Optimum Integrated Link Scheduling and Power Control for Ad Hoc Wireless Networks

Arash Behzad, Izhak Rubin, and Pallavi Chakravarty

***Abstract*—In this paper, we develop a new mathematical programming formulation for minimizing the schedule length in ad hoc wireless networks based on the optimal joint scheduling of transmissions across the multi-access communication links and allocation of transmit power levels, while meeting the requirements on the signal-to-interference and noise ratio (SINR) at intended receivers. We prove that the problem can be represented as a mixed integer linear programming (MILP) and show that the latter yields a solution that consists of transmit power levels that are *strongly Pareto optimal*. We demonstrate that our MILP formulation can be used effectively to derive optimal scheduling and power levels for networks with as many as 30 designated communication links. We exhibit that the MILP formulation can be also effectively solved to provide tight upper and lower bounds (corresponding to an approximation factor Δ) for the optimum schedule length of networks with as many as 100 designated links. We prove that the integrated link scheduling and power control problem is NP-complete. Consequently, we develop and investigate a heuristic algorithm of polynomial complexity (O|L|^5) for solving the problem in a timely and practical manner. Our algorithm is based on the properties of a novel interference graph (the *Power-based Interference Graph)* that we have introduced,. We demonstrate that the frame length of schedules realized by our heuristic schemes reside in the 25 percentile of those attained by the optimal mechanism for randomly generated topologies.**

***Index Terms*—Ad hoc wireless networks, graph theory, mathematical programming/optimization, medium access control (MAC), power control.**

I. INTRODUCTION

One major widespread feature of wireless networks is the scarcity of spectrum. An important issue is therefore to design multiple access mechanisms to control channel utilization efficiently. This has motivated the need for channel spatial reuse, i.e. having users sufficiently apart use the same time slot, frequency band, or code.

A prevalent medium access scheme for channel spatial reuse in ad hoc wireless networks is *spatial time division multiple access* (STDMA), in which time is divided into fixed length slots that are organized cyclically ([1]-[7]). STDMA schemes (with no power control) proposed in the literature can be classified into two focal categories: *link scheduling* ([1]-[6]) and *node scheduling* ([4]-[7]). In each cycle, or timeframe, every time slot is allocated to different designated communication links (under link scheduling) or to different designated user nodes (under node scheduling) such that all transmissions are received *successfully* at their intended receivers. The schedule length is the most pertinent criterion of the performance of the scheduling algorithm and has been used widely in the literature ([2], [4]-[7]). The majority of the previous approaches (except for [3] and [6]) are based on the Protocol Interference Model [8] (or one of its variants) that assumes a transmission range is limited (typically circular) and beyond that range no interference is caused. The latter assumption generically leads to transformation of the node/link scheduling problem to a graph-theoretic problem (such as graph coloring), which, in turn, facilitates addressing the scheduling problem. However, one common limitation among the latter studies is that the Protocol Interference Model does not generally provide a comprehensive scrutiny of reality due to aggregate effect of interference in wireless networks and relativity of received power levels, among other reasons ([1], [3], [9], [14]). We note that in all of the above-mentioned studies the power level is assumed to be fixed.

In turn, power control has been widely used in the literature for multitude of purposes, including routing and topology control. Specifically, power control has been recently employed as part of the medium access control (MAC) in ad hoc wireless networks ([10]-[11], [13]-[14]). However, only a few of such power control-based MAC schemes are scheduling oriented: in [14], a simple heuristic is presented for the joint scheduling and power control operations that searches for an admissible set of communication links along with their transmission powers via two alternating phases. In phase one, the algorithm sequentially examines a set of constraints, including the *spatial separation* constraint (i.e., a node receiving from a neighboring node should be spatially separated from any other transmitter by at least a distance *D*), over the set of designated links to eliminate *strong interferences*. Phase two of the latter scheme employs power control operation to determine the admissible set of powers that could be used by the scheduled links, if one exists. However, the sequence of examining the latter constraints and the sequence of examining the communication links in phase one are not addressed in the paper, though they have a momentous effect on the performance of the algorithm. Moreover, the choice of parameter *D* is not directly addressed, though, as the authors indicate, the value of *D* has a significant impact on the performance of the algorithm. The behavior of the proposed

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algorithm is finally evaluated for a network consisting of only seven nodes.

The problem of finding an optimal link scheduling and power control policy that minimizes the total average transmission power in the ad hoc wireless network, subject to given constraints regarding the minimum average data rate per link, is addressed in [13]. Under the assumption that the achieved data rate on a link is a linear function of SINR and the available bandwidth (as an approximation to the Shannon capacity), it is shown that the problem can be reduced to a convex optimization problem using a duality approach. However, as pointed out by the authors, the complexity of the proposed approach is exponential as their numerical analysis is limited to few examples with no more than nine links.

In this paper, we develop a new mathematical programming formulation for minimizing the schedule length in ad hoc wireless networks based on the optimal joint scheduling of transmissions across the multi-access communication links and allocation of transmit power levels, while meeting the requirements on the signal-to-interference and noise ratio (SINR) at intended receivers. We prove that the problem can be represented as a mixed integer linear programming (MILP) and show that the latter yields a solution that consists of transmit power levels that are *strongly Pareto optimal*. We demonstrate that our MILP formulation can be used effectively to derive optimal scheduling and power levels for networks with as many as 30 designated communication links. We exhibit that the MILP formulation can be also effectively solved to provide tight upper and lower bounds (corresponding to an approximation factor Δ) for the optimum schedule length of networks with as many as 100 designated links. Our results provide important benchmarks for evaluation of heuristic scheduling algorithms in ad hoc wireless networks.

We prove that the integrated link scheduling and power control problem is NP-complete. Consequently, we develop and investigate an algorithm of polynomial complexity ($O|L|^5$, where |L| is the number of designated links) for solving the problem in a timely and practical manner. The heuristic algorithm is based on the properties of a novel interference graph (the *Power-based Interference Graph)* that we have introduced. Our algorithm satisfies the requirement that a minimum SINR level is met at all intended receivers, which directly translates into quality of service (QoS) in terms of bit error rate (BER) at individual receivers. We demonstrate that the frame length of schedules realized by our heuristic scheme reside in the 25 percentile of those attained by the optimal mechanism for random topologies.

The rest of the paper is structured as follows. In section II, we introduce the system model. The optimization model is presented in section III. We introduce the Power-based Interference Graph in section IV. Our heuristic algorithm is elaborated in section V. Numerical analysis and conclusions are discussed in section VI and section VII, respectively.

II. System Model

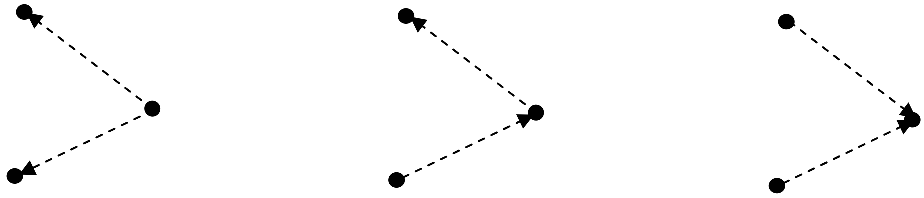
We consider a ad hoc wireless network with a set of nodes N, $N = \{1,2,\ldots,n\}$. During the period of operation under consideration in this paper, we assume network nodes to be immobile. Each node is capable of adjusting its transmit power continuously in a given range $[0,P_{max}]$ and in a packet-by-packet fashion. Every node, when scheduled to access the communication channel, transmits at a fixed data rate of W bits per second, and variations in transmission power merely affect the transmission range. A single transmission is intended for exactly one receiver (the unicasting constraint; Fig. 1.a). All nodes are equipped with identical half-duplex radios (the half-duplexing constraint; Fig. 1.b) and omni-directional antennas. We assume every transmission to occupy the entire bandwidth of the system under consideration. Channel time is slotted into identical synchronized time slots. Slot duration τ is assumed to be equal to the transmission time of a packet plus some overhead duration that includes the maximum propagation delay. A node can successfully receive from at most one other node in the same time slot (the receptivity constraint; Fig. 1.c). We are concerned with the fixed assignment of transmissions for the designated links in a frame. Thus, once the optimal transmission patterns (i.e., the arrangement of transmissions and the associated transmit power levels) are determined, the frame is repeated in the time axis.

A directed communication link (or simply a link) l_{ij} can be established from node i (or equivalently transmitter i) to node j (or equivalently receiver j) if there exists a power P, $P \in [0, P_{\max}]$, under which the *signal-to-noise ratio* (SNR) at node j is not less than a threshold γ, i.e.

$$G_{ij} P / \eta_j \geq \gamma, \tag{1}$$

in which G_{ij} is the propagation gain (incorporating the effects of link loss phenomena such as fading and shadowing) for direct transmission from node i to node j, and η_j is the thermal noise power at receiver j [16]. It has been commonly assumed that G_{ij} is equal to G_{ji} ([2], [7], [11], [14]). We do not make this assumption in this paper, since our mathematical model and algorithm apply also to the general case. The set of all designated communication links that we need to assign a time slot (based on the network layer and underlying routing considerations) is denoted by L. N^{Tx} represents a subset of nodes in N that are the transmitter associated with at least one of the links in L. Similarly, N^{Rx} denotes a subset of nodes in N that are the receiver associated with at least one of the links in L. $G = [G_{ij}]$ is the *propagation gain matrix* representing the propagation gain from each of the nodes in N^{Tx} to each of the nodes in N^{Rx}.

Let $i \to j$ and $P_{ij}^{(t)}$ denote a direct transmission over the link l_{ij} and the corresponding transmit power level in time slot t, respectively, $P_{ij}^{(t)} \in [0, P_{\max}]$. A *transmission scenario* $S(t) = \{i_1 \to j_1, i_2 \to j_2, \cdots, i_M \to j_M\}$ is defined as a candidate set of transmissions that are considered to all take place at time slot t, where all transmitting and receiving nodes



1.a. Simultaneous infeasible transmissions based on the unicasting constraint.

1.b. Simultaneous infeasible transmissions based on the half-duplexing constraint.

1.c. Simultaneous infeasible transmissions based on the receptivity constraint.

Figure 1. Simultaneous infeasible transmissions under different constraints.

are distinct [12]. Note that the distinction of all transmitting and receiving nodes ensures that the unicasting, half-duplexing, and receptivity constraints are satisfied in every transmission scenario (Fig. 1).

For such a transmission scenario *S(t),* under power vector $P(t)=(P^{(t)}_{i_1 j_1}, P^{(t)}_{i_2 j_2},\ldots,P^{(t)}_{i_M j_M})$, $0 \le P^{(t)}_{i_k j_k} \le P_{\max}, k=1,2,\ldots,M$, we say that the transmission from i_k is *successful* if the *signal-to- interference and noise ratio* (SINR) at j_k is not less than the threshold γ, i.e.

$$G_{i_k j_k} P^{(t)}_{i_k j_k} /(\eta_{j_k} + \sum_{\substack{r=1 \\ r\neq k}}^{M} G_{i_r j_k} P^{(t)}_{i_r j_r}) \ge \gamma, \quad k=1,\cdots,M, \qquad (2)$$

or equivalently,

$$G_{i_k j_k} P^{(t)}_{i_k j_k} - \gamma \sum_{\substack{r=1 \\ r\neq k}}^{M} G_{i_r j_k} P^{(t)}_{i_r j_r} \ge \gamma \eta_{j_k}, \quad k=1,\cdots,M. \qquad (3)$$

The value of the prespecified threshold γ depends on the acceptable BER, detector structure, modulation/demodulation scheme, and channel coding/decoding algorithm [16]. We refer to such a model for successful reception of a packet as the *SINR-based Interference Model* [9]. The left-hand-side of every inequality in relation (3) is linear in terms of $P^{(t)}_{i_1 j_1}, P^{(t)}_{i_2 j_2},\ldots,P^{(t)}_{i_M j_M}$, which defines an M-dimensional half-space. The intersection of all of these half-spaces is described geometrically by an unbounded polyhedral "cone" in Euclidean space. The apex of the "cone" is the power vector $P^{AP}(t)=(P^{(AP,t)}_{i_1 j_1}, P^{(AP,t)}_{i_2 j_2},\ldots,P^{(AP,t)}_{i_M j_M})$ that satisfies system (3) of linear inequalities in equality form. We refer to the latter vector as the *apex solution* of the system of linear inequalities. Based on the *Perron-Frobenious theorem*, it can be shown that the following statements are equivalent ([17]-[20]):

1) System (3) of linear inequalities has a nonnegative solution (i.e., each of its components is nonnegative).

2) The apex solution of system (3) of linear inequalities is nonnegative (component-wise).

Also, if statement 1 (or statement 2) is valid, then $P^{'}(t) \ge P^{AP}(t)$ (component-wise), where $P'(t)=(P^{'(t)}_{i_1 j_1}, P^{'(t)}_{i_2 j_2},\ldots,P^{'(t)}_{i_M j_M})$ is an arbitrary nonnegative solution of system (3) of linear inequalities ([19]-[20]). We, henceforth, refer to the above-mentioned features of system (3) of inequalities as *Fact 1*.

*Definition 1***.** We define a transmission scenario $S(t)=\{i_1 \to j_1, i_2 \to j_2, \cdots, i_M \to j_M\}$ to be feasible under power vector $P(t)=(P^{(t)}_{i_1 j_1}, P^{(t)}_{i_2 j_2},\ldots,P^{(t)}_{i_M j_M})$, $0 \le P^{(t)}_{i_k j_k} \le P_{\max}, k=1,2,\ldots,M$, if all the transmissions of *S(t)* are successful under *P(t)*. We further define the transmission scenario *S(t)* to be feasible if there exists at least one power vector $P(t)=(P^{(t)}_{i_1 j_1}, P^{(t)}_{i_2 j_2},\ldots,P^{(t)}_{i_M j_M})$, $0 \le P^{(t)}_{i_k j_k} \le P_{\max}, k=1,2,\ldots,M$, under which all the transmissions of *S(t)* are successful.

Definition 2. We define the power vector $P(t)=(P^{(t)}_{i_1 j_1}, P^{(t)}_{i_2 j_2},\ldots,P^{(t)}_{i_M j_M})$, $0 \le P^{(t)}_{i_k j_k} \le P_{\max}, k=1,2,\ldots,M$, to be strongly Pareto optimal with respect to the transmission scenario $S(t)=\{i_1 \to j_1, i_2 \to j_2, \cdots, i_M \to j_M\}$ if

1) *S(t)* is a feasible transmission scenario under *P(t),* and

2) Any other power vector $P'(t)=(P^{'(t)}_{i_1 j_1}, P^{'(t)}_{i_2 j_2},\ldots,P^{'(t)}_{i_M j_M})$, $0 \le P^{'(t)}_{i_k j_k} \le P_{\max}, k=1,2,\ldots,M$, satisfying condition (1) would require at least as much power from every transmitter, i.e. $P'(t) \ge P(t)$ component-wise.

Similar to previous studies in the literature ([4]-[7]), we assume that the transmission requirements in the network are uniform, i.e. (at least) one packet is required to be transmitted across each designated link at every timeframe. As it will be shown later in the paper, this assumption can be easily relaxed to accommodate nonuniform traffic requirements; we retain it here for presentation simplicity. Our objective in this paper is to design a timeframe with the minimum schedule length that satisfies the following conditions:

1) The timeframe includes at least one time slot for transmission across every designated link $l_{ij}, l_{ij} \in L$, $i=1,\cdots,n, j=1,\cdots,n, i \ne j$.

2) The power vector allocated to transmissions at every time slot is strongly Pareto optimal with respect to the underlying transmission scenario.

We refer to this problem as the *integrated link scheduling and power control problem* (ILSP). Further, we refer to the above two conditions as the *feasibility conditions* for the ILSP problem.

III. Linear Programming Formulation for the ILSP Problem

In this section, we develop and investigate a mixed integer linear programming (MILP) formulation for the integrated link scheduling and power control problem. The input for the optimization problem is the set of designated links (L) and the associated physical-layer (PHY) parameters consisting of propagation gain matrix *(G)*, maximum transmit power level ($P_{\max}$), minimum required SINR (γ), and thermal noise at every receiver j ($\eta_j, j \in N^{Rx}$). It is apparent that the optimum length of the timeframe cannot be larger than the cardinality of set L (denoted by $|L|$).[1] For every designated link l_{ij} and every time slot t, we define the binary variable $X_{ij}^{(t)}$ as follows:

$$X_{ij}^{(t)} = \begin{cases} 1, & \text{if time slot } t \text{ is allocated to link } l_{ij} \\ 0, & \text{otherwise} \end{cases} \quad ,(i,j) \in L, t = 1,\cdots,|L|. \quad (4)$$

The decision variables in our mathematical modeling are $X_{ij}^{(t)}$'s and $P_{ij}^{(t)}$'s. Therefore, every *solution* of the MILP formulation (and the ILSP problem) can be represented as $(\mathcal{X}, \mathcal{P})$, where $\mathcal{X} = \{X_{ij}^{(t)}, (i,j) \in L, t = 1,\cdots,|L|\}$ and $\mathcal{P} = \{P_{ij}^{(t)}, (i,j) \in L, t = 1,\cdots,|L|\}$. It can be seen that the constraint

$$\sum_{(i,j)\in L} X_{ij}^{(t)} + \sum_{(j,k)\in L} X_{jk}^{(t)} \le 1, t = 1,\ldots,|L|, j \in N^{Tx} \cup N^{Rx}, \quad (5)$$

guarantees that node j is either the transmitter or the receiver of at most one of the transmissions scheduled for time slot t. This feature simultaneously imposes the unicasting, half-duplexing, and receptivity constraints at every time slot t. Also, the set of quadratic constraints

$$G_{ij} P_{ij}^{(t)} - \gamma \sum_{(r,s)\in L-(i,j)} G_{rj} P_{rs}^{(t)} X_{rs}^{(t)} - \gamma \eta_j \ge \Phi(X_{ij}^{(t)} - 1) \quad ,(i,j) \in L, t = 1,\ldots,|L|, \quad (6)$$

imposes the SINR requirement for a transmission over link l_{ij} at time slot t (see relation (3)), where Φ is a sufficiently large positive number. Note that if no transmission is scheduled to take place over link l_{ij} at time slot t (i.e., $X_{ij}^{(t)} = 0$), the associated constraint becomes *redundant*.

We next (through Lemmas 1-2 and Theorems 3-5) prove that the integrated link scheduling and power control problem can be linearly formulated as the following (which we refer to, henceforth, as the *MILP formulation*):

$$\textit{Minimize} \quad Z(\mathcal{X},\mathcal{P}) = \sum_{t=1}^{|L|} \sum_{(i,j)\in L} (c_t X_{ij}^{(t)} + \varepsilon P_{ij}^{(t)}) \quad (7)$$

s.t.

$$\sum_{t=1}^{|L|} X_{ij}^{(t)} \ge 1, \quad t = 1,\ldots,|L|, \quad (8)$$

$$\sum_{(i,j)\in L} X_{ij}^{(t)} + \sum_{(j,k)\in L} X_{jk}^{(t)} \le 1, \quad t = 1,\ldots,|L|, j \in N^{Tx} \cup N^{Rx}, \quad (9)$$

$$G_{ij} P_{ij}^{(t)} - \gamma \sum_{(r,s)\in L-(i,j)} G_{rj} P_{rs}^{(t)} - \gamma \eta_j \ge \Phi(X_{ij}^{(t)} - 1), \quad (i,j) \in L, t = 1,\ldots,|L|, \quad (10)$$

$$0 \le P_{ij}^{(t)} \le P_{\max}, \quad (i,j) \in L, t = 1,\ldots,|L|, \quad (11)$$

$$X_{ij}^{(t)} = 0 \text{ or } 1, \quad (i,j) \in L, t = 1,\ldots,|L|, \quad (12)$$

whereby ε is a sufficiently small positive number, and c_t's are positive constants defined as

$$c_t = t.|L|.c_{t-1}, t = 2,\ldots,|L|, \quad (13)$$

and $c_1 = 1$. Note that the quadratic constraint (6) is changed into the linear constraint (10) by excluding the $X_{rs}^{(t)}$ variables.

Lemma 1. Every feasible solution of the MILP formulation yields a feasible transmission scenario at each time slot.

Proof. Let's consider an arbitrary time slot t under a feasible solution $(\mathcal{X},\mathcal{P})$ of the MILP formulation, where $\mathcal{X} = \{X_{ij}^{(t)}, (i,j) \in L, t = 1,\cdots,|L|\}$ and $\mathcal{P} = \{P_{ij}^{(t)}, (i,j) \in L, t = 1,\cdots,|L|\}$. Relation (9) guarantees that all the transmitting and receiving nodes in the set of transmissions in time slot t are distinct, i.e. $S(t) = \{i \to j \mid X_{ij}^{(t)} = 1\}$ forms a transmission scenario. Assume *S(t)* is equal to $\{i_1 \to j_1, i_2 \to j_2, \cdots, i_M \to j_M\}$. Now, we claim that transmission scenario *S(t)* under power vector $P(t) = (P_{i_1 j_1}^{(t)}, P_{i_2 j_2}^{(t)}, \ldots, P_{i_M j_M}^{(t)})$ is feasible: Let's consider an arbitrary transmission $i_k \to j_k$ in *S(t)*. Since $X_{i_k j_k}^{(t)} = 1$, based on relation (10) (associated with the transmission over link $l_{i_k j_k}$ in time slot t) we have

$$G_{i_k j_k} P_{i_k j_k}^{(t)} - \gamma \sum_{(r,s)\in L-(i_k,j_k)} G_{rj_k} P_{rs}^{(t)} - \gamma \eta_{j_k} \ge 0. \quad (14)$$

Since $\mathcal{X}(t) \subseteq L$, we have

$$\sum_{(r,s)\in L-(i_k,j_k)} G_{rj_k} P_{rs}^{(t)} \ge \sum_{(r,s)\in \mathrm{X}(t)-(i_k,j_k)} G_{rj_k} P_{rs}^{(t)} \quad (15)$$

$$= \sum_{\substack{r=1 \\ r\ne k}}^{M} G_{i_r j_k} P_{i_r j_r}^{(t)}. \quad (16)$$

Based on relations (14)-(16), we conclude

$$G_{i_k j_k} P_{i_k j_k}^{(t)} - \gamma \sum_{\substack{r=1 \\ r\ne k}}^{M} G_{i_r j_k} P_{i_r j_r}^{(t)} \ge \gamma \eta_{j_k}, \quad (17)$$

which (along with relation (11)) indicates that the arbitrary transmission $i_k \to j_k$ of the transmission scenario *S(t)* under power vector *P(t)* is successful. Consequently, transmission scenario *S(t)* is feasible under power vector *P(t)*. Hence, every optimum solution of the MILP formulation yields a feasible transmission scenario at each time slot t, $t = 1,\cdots,|L|$. ■

[1] The latter bound is simply achieved by assigning exactly one link to every time slot and by selecting a power level for every transmission that satisfies relation (1) in equality form.

Following Lemma 2 in Appendix A, we prove the subsequent theorem. Please see Appendix A for details.

Theorem 3. Every optimum solution of the MILP formulation yields a strongly Pareto optimal power vector with respect to the underlying transmission scenario at each time slot.[2]

Theorem 4. Every optimum solution of the MILP formulation satisfies the feasibility conditions of the ILSP problem.

Proof. Since every feasible solution of the MILP formulation satisfies relation (8), it is guaranteed that at least one time slot is allocated to every link l_{ij} under an optimum solution of the MILP formulation, $l_{ij} \in L$. Consequently, every optimum solution of the MILP formulation satisfies the first feasibility condition of the ILSP problem. Moreover, based on Theorem 3, every optimum solution of the MILP formulation satisfies the second feasibility condition of the ILSP problem, which completes the proof. ■

Note that the costs c_t's assigned to different time slots are monotonically increasing function of the slot index (relation (13)). This feature motivates the mathematical model to make the timeframe shorter. In Theorem 5, we prove that selecting the values of c_t's according to relation (13) leads to the minimization of the frame length.

Theorem 5. Every optimum solution of the MILP formulation is an optimum solution of the ILSP problem.

Proof. Let $(\mathcal{X}', \mathcal{P}')$ and T' represent an optimum solution of the ILSP problem and the associated (minimum) frame length, respectively, where $\mathcal{X}' = \{X_{ij}^{'(t)}, (i,j) \in L, t = 1,\cdots,|L|\}$ and $\mathcal{P}' = \{P_{ij}^{'(t)}, (i,j) \in L, t = 1,\cdots,|L|\}$. We have

$$Z(\mathcal{X}',\mathcal{P}') = \sum_{t=1}^{T'} \sum_{(i,j)\in L} (c_t X_{ij}^{'(t)} + \varepsilon P_{ij}^{'(t)}) \quad (18)$$

$$\leq \sum_{t=1}^{T'} (c_t |L| + \varepsilon \sum_{(i,j)\in L} P_{ij}^{'(t)}) \quad (19)$$

$$\leq c_{T'} |L| T' + \varepsilon \sum_{t=1}^{T'} \sum_{(i,j)\in L} P_{ij}^{'(t)}, \quad (20)$$

where inequality (19) is deduced by the fact that there cannot be more than $|L|$ simultaneous transmissions in every time slot. Since ε is a sufficiently small number, then independent of L and independent of the values of $|L|$, T', and $P_{ij}^{'(t)}, t = 1,\cdots,T'$, we have

$$\varepsilon \sum_{t=1}^{T'} \sum_{(i,j)\in L} P_{ij}^{'(t)} \leq c_{T'} |L|. \quad (21)$$ [3]

By considering relations (13), (20), and (21), we conclude

$$Z(\mathcal{X}', \mathcal{P}') \leq c_{T'+1}. \quad (22)$$

Now, suppose there is an optimum solution of the MILP formulation $(\mathcal{X}^* = \{X_{ij}^{*(t)}, (i,j) \in L, t = 1,\cdots,|L|\}$, $\mathcal{P}^* = \{P_{ij}^{*(t)}, (i,j) \in L, t = 1,\cdots,|L|\})$ that yields a frame length T^*, where $T^* > T'$. We have

$$Z(\mathcal{X}^*,\mathcal{P}^*) = \sum_{t=1}^{T^*} \sum_{(i,j)\in L} (c_t X_{ij}^{*(t)} + \varepsilon P_{ij}^{*(t)}) \quad (23)$$

$$\geq \sum_{t=1}^{T'} \sum_{(i,j)\in L} (c_t X_{ij}^{*(t)} + \varepsilon P_{ij}^{*(t)}) + c_{T'+1}, \quad (24)$$

which relation (24) is valid due to the assumption that $T^* > T'$. Therefore,

$$Z(\mathcal{X}^*, \mathcal{P}^*) > c_{T'+1}. \quad (25)$$

Considering relations (22) and (25), we have

$$Z(\mathcal{X}', \mathcal{P}') < Z(\mathcal{X}^*, \mathcal{P}^*), \quad (26)$$

which contradicts the optimality of $(\mathcal{X}^*, \mathcal{P}^*)$ for the MILP formulation (noting that based on the definition of the ILSP problem, $(\mathcal{X}', \mathcal{P}')$ is a feasible solution for the MILP formulation). Therefore, T^* cannot be strictly greater than T'. Consequently, based on Theorem 4, we conclude that every optimum solution of the MILP formulation is an optimum solution of the ILSP problem. ■

Corollary 5.1. The minimum frame length is equal to

$$T^* = \underset{(i,j)\in L}{Max} \{t : X_{ij}^{*(t)} = 1\}, \quad (27)$$

where $X_{ij}^{*(t)}$'s are the optimum values of $X_{ij}^{(t)}$ variables derived by solving the MILP formulation.

As illustrated in section VI, based on Theorem 5, the MILP formulation can be used to derive an optimal solution of the ILSP problem as well as to attain non-trivial tight upper and lower bounds for the optimal solution of the ILSP problem for dozens of designated links. Yet, in the following theorem, we prove that the ILSP problem is NP-complete. This feature exhibits the need for a heuristic algorithm to provide an acceptable solution to the ILSP problem in a timely manner for networks with hundreds of designated links.

Theorem 6. The integrated link scheduling and power control problem is NP-complete.

Proof. Please see Appendix B for the proof.

IV. The Power-based Interference Graph

In this section, we introduce the notion of the *Power-based Interference Graph*, which is used as the basic building block of our heuristic algorithm in the subsequent section.

Lemma 7. Transmission scenario $S(t) = \{i_1 \to j_1, i_2 \to j_2\}$ is feasible if and only if

$$0 \leq (\gamma\eta_{j_1} G_{i_2 j_2} + \gamma^2 \eta_{j_2} G_{i_2 j_1}) / (G_{i_1 j_1} G_{i_2 j_2} - \gamma^2 G_{i_2 j_1} G_{i_1 j_2}) \leq P_{\max} \quad (28)$$

and

$$0 \leq (\gamma\eta_{j_2} G_{i_1 j_1} + \gamma^2 \eta_{j_1} G_{i_1 j_2}) / (G_{i_1 j_1} G_{i_2 j_2} - \gamma^2 G_{i_2 j_1} G_{i_1 j_2}) \leq P_{\max} \quad (29)$$

Proof. Suppose that transmissions $i_1 \to j_1$ and $i_2 \to j_2$ are the only transmissions in the network and i_1, j_1, i_2, j_2 are distinct nodes. Based on relation (3), transmissions $i_1 \to j_1$ and $i_2 \to j_2$ under power vector $P(t) = (P_{i_1 j_1}^{(t)}, P_{i_2 j_2}^{(t)})$ are

[2] Note that Theorem 3 is not necessarily valid for every feasible solution of the MILP formulation.

[3] For instance, ε can be any positive number less than $[|L|^2 P_{\max}]^{-1}$. The latter ensures that the left-hand-side of relation (21) is less than unity, and therefore, relation (21) is satisfied.

successful if and only if *P(t)* satisfies the following system of linear inequalities:

$$\begin{cases} G_{i_1 j_1} P^{(t)}_{i_1 j_1} - \gamma G_{i_2 j_1} P^{(t)}_{i_2 j_2} \geq \gamma \eta_{j_1} \\ -\gamma G_{i_1 j_2} P^{(t)}_{i_1 j_1} + G_{i_2 j_2} P^{(t)}_{i_2 j_2} \geq \gamma \eta_{j_2} \end{cases} . \quad (30)$$

Moreover, based on Fact 1, system (30) of linear inequalities has a solution $P(t) = (P^{(t)}_{i_1 j_1}, P^{(t)}_{i_2 j_2})$, $0 \leq P^{(t)}_{i_k j_k} \leq P_{\max}, k = 1,2$, if and only if the components of the apex solution of system (30) of linear inequalities ($P^{(AP,t)}_{i_1 j_1}$ *and* $P^{(AP,t)}_{i_2 j_2}$) satisfies the following conditions:

$$\begin{cases} 0 \leq P^{(AP,t)}_{i_1 j_1} \leq P_{\max} \\ 0 \leq P^{(AP,t)}_{i_2 j_2} \leq P_{\max} \end{cases} . \quad (31)$$

But, the values of $P^{(AP,t)}_{i_1 j_1}$ and $P^{(AP,t)}_{i_2 j_2}$ can be computed as

$$P^{(AP,t)}_{i_1 j_1} = (\gamma \eta_{j_1} G_{i_2 j_2} + \gamma^2 \eta_{j_2} G_{i_2 j_1}) / (G_{i_1 j_1} G_{i_2 j_2} - \gamma^2 G_{i_2 j_1} G_{i_1 j_2}) \quad (32)$$

and

$$P^{(AP,t)}_{i_2 j_2} = (\gamma \eta_{j_2} G_{i_1 j_1} + \gamma^2 \eta_{j_1} G_{i_1 j_2}) / (G_{i_1 j_1} G_{i_2 j_2} - \gamma^2 G_{i_2 j_1} G_{i_1 j_2}), \quad (33)$$

which completes the proof. ■

The *Power-based Interference Graph* is defined as an undirected graph *G(V,E)*, in which *V* and *E* are the set of vertices and the set of edges of graph *G*, respectively. Every vertex in *V* is represented by an ordered pair *(i, j)*. Vertex *(i, j)* belongs to *V* if and only if the communication link *(i, j)* belongs to *L* (Therefore, there is a one-to-one correspondence between every vertex in *V* and every communication link in *L*).[4] Vertices (i_1, j_1) and (i_2, j_2) are connected to each other by an edge in *G* if and only if one of the following conditions is satisfied:

1) Nodes i_1, j_1, i_2, and j_2 are not distinct.

2) Transmission scenario $S(t) = \{i_1 \to j_1, i_2 \to j_2\}$ does not satisfy relation (28) or relation (29) (i.e., transmission scenario $S(t) = \{i_1 \to j_1, i_2 \to j_2\}$ is not feasible).

Consequently, based on Lemma 7, every two adjacent vertices in the Power-based Interference Graph have the property that successful simultaneous transmissions over the associated links under any power allocation is impossible.

Theorem 8. Let $MIS = \{(i_1, j_1), (i_2, j_2), \cdots, (i_M, j_M)\}$ denote an arbitrary maximal independent set of the Power-based Interference Graph. Then, $S(t) = \{i_1 \to j_1, i_2 \to j_2, \cdots, i_M \to j_M\}$ is a transmission scenario with the property that it is not a proper subset of any feasible transmission scenario.

Proof. First, we note that the set $S(t) = \{i_1 \to j_1, i_2 \to j_2, \cdots, i_M \to j_M\}$ is a transmission scenario, since all nodes $i_1, j_1, i_2, j_2, \cdots, i_M, j_M$ are distinct. Now, assume that $S'(t) = \{i_1 \to j_1, \cdots, i_M \to j_M, \cdots, i_{M+K} \to j_{M+K}\}$ is a feasible transmission scenario, $K > 1$ (Clearly, *S(t)* is then a proper subset of *S'(t)*). By definition of a feasible transmission scenario, then there exists a power vector $P(t) = (P'^{(t)}_{i_1 j_1}, \ldots, P'^{(t)}_{i_M j_M}, \cdots, P'^{(t)}_{i_{M+K} j_{M+K}})$, $0 \leq P'^{(t)}_{i_k j_k} \leq P_{\max}, k = 1,2,\ldots,M+K$, under which simultaneous transmission of all the elements in *S'(t)* yields in acceptable SINR value (i.e., an SINR value that is larger than or equal to γ) at each intended receiver $j_k, k = 1,2,\ldots,M+K$. Therefore, under the same transmission power vector *P'(t),* simultaneous transmission of every pair of transmissions in *S'(t)* is successful (i.e., it yields an acceptable SINR values at the receivers). Consequently, the set $\{(i_1, j_1), \cdots, (i_M, j_M), \cdots, (i_{M+K}, j_{M+K})\}$ is also an independent set of the Power-based Interference Graph. But, the latter contradicts the maximality of the maximal independent set $MIS = \{(i_1, j_1), (i_2, j_2), \cdots, (i_M, j_M)\}$ and the proof is complete. ■

V. THE INTEGRATED SCHEDULING AND POWER CONTROL ALGORITHM (ISPA)

Based on Theorem 6, it is quite unlikely to find an efficient algorithm that can attain an optimal solution of the ILSP problem for networks with large number of designated links in a reasonable time. This motivates the need for an efficient heuristic algorithm that provides an acceptable solution to any instance of the problem in a polynomial time. In this section, we introduce a novel polynomial heuristic (ISPA) that efficiently solves the ILSP problem.

The *Integrated Scheduling and Power Control Algorithm* (ISPA) (Fig. 2) initially generates the Power-based Interference Graph as illustrated in section IV. Then, by using the *Minimal Degree Greedy Algorithm* (MDGA) [23], ISPA finds a maximal independent set $MIS = \{(i_1, j_1), (i_2, j_2), \cdots, (i_M, j_M)\}$ of the Power-based Interference Graph for possible allocation to slot 1. Based on Theorem 8, the set $S(1) = \{i_1 \to j_1, i_2 \to j_2, \cdots, i_M \to j_M\}$ has the property that it satisfies the half-duplexing, unicasting, and receptivity constraints. Furthermore, based on Theorem 8, transmission scenario *S*(1) is maximal with respect to the property that it is not a proper subset of any feasible transmission scenario. Moreover, based on the definition of the Power-based Interference Graph, every subset of *S*(1) with cardinality equal to two is a feasible transmission scenario, which, in turn, implies that there is also a high compatibility among all the transmissions in *S*(1) (i.e., there is a good chance that *S*(1) (or a large subset of *S*(1)) is also a feasible transmission scenario). Consequently, ISPA utilizes transmission scenario *S*(1) as a suitable initial point for its search for a maximal feasible transmission scenario for allocation to time slot 1. This search consists of two consecutive stages (i.e., Pruning stage and Maximality stage). After performing the latter search, the resulting feasible transmission scenario is allocated to the first time slot. Subsequently, the nodes (and the incident edges) associated with the allocated transmissions are removed from

[4] For the nonuniform traffic case, wherein every timeframe $R_{i,j}$ packets are required to be transmitted across *(i, j)*, it suffices to define $R_{i,j}$ vertices in *V* associated with the communication link *(i, j)*.

Input: An instance of the ILSP problem
Output: A near optimal solution for the ILSP problem

The Integrated Scheduling and Power Control Algorithm [ISPA]

Step 1. Construct the Power-based Interference Graph *G=(V,E)*. Let *H = G*.
Step 2. Find a maximal nodal independent set of graph *H* using the Minimum Degree Greedy Algorithm (MDGA).
Step 3. (Pruning stage) Find a feasible transmission scenario *S** of graph *H* by using the SMIRA algorithm.
Step 4. (Maximality stage) Find a maximal feasible transmission scenario *MS* that is a superset of the feasible transmission scenario *S**.
Step 5. Let *H'* denote the subgraph of *H induced* by *MS* [22]. If *H'* is a trivial graph, stop; otherwise, set *H = H'* and return to Step 2.

Figure 2. Outline of heuristic ISPA for the integrated link scheduling and power control problem.

the Power-based Interference Graph. Next, the algorithm iterates the same procedure for the next time slot. ISPA terminates when the residual Power-based Interference Graph becomes a *trivial* graph [22].

We note that MDGA is a delightfully simple algorithm, which has been proven to be much better than previously claimed. In particular, MDGA almost always yields a solution that is at least half the independence number of a random graph ([24]-[25]). In the following, we explain the Pruning and Maximality stages in more detail:

A. Pruning Stage

The Pruning Stage aims to find a maximum feasible transmission scenario over all the subsets of transmission scenario $S(t)=\{i_1 \rightarrow j_1, i_2 \rightarrow j_2, \cdots, i_M \rightarrow j_M\}$. This problem is known to be an NP-complete problem [26]. We adopt the *Stepwise Maximum Interference Removal Algorithm* (SMIRA) that was originally introduced for the downlink connection removal of the cellular radio systems [27]. In the SMIRA algorithm, at every step a transmission is removed from the group of potential transmissions, which on average causes most interference to other receivers (i.e., the non-intended receivers) or is most sensitive to interference from other transmissions. SMIRA iterates this process until the resulting transmission scenario is feasible. It has been shown that SMIRA achieves a close to optimum performance ([26]-[27]), and it outperforms other removal schemes (such as the *Stepwise Removal Algorithm*), which have been earlier proposed. The complexity of the algorithm is $O(M^4)$, which is dominated by solving an eigenproblem for an $(M-k+1) \times (M-k+1)$ matrix at the k-th transmission removal, $k = 1, \ldots, M-1$.

B. Maximality Stage

The feasible transmission scenario *S** induced by the pruning stage is not necessarily maximal with respect to the underlying residual Power-based Interference Graph. This is due to the fact that inclusion of some of the remaining transmissions (i.e., transmissions which are not yet allocated to the previous time slots) into the set *S** might yield a new feasible transmission scenario (which is obviously a superset of set *S**). The purpose of the maximality stage is to ensure that the set of transmissions allocated to every time slot forms a maximal feasible transmission scenario. This stage is performed in an iterative fashion by considering the remaining transmissions for possible inclusion in the transmission scenario *S**.

The computational complexity of ISPA is $O(|L|^5)$ and is dominated by Step 3 and Step 4, which have a complexity of $O(|L|^4)$ at every time slot.

VI. NUMERICAL ANALYSIS

In our numerical analysis, nodes in the network are immobile and distributed independently and uniformly in 2500 x 2500 meters square. Thermal noise power at every receiver (η_j) is equal to -90 *dBm*. The minimum required SINR level (γ) and the maximum transmit power level ($P_{\max}$) are set to 10 *dB* and 300 *mW*, respectively. We assume that the propagation gain between every two nodes to be inversely proportional to the fourth order of the distance between them. The length of every time slot is 550 μsec.

In Fig. 3, we depict the optimum schedule lengths (in milli-seconds) derived by solving the MILP formulation (in ILOG CPLEX 7.0 software) for various randomly generated topologies. In Fig. 3, we also illustrate the length of the schedules attained by applying the ISPA algorithm to the same random topologies. As expected, the difference between the two curves is a monotonically increasing function of the number of designated links. Yet, we observe that the frame length of schedules realized by the ILSP algorithm reside in the 25 percentile of those attained by the optimal mechanism for the randomly generated topologies.

In Fig. 4, we depict the lower and upper bounds derived by solving the MILP formulation in ILOG CPLEX 7.0 for various randomly generated topologies with as many as 100 designated links. For networks with as many as 30 designated links (i.e. $|L| \leq 30$), the derived lower bounds and upper bounds are identical. This is due to the fact that the optimum solutions of the MILP formulation corresponding to the latter networks are achievable (see Fig. 3). For |L|>30, the upper bound of the optimum frame length is associated with a

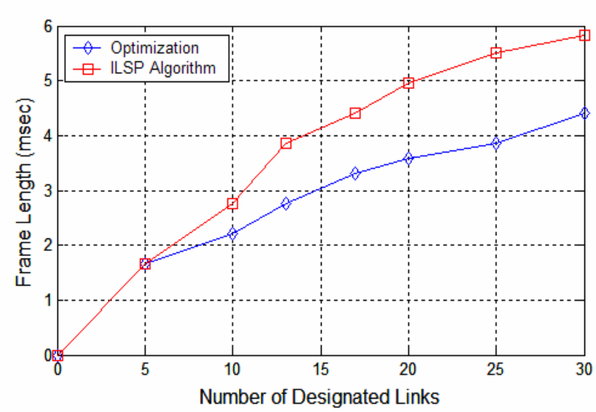


Figure 3. Comparison between the performance of the ILSP algorithm and the optimum performance for various number of designated links.

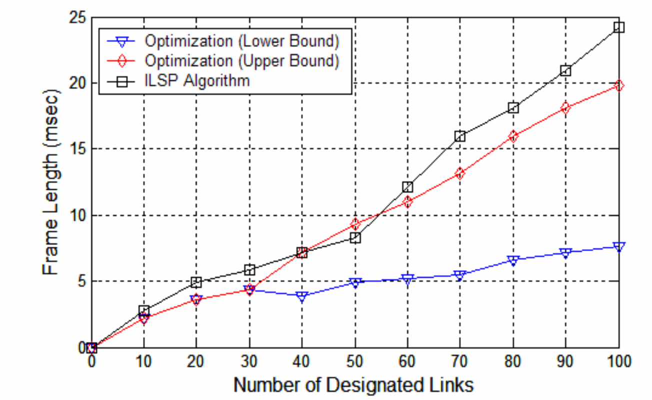


Figure 4. Illustration of the lower and upper bounds of the optimum frame length and the schedule length attained by the ILSP algorithm.

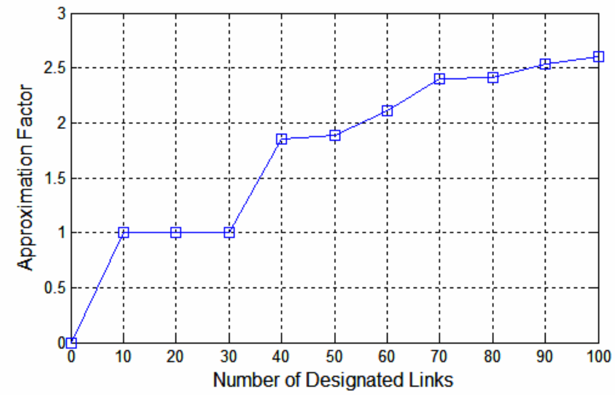


Figure 5. Illustration of the approximation factor corresponding to the feasible solution of the MILP formulation for various number of designated links.

feasible (but suboptimal) solution of the MILP formulation, while the lower bound corresponds to an infeasible (but superoptimal) solution of the MILP formulation. In Fig. 4, we also illustrate the frame length attained based on the ILSP algorithm for the corresponding topologies. We note the proximity of the frame length attained by the ILSP algorithm and the upper bound for the optimum frame length.

In Fig. 5, we illustrate the approximation factor Δ corresponding to the feasible solution provided by solving the MILP formulation. The above factor is calculated for the underlying topologies by dividing the upper bound of the optimum frame length by the corresponding lower bound. As expected, Δ is an increasing function of number of designated links. For randomly generated topologies with 100 designated links, we observe an approximation factor of 2.6 attained by solving the MILP formulation.

VII. Conclusions

In this paper, we develop a new mathematical programming formulation for minimizing the schedule length in ad hoc wireless networks based on the optimal joint scheduling of transmissions across the multi-access communication links and allocation of transmit power levels, while meeting the requirements on the signal-to-interference and noise ratio (SINR) at intended receivers. We prove that the problem can be represented as a mixed integer linear programming (MILP). We demonstrate that the MILP formulation can be used to derive an optimal solution of the ILSP problem as well as to attain non-trivial tight upper and lower bounds for the optimal solution of the ILSP problem for dozens of designated links. We prove that the integrated link scheduling and power control problem is NP-complete. Consequently, we develop and investigate a heuristic of polynomial complexity for solving the problem in a timely and practical manner.

Appendix A

Lemma 2. Every optimum solution of the MILP formulation $(\mathcal{X}^*=\{X_{ij}^{*(t)},(i,j)\in L, t=1,\cdots,|L|\},\quad \mathcal{P}^*=\{P_{ij}^{*(t)},(i,j)\in L, t=1,\cdots,|L|\})$, satisfies the following two features:

1) $$P_{rs}^{*(t)}=0,\ if\ X_{rs}^{*(t)}=0,\quad (r,s)\in L, t=1,\cdots,|L|, \qquad (34)$$

and

2) $$\sum_{k=1}^{M} P_{i_k j_k}^{*(t)} \le \sum_{k=1}^{M} P_{i_k j_k}^{'(t)},\quad t=1,\cdots,|L|, \qquad (35)$$

where $S^*(t)=\{i_1\to j_1, i_2\to j_2,\cdots,i_M\to j_M\}$ represent the set of transmissions at time slot t under $(\mathcal{X}^*,\mathcal{P}^*)$, and $P(t)=(P_{i_1j_1}^{'(t)},P_{i_2j_2}^{'(t)},\ldots,P_{i_Mj_M}^{'(t)})$, $0\le P_{i_kj_k}^{'(t)}\le P_{\max}, k=1,2,\ldots,M$, represent an arbitrary power vector under which transmission scenario $S^*(t)$ is feasible.

Proof. Suppose there exists a link (r, s) and a time slot t such that $P_{rs}^{*(t)}>0$ and $X_{rs}^{*(t)}=0$, $(r,s)\in L, t=1,\cdots,|L|$. Now, let's consider the solution $(\mathcal{X}_1, \mathcal{P}_1)$, wherein $P_{rs}^{(t)}$ is equal to zero and the rest of the decision variables attain the similar values as the optimum solution $(\mathcal{X}^*, \mathcal{P}^*)$. $(\mathcal{X}_1, \mathcal{P}_1)$ is also a feasible solution of the MILP formulation. The latter is due do the fact that by assigning a value of zero to $P_{rs}^{(t)}$, the left-hand-side of constraints in relation (10) (which are not associated with transmission over link (r, s) in time slot t) becomes even larger. Clearly, $(\mathcal{X}_1, \mathcal{P}_1)$ also satisfies the remaining of the constraints of the MILP formulation.[5] Moreover, based on relation (7), $Z(\mathcal{X}_1, \mathcal{P}_1)$ is strictly less than $Z(\mathcal{X}^*, \mathcal{P}^*)$, which contradicts the optimality of solution $(\mathcal{X}^*, \mathcal{P}^*)$. Hence, the proof of relation (34) is complete.

Now, assume $\sum_{k=1}^{M} P_{i_k j_k}^{*(t)} \le \sum_{k=1}^{M} P_{i_k j_k}^{'(t)}$ is not valid. Let's consider the solution $(\mathcal{X}_2, \mathcal{P}_2)$, wherein all the variables attain the similar values as the optimum solution $(\mathcal{X}^*, \mathcal{P}^*)$, except for the transmit power levels in time slot t which are defined as the following:

$$P_{ij}^{(t)}=\begin{cases} P_{ij}^{'(t)}, if\ X_{ij}^{*(t)}=1 \\ 0, if\ X_{ij}^{*(t)}=0 \end{cases}. \qquad (36)$$

Similar to the argument in part (i), it can be seen that $(\mathcal{X}_2, \mathcal{P}_2)$ is a feasible solution of the MILP formulation and $Z(\mathcal{X}_2, \mathcal{P}_2)$ is strictly less than $Z(\mathcal{X}^*, \mathcal{P}^*)$, which contradicts the optimality of solution $(\mathcal{X}^*, \mathcal{P}^*)$. Hence, the proof of relation (35) is complete. ■

Proof of Theorem 3. The proof is based on Lemma 1, Lemma 2, and Fact 1. Details are omitted here due to space limitation.

[5] Note that the constraint in relation (10) which is associated with the transmission over link (r, s) in time slot t is redundant under $(\mathcal{X}_1,\mathcal{P}_1)$.

APPENDIX B

Proof of Theorem 6. The ILSP problem can be described as a decision problem as the following:

Instance: The integrated link scheduling and power control problem for a set of designated links L, the set of PHY parameters, and a positive integer K, $K \leq |L|$.

Question: Is there a schedule which has the minimum frame length K or less that satisfies the feasibility conditions of the ILSP problem?

First, we prove that the ILSP problem belongs to NP: Initially, we need to *guess* a frame schedule with an arbitrary frame length T ($T \leq |L|$) and an arbitrary power allocation. It is easy to see that the decision problem verifying whether such a guess satisfies the feasibility conditions (of the ILSP problem), and whether the associated frame length is less than or equal to the constant K, can be completed in polynomial time.[6]

On the other hand, it is known that the edge coloring problem (EC) is an NP-complete problem [21]. Therefore, in order to show the ILSP problem is NP-complete, it suffices to introduce a polynomial-time reduction from any instance of the EC problem to an instance of the ILSP problem, the solution to which provides a solution to the instance of the EC problem. The edge coloring problem is to determine, given an undirected graph G and integer K, whether all edges of $G = (V, E)$ can be colored by less than or equal to K colors [22]. Let the set of links in the instance of the ILSP problem be defined as $L = \{(i, j) : \{i, j\} \in E, i < j\}$ (So, there is a one-to-one correspondence between every edge in G and every communication link in the ILSP problem). Furthermore, let $\gamma = 0$, $\eta_j > 0, j = 1, \cdots, n,$ and $P_{\max} = 0$, where the latter indicates that all the transmit power levels should be equal to zero.

We now claim that all edges of graph G can be colored by k colors if and only if there is a (feasible) frame for the associated ILSP problem whose length is equal to k. Let's consider an arbitrary k-coloring of graph G. Suppose that all the links whose associated edge in G have the same color are assigned to the same time slot. Clearly, the latter approach yields a schedule with length k such that: i) the half-duplexing, unicasting, and receptivity constraints are satisfied, ii) every link in L is allocated exactly once, iii) the SINR condition is satisfied for every link at every slot, vi) the transmit power levels are strongly Pareto optimal with respect to the underlying transmission scenario. Therefore, if G can be colored by k colors, then there is a timeframe for the associated ILSP problem whose length is equal to k. Conversely, let assume that a feasible frame to the ILSP problem is given whose length is k. Then, by assigning the same color to the edges in G whose associated link in the ILSP problem are allocated to the same slot, we obtain a (feasible) k-coloring of graph G. The above-mentioned reduction can obviously be performed in a polynomial time, since it requires only the construction of set L from G, which completes the proof. ■

[6] Note that verifying whether a power vector is strongly Pareto optimal with respect to the underlying transmission scenario has the same complexity as finding the apex solution of the associated system of linear inequalities. The latter has a complexity of $O(M^3)$, where M is the cardinality of the transmission scenario [21].

REFERENCES

[1] A. Behzad and I. Rubin, "On the Performance of Graph-based Scheduling Algorithms for Packet Radio Networks," in *Proceedings of IEEE GLOBECOM*, San Francisco, CA, December 2003.

[2] B. Hajek and G. Sasaki, "Link Scheduling in Polynomial Time," *IEEE Transactions on Information Theory*, September 1988.

[3] J. Grönkvist and A. Hansson, "Comparison between Graph-based and Interference-based STDMA Scheduling," in *Proceedings of ACM MOBIHOC,* 2002.

[4] G. Prohazka, "Decoupling Link Scheduling Constraints in Multihop Packet Radio Networks," *IEEE Trans. Comp.*, March 1989.

[5] R. Ramanathan, "A Unified Framework and Algorithm for Channel Assignment in Wireless Networks," *Wireless Networks* (5), 1999.

[6] P. Bjorklund, P. Varbrand, and D. Yuan, "Resource Optimization of Spatial TDMA in Ad Hoc Radio Networks: A Column Generation Approach," in *Proceedings of IEEE INFOCOM*, 2003.

[7] G. Wang and N. Ansari, "Optimal Broadcasting Scheduling in Packet Radio Networks using Mean Field Annealing," *IEEE Journal on Selected Areas in Communications*, Vol. 15, No. 2, February 1997.

[8] P. Gupta and P. R. Kumar, "The Capacity of Wireless Networks," *IEEE Transactions on Information Theory,* March 2000.

[9] A. Behzad and I. Rubin, "High Transmission Power Increases the Capacity of Ad Hoc Wireles Networks," to appear in *IEEE Transactions on Wireless Communications*.

[10] E. S. Jung and N. H. Vaida, "A Power Control MAC Protocol for Ad Hoc Networks," in *Proceedings of MOBICOM,* 2002.

[11] J. P. Monks, V. Bharghavan, and W. W. Hwu, "A Power Controlled Multiple Access Protocol for Wireless Packet Networks," in *Proceedings of IEEE INFOCOM,* 2001.

[12] A. Behzad and I. Rubin, "Impact of Power Control on the Performance of Ad Hoc Wireless Networks," to appear in *Proceedings of IEEE INFOCOM,* March 13-17, 2005.

[13] R. L. Cruz, and A. V. Santhanam, "Optimal Routing, Link Scheduling and Power Control in Multi-Hop Wireless Networks," in *Proceedings of IEEE INFOCOM*, 2003.

[14] T. ElBatt and A. Ephremides, "Joint Scheduling and Power Control for Wireless Ad Hoc Networks," in *Proceedings of IEEE INFOCOM, 2002.*

[15] W. C. Y. Lee, *Mobile Cellular Telecommunication Systems*," Mcgraw-Hill, Inc., 1989.

[16] T. S. Rappaport. *Wireless Communications: Principles and Practice,* Second Edition, Prentice Hall, 2002.

[17] B. Noble and J. Daniel, *Appied Linear Algebra*, 3rd ed., Englewood Cliff, NJ: Prentice-Hall, pp. 375-376, 1988.

[18] Henryk Minc, *Nonnegatice Matrices*, New York: Wiley, 1988.

[19] N. Bambos, S. C. Chen, G. J. Pottie, "Radio Link Admission Algorithms for Wireless Networks with Power Cotnrol and Active Link Quality Protoction," in *Proceedings of IEEE INFOCOM,* 1995.

[20] Debasis Mitra, "An Asynchronous Distributed Algorithm for Power Control in Cellular Radio Systems," *in Proceedings of Fourth Winlab Workshop on Third Generation Wireless Information Network*, October 1993.

[21] T. H. Corman, C. E. Leiserson, R. L. Rivest, and C. Stein, *Introduction to Algorithms*, The MIT Press, 2001.

[22] M. Behzad, G. Chartrand, and L. L. Foster, *Graphs and Digraphs*, Wadsworth International mathematics Series, Boston, 1981.

[23] M. M. Halldorsson and J. Radhakrishnan, "Greed is Good: Approximating Independent Sets in Sparse and Bounded-Degree Graphs," *Algorithmica*, (18) 1997.

[24] S. Janson, T. Luczak, and A. Rucinski, *Random Graphs*, John Wiley & Sons, 2000.

[25] C. McDiarmid,"*Colouring Random Graphs*," Annals of Operations Research, (1) 1984.

[26] M. Andersin, Z. Rosberg, and J. Zander, "Gardual Removals in Cellular PCS with Constrained Power Control and Noise," *Wireless Networks*, Vol. 2, pp. 27-43, 1996.

[27] T. H. Lee, J. C. Lin, and Y. T. Su, "Downlink Power Control Algorithms for Cellular Radio Systems," *IEEE Trans. Veh. Tech.*, Vol. 44, No. 1, pp. 89-94, February 1995.