transmission scenario $S_{(M)}$ (with more than one transmission) under $P_{(M)}$ converges to zero as β becomes sufficiently large.

Theorem 2. If $P' = (P'_1, \cdots, P'_n)$ is relatively maximized with respect to $P = (P_1, \cdots, P_n)$, then $\lambda(P) \le \lambda(P')$, independent of nodal distribution and traffic pattern.

Proof. Let P' be relatively maximized with respect to P. Based on Lemma 1, every feasible transmission scenario $S_{(M)} = \{i_1 \xrightarrow{s_1} j_1, \ldots, i_M \xrightarrow{s_M} j_M\}$ under power vector $P_{(M)} = (P_{i_1}, \cdots, P_{i_M})$ is also a feasible transmission scenario under power vector $P'_{(M)} = (P'_{i_1}, \cdots, P'_{i_M})$. Therefore, based on the definition of scenario sequence, every scenario sequence under P is also a scenario sequence under P'.

Now, let N_{i,p_i} represent the set of all nodes j in which there is a communication link from node i to node j under power level P_i, $i = 1,\ldots,n$. Since $P'_i \geq P_i$, node i may be able to directly communicate with some additional nodes under P'_i, i.e. $N_{i,p_i} \subseteq N_{i,p'_i}, i = 1,\ldots,n$. As a result, under power vector P', additional routes may be explored, which translates into supplementary scenario sequences. Therefore, $X(P) \subseteq X(P')$.

Assume the *ith* scenario sequence under power vector P is the same as the *jth* scenario sequence under power vector P', i.e. $SQ_i(P) \equiv SQ_j(P')$. Since, every scenario sequence consists of a sequence of feasible transmission scenarios, we have $\lambda_{SQ_i(P)} = \lambda_{SQ_j(P')}$. Consequently, since $X(P) \subseteq X(P')$ and based on relation (4), we conclude that $\lambda(P) \le \lambda(P')$, independent of nodal distribution and traffic pattern. ■

Theorem 3. Independent of the underlying nodal distribution and traffic pattern, there exists a relatively maximum power vector that maximizes the throughput capacity of an ad hoc wireless network.

Proof. Let assume that there is no optimum relatively maximum power vector. Then, the optimum power vector P^* is not relatively maximum. Let λ^* and P' denote the optimum throughput capacity of the underlying network and the relatively maximized power vector with respect to P^*, respectively. But, based on Theorem 2, $\lambda^* \le \lambda(P')$. Clearly, the latter contradicts the suboptimality of every relatively maximum power vector, which completes the proof. ■

In general, an optimum power vector is a function of nodal distribution and traffic pattern. However, based on Theorem 3, there always exists an optimum power vector that is relatively maximum. Intuitively, this is due to the fact that relative maximality provides a higher *combinatorial diversity* (i.e. higher degree of freedom in terms of the optimization of the spatial and temporal joint scheduling scheme.) In fact, as we illustrate in our numerical analysis (section IV), the latter property leads to significant increase in the capacity of ad hoc wireless networks.

The following conclusions follow directly from the latter theorem.

Corollary 3.1. Under the special case that the maximum transmission power is sufficiently high, the fully connected topology is an optimum topology of an ad hoc wireless network, independent of nodal distribution and traffic pattern.

Corollary 3.2. Under the special case that the transmission power of all nodes is assumed to be identical, the power vector $P = (P_1 = P_{\max}, \cdots, P_n = P_{\max})$ maximizes the capacity of an ad hoc wireless network, independent of nodal distribution and traffic pattern.

B. Linear Programming Formulation

We provide in this section a linear programming (LP) formulation for analysis of the throughput capacity of ad hoc wireless networks over the operational period T that can represent an unlimited duration, for an infinite horizon operation, or it can denote a finite sufficiently long operational period. We assume the nodes to operate under a given nodal transmit power vector. Assuming N'_S denotes the total number of feasible transmission scenarios for the underlying ad hoc wireless network under power vector P,[3] we define $s_{ij}^{(k)}$ as the following:

[3] Note that we keep all the feasible transmission scenarios in the same order, over all time slots.

$$s_{ij}^{(k)} = \begin{cases} 1, & if\ in\ the\ kth\ feasible\ transmission \\ & scenario\ under\ P, j\ is\ the\ receiver\ of \\ & a\ packet\ whose\ source\ is\ i, i \neq j \\ -1, & if\ in\ the\ kth\ feasible\ transmission \\ & scenario\ under\ P, j\ is\ the\ transmitter\ of \\ & a\ packet\ whose\ source\ is\ i, i \neq j \\ 0, & otherwise \end{cases}, k = 1,...,N'_S, \quad (10)$$

Let $\bar{a} = (a_1, \cdots, a_{N'_S})$, where $a_k, k = 1, \cdots, N'_S$, represents the fraction of time over an arbitrary finite positive period T_{rep} allocated to the *kth* feasible transmission scenario, $\sum_{k=1}^{N'_S} a_k = 1, a_k \geq 0$ .Assuming $A = \{(a_1, \cdots, a_{N'_S}) : \sum_{k=1}^{N'_S} a_k = 1, a_k \geq 0\}$ and $\{(i_1, j_1), ..., (i_\Phi, j_\Phi)\}$ to represent the set of all given source-destination pairs, we first define the following nonlinear programming problem:

$$\underset{a \in A}{Max} \underset{l \in \{1,...,\Phi\}}{Min} (\sum_{k=1}^{N'_S} s_{i_l j_l}^{(k)} a_k) \quad (11)$$

$s.t.$

$$\sum_{k=1}^{N'_S} s_{ij}^{(k)} a_k \geq 0, \quad i = 1, \cdots, n, j = 1, \cdots, n, \quad (12)$$

$$\sum_{k=1}^{N'_S} a_k = 1, \quad (13)$$

$$a_k \geq 0, k = 1, \cdots, N'_S, \quad (14)$$

where a_k's are the decision variables and constraint (12) describes the *flow conservation* requirement at every node (i.e., the amount of outgoing *flow* cannot be larger than the amount of incoming *flow*).

By defining a single non-negative dummy variable λ and substituting (11) with

$$\underset{a \in A}{Max}\ \lambda \quad (15)$$

$s.t.$

$$\sum_{k=1}^{N'_S} s_{i_l j_l}^{(k)} a_k - \lambda \geq 0, \quad l = 1, \cdots, \Phi, \quad (16)$$

it can be clearly seen that the non-linear optimization problem is transformed into an equivalent linear programming problem.

We note that within the period T_{rep}, an intermediary node may relay a packet from another node before that packet actually arrives. This situation (i.e., a non-causal routing) can be resolved by considering a transient period with finite duration for backlogging packets at intermediary nodes. Due to the finite duration of the transient period, the result of the optimization model is not affected, in the limit of large number of cyclic repetitions of the period T_{rep}.

From computation point of view, when a large number of nodes/flows is involved, we note that the computational limitation of calculating the throughput capacity of ad hoc wireless networks based on the LP formulation is in the verification of the feasibility of all transmission scenarios, the number of which grows factorially fast as the number of nodes increases.

IV. NUMERICAL ANALYSIS

In this section, we use the LP optimization model to evaluate our theoretical results derived in the previous section. Let $P_{\min}$ denote the minimal common transmission power that maintains connectivity in the underlying network ([1], [5]). The Physical Interference Model [5] is used as the measure for successful reception of transmissions, and every node is assumed to be the source node of a transmission. In particular, the path loss exponent is set to be 4, noise power is -90 *dBm*, and the minimum required SINR is 10 *dB*. All transmissions are performed at *W* = 12 *Mbps*. $P_{\max}$ is set to be 5 Watt in order to maintain the fully connected topology, independent of the distribution of nodes, while $P_{\min}$ is selected based on the underlying nodal distribution.

Though we restrict our numerical analysis to networks with at most 10 nodes to be able to compute all the possible feasible transmission scenarios, it adequately captures the trade-offs addressed in this paper. For each *n*, $2 < n \leq 10$, we randomly and uniformly generated 100 layout realizations for nodes in a 500 x 500 meters square. For each realization, the destination of each source node was uniformly and independently selected. The throughput capacity for each layout was then calculated by solving the LP optimization model using the ILOG CPLEX 7.0 software for two cases: i) the transmit power of all nodes was equal to $P_{\min}$; and ii) the transmit power of all nodes was equal to $P_{\max}$. Interestingly, for all 100 instants of each *n*, $n > 5$, the capacity under $P_{\max}$ was strictly greater than that under $P_{\min}$. The average capacity for each *n* was then calculated over the 100 topologies under $P_{\min}$ ($\lambda(P_{\min})$) and under $P_{\max}$ ($\lambda(P_{\max})$).

In Fig. 1 we depict the average throughput capacity (recalling it to be defined as the guaranteed data rate per source-destination pair) of an ad hoc wireless network under $P_{\min}$ and $P_{\max}$ as a function of the number of nodes. We observe the monotonic decrease in the throughput capacity under both $P_{\min}$ and $P_{\max}$, which is consistent with the results in [5] regarding the reduction in capacity with the increase in the number of nodes (under the aforementioned assumptions). As illustrated in Fig. 1, the capacity under $P_{\max}$ is markedly greater than that under $P_{\min}$.

In Fig. 2, we show the percentage increase in capacity under $P_{\max}$ with respect to that under $P_{\min}$, as a function of the number of nodes. Clearly, it can be seen that $\lambda(P_{\max})/\lambda(P_{\min})$ (or equivalently, $[\lambda(P_{\max})/\lambda(P_{\min})] - 1$) is a monotonically increasing function of *n*. For an ad hoc wireless network with 10 nodes (averaging over randomly selected 100 Random Networks), a 78% capacity gain is observed under high transmission power.

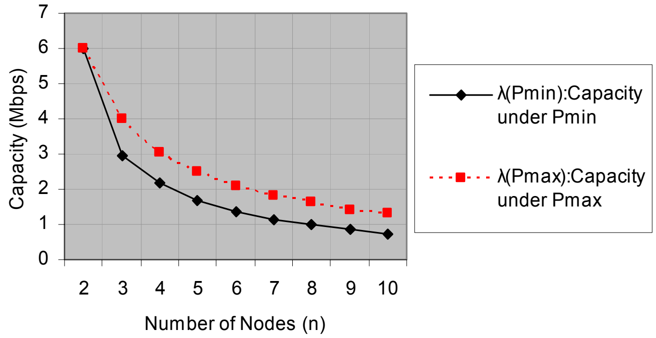

Figure 1. Capacity under low and under high transmit power levels.

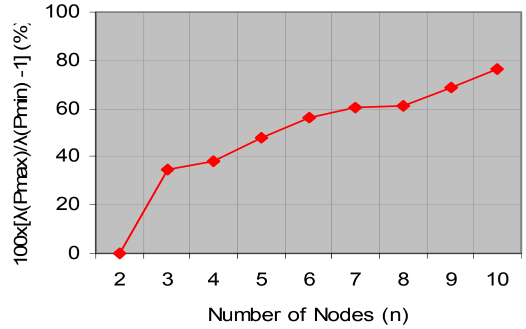

Figure 2. The capacity gain under high transmission power.

In Table I we analyze the reasons behind the significant difference between $\lambda(P_{\min})$ and $\lambda(P_{\max})$. Based on Theorem 2 and Theorem 3, this difference is rooted in the higher number of feasible transmission scenarios achievable under $P_{\max}$. In this table, we compare the (maximum) number of transmission scenarios (N_S) and the average number of feasible transmission scenarios (averaging over 100 layouts) under $P_{\min}$ ($N'_S(P_{\min})$) and under $P_{\max}$ ($N'_S(P_{\max})$) as a function of the number of nodes. In particular, we note the major increase in difference between $N'_S(P_{\min})$ and $N'_S(P_{\max})$ as the number of nodes increases. This is to a large extent due to the fact that as the number of nodes increases, $P_{\min}$ decreases (by definition), while $P_{\max}$ remains constant. The increase in the difference between $N'_S(P_{\min})$ and $N'_S(P_{\max})$ as a function of n results in higher capacity gain under $P_{\max}$ as n grows, which explains the result depicted in Fig. 2. In particular, we note that for an ad hoc wireless network with 10 nodes there are (in average) more than 24,000 additional feasible transmission scenarios under the high transmission power level, which in turn, leads to astronomically higher number of supplementary scenario sequences under the high transmission power level.

VI. CONCLUSIONS

In this paper, we analyze the effect of transmission power on the capacity of finite ad hoc wireless networks. We prove that, independent of nodal distribution and traffic pattern, the throughput capacity of an ad hoc wireless network is maximized by properly increasing the nodal transmit power level. This is mainly due to the fact that high transmission power provides a higher combinatorial diversity (i.e. higher degree of freedom in terms of the optimization of the spatial and temporal joint scheduling scheme).

We note that our selection of the optimal power vector to have the relative maximality feature also provides a high level of robustness under dynamic topologies (induced by mobility). We further note that we did not include energy consumption as an objective for the networks under consideration in this paper. The analysis of the trade-offs among capacity, energy, and robustness under the relative maximality feature is part of our ongoing research.

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Table I. Comparison between (maximum) number of transmission scenarios (N_S) and the average number of feasible transmission scenarios under $P_{\max}$ ($N'_S(P_{\max})$) and under $P_{\min}$ ($N'_S(P_{\min})$).

n (Number of nodes)	2	3	4	5	6	7	8	9	10
N_S	2	12	144	1040	19650	196,812	5,227,712	67,189,824	2,300,229,090
$N'_S(P_{\max})$	2	12	30.16	92.97	290.82	1170.46	12,375.54	112,588.90	504,668.96
$N'_S(P_{\min})$	2	6	17.79	58.83	194.65	934.03	7453.29	100,541.71	480,041.33
$N'_S(P_{\max}) - N'_S(P_{\min})$	0	6	12.37	34.14	96.17	236.43	4922.25	12,047.19	24,627.63

