An Alternative Analysis of Noise Folding in Fractional-N Synthesizers

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Abstract

A new method of analyzing the effect of charge pump mismatches upon the phase noise of ΣΔ fractional-N synthesizers is proposed. This approach produces a simple, universal relationship between the mismatch and the ratio of the noise floor to the peak of the quantization noise spectrum. Simulations confirm that the ratio is relatively independent of other synthesizer parameters such as the ΣΔ modulator order, the shape of the spectrum, and the maximum phase excursion at the feedback divider output.

I. Introduction

Type-II frequency synthesizers must deal with various imperfections related to the phase/frequency detector (PFD) and the charge pump (CP). Among these, the mismatch between the CP's up and down currents proves particularly troublesome in fractional-N loops as it folds the high-frequency quantization noise to in-band components [1, 2, 3]. The mismatches can be random or, more importantly, can arise from channel-length modulation, a severe difficulty in today's designs. This phenomenon has been analyzed using different approaches that assume a Gaussian distribution for the phase error, ΔT, arriving at the PFD [4, 5, 6].

In this paper, we propose an intuitive method of formulating the folding mechanism due to the CP static mismatch while assuming no particular distribution for ΔT. Our objective is to derive a simple expression for the folded noise that designers can readily utilize in deciding the acceptable mismatch. We also show that, to the first order, the difference in dB between the peak of the shaped ΣΔ noise and the folded floor is independent of the variance of the quantization noise, the shape of the noise spectrum, the order of the ΣΔ modulator, the absence or presence of dither, and the reference frequency.

II. CP Nonlinearity Modeling

We begin with the generic fractional-N synthesizer shown in Fig. 1, where, due to the ΣΔ modulator action, the divider generates a pulsewidth-modulated output with phase excursions equal to an integer multiple of the VCO period, T_{VCO}. We assume the CP currents exhibit a mismatch of $\Delta I \ll I_P$ and study the spectrum of the CP output current, I_{CP}, seeking the ratio of the peak to the folded floor, K.

The principal challenge in the analysis of noise folding stems from the slope discontinuity of the CP characteristic at the

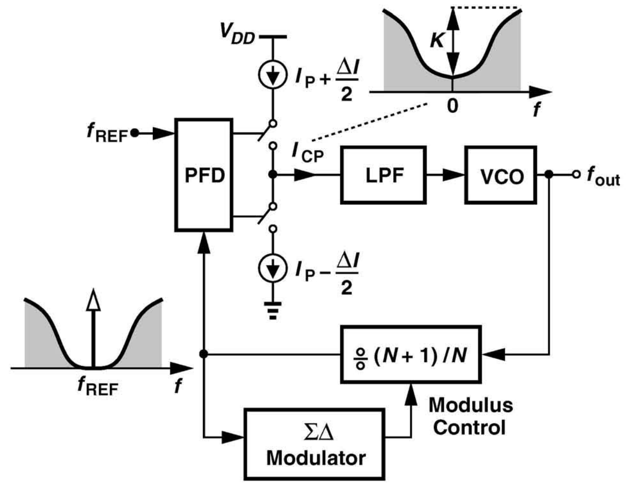

Fig. 1. Generic fractional-N synthesizer showing the effect of CP mismatch.

origin. Specifically, with a mismatch of ΔI between the up and down currents, the charge delivered by the CP exhibits different gains for positive and negative phase errors [2] [Fig. 2(a)]. Here, ΔT_{max} is the maximum phase excursion produced

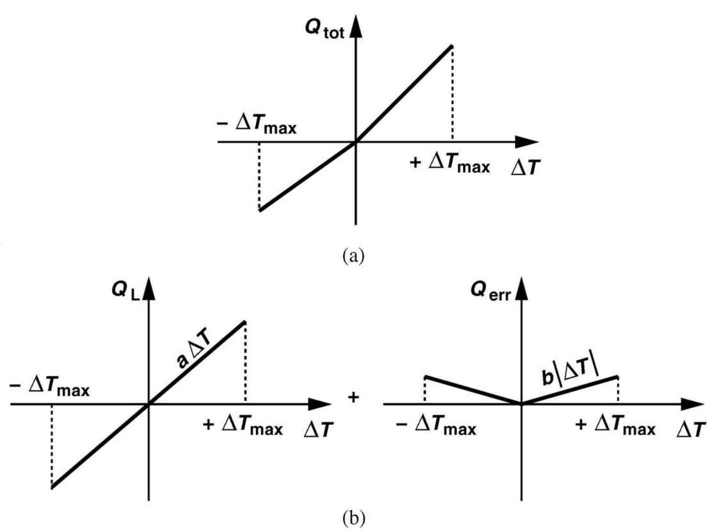

Fig. 2. (a) CP transfer characteristic in the presence of charge pump mismatch, and (b) decomposition of (a) into a linear and an absolute value term.

by the synthesizer's feedback divider. This characteristic can be decomposed into a linear term, Q_L, and an absolute value function, Q_{err} [2, 4] [Fig. 2(b)]. It is the latter that makes the

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analysis difficult.

In [4], the nonlinearity is analyzed in the time domain and the variance of the absolute value function's output is related to that of the input, assuming a Gaussian distribution for the latter. The computations in [5] assume that the spectrum of $|\Delta T|$ is known. The analysis in [6] employs Price's theorem [7] to relate the autocorrelations of the input and output of a nonlinear system with a Gaussian input.

In this paper, we wish to directly approximate $Q_{err} = b|\Delta T|$ by a polynomial. If successful, such an endeavor could then readily follow the standard nonlinearity analysis methods used in RF design.

Let us consider the derivative of the absolute value function. As shown in Fig. 3, dy/dt can be approximated by a hyperbolic

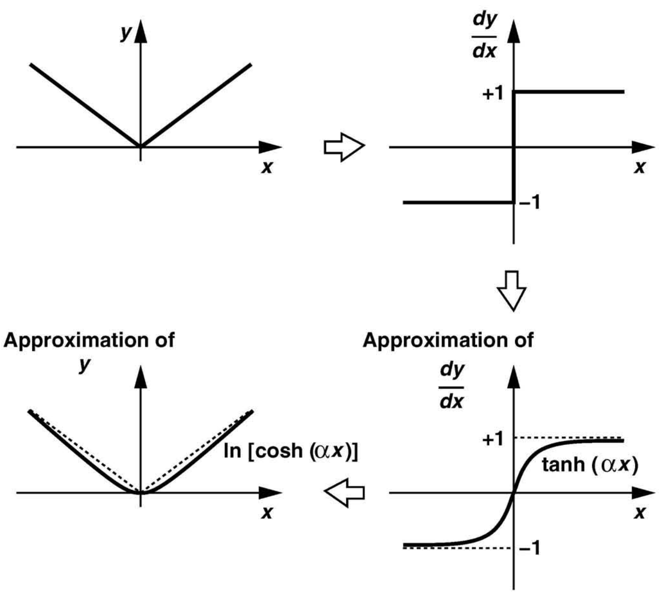

Fig. 3. Method of approximating the absolute value function.

tangent, tanh(αx), where α is chosen large enough to obtain the necessary accuracy. It follows that $y = |x|$ can be approximated by the integral of tanh αx, namely, $(1/\alpha)$ ln[cosh(αx)]. We also note that this result can be expanded in a Taylor series:

$$\frac{1}{\alpha}\ln[\cosh(\alpha x)] \approx \frac{\alpha}{2}x^2 - \frac{\alpha^3}{12}x^4 + \frac{\alpha^5}{45}x^6 + \cdots. \quad (1)$$

These observations justify the use of a polynomial to approximate Q_{err} in Fig. 2(b). In practice, we can choose α and the number of terms in Eq. (1) to model Q_{err} with a desired accuracy in the range of interest, namely, from $-\Delta T_{max}$ to $+\Delta T_{max}$ (Fig. 4). As shown in Section III, the final result is, in fact, independent of ΔT_{max}.

In this paper, we demonstrate that even the first term in Eq. (1) provides sufficient accuracy. According to the fitting scheme in Fig. 4, we have

$$Q_{err} = b|\Delta T| \quad (2)$$

$$\approx \frac{b}{\Delta T_{max}}\Delta T^2. \quad (3)$$

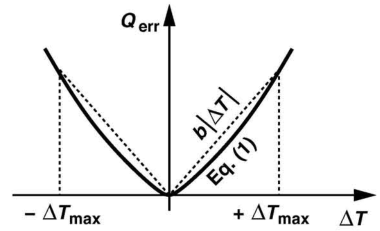

Fig. 4. Fitting a polynomial to the absolute value function according to Eq. (1).

Thus, Q_{tot} in Fig. 2(a) can be written as

$$Q_{tot} \approx a\Delta T + \frac{b}{\Delta T_{max}}\Delta T^2. \quad (4)$$

We must express a and b in terms of the nominal CP current, I_P, and the mismatch, ΔI. We note that $a = I_P$ and $b = \Delta I/2$. Since Q_{tot} and the CP current, I_{CP} in Fig. 1, are related by a factor of $T_{REF} = 1/f_{REF}$, we can still proceed with the expression for Q_{tot} even though we are interested in the spectrum of I_{CP}.

III. Noise Folding Analysis

Let us consider the shaped phase noise spectrum at the divider output (Fig. 5), recognizing that strong, closely-spaced

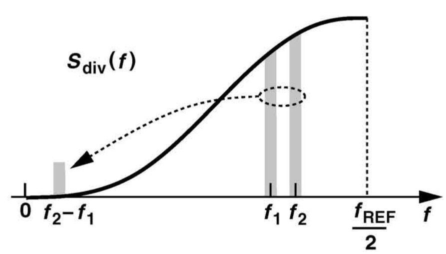

Fig. 5. Approximation of shaped noise by two impulses.

components near $f_{REF}/2$ are downconverted if they experience even-order nonlinearity. As an approximation, we assume that the entire noise can be modeled by two impulses at f_1 and f_2. We express these components in the time domain as $A\cos\omega_1 t + A\cos\omega_2 t$. This is similar to the use of two-tone tests for measuring the nonlinearity in RF circuits; even though the tones are unmodulated, they accurately reveal the behavior for modulated signals as well.

We recognize that the peak value of the sum $A\cos\omega_1 t + A\cos\omega_2 t$ must be equal to the maximum phase excursion at the divider output, ΔT_{max}. For example, $\Delta T_{max} = 3T_{VCO}$ for some third-order $\Sigma\Delta$ modulators. Thus, $2A = \Delta T_{max}$. It is interesting to interpret this assumption for Gaussian noise. Since the power of a Gaussian process is given by its variance, σ^2, and since the two tones, $A\cos\omega_1 t$ and $A\cos\omega_2 t$, are assumed to carry this power, we have $2(A^2/2) = \sigma^2$ and hence $\Delta T_{max} = 2\sigma$. That is, our calculation is equivalent to assuming a peak value of $\pm 2\sigma$ for a Gaussian variable, a reasonable approximation as 95% of the values fall within this range.

Subjecting the two tones to the transfer characteristic given

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by (4), we have

$$Q_{tot} \approx a\frac{\Delta T_{max}}{2}(\cos\omega_1 t + \cos\omega_2 t) + \frac{b}{\Delta T_{max}}\frac{\Delta T_{max}^2}{4}(\cos\omega_1 t + \cos\omega_2 t)^2. \quad (5)$$

The folded component is therefore equal to $(b\Delta T_{max}/4)\cos(\omega_1 - \omega_2)t$, whose amplitude can be normalized to that of the main components, $a\Delta T_{max}/2$, yielding $K = b/(2a)$. With the a and b values found in Section II, we have

$$K = \frac{\Delta I}{4I_P}. \quad (6)$$

For example, a 5% mismatch produces a floor 38 dB below the peak.

This universal expression reveals that, to the first order, K is independent of other design parameters such as the maximum phase fluctuation at the divider output, the exact shape of the quantization noise spectrum, the ΣΔ modulator order, and the reference frequency.

In practice, we are also interested in the absolute in-band noise floor at the charge pump output. Since the peak of the quantization noise spectrum is known [4], it can be simply multiplied by K^2 to obtain the phase noise floor.

IV. Simulation Results and Discussion

A fractional-N synthesizer has been simulated in the time domain using behavioral models. We have $f_{REF} = 40$ MHz, $f_{VCO} = 4.0104$ GHz, $I_P = 40$ μA, and $K_{VCO} = 250$ MHz/V. The loop bandwidth is chosen around 10 kHz so that the phase noise at higher offsets is not affected by the loop action. The ΣΔ modulator is a third-order MASH 1-1-1 topology having a multi-bit output. In all cases, the divide ratio is chosen equal to 100.26 and the CP output current is directly monitored.

Figure 6 plots the spectra of the CP output current for three cases. The plot on top corresponds to no CP mismatch. The plot in the middle is obtained for an up and down current mismatch of 2.5%. The noise floor is 44 dB below the peak at $f_{REF}/2 = 20$ MHz, in agreement with $20\log K = 20\log(2.5\%/4) = 44$ dB. The bottom plot in Figure 6 repeats the experiment with a 5% mismatch, indicating a floor 37 dB below the peak and closely agreeing with $20\log K = 38$ dB.

Our assertion that K is relatively independent of the synthesizer parameters is also corroborated by the published data. Specifically, as shown in Fig. 7, the four cases simulated in [4] with $\Delta I/I_P = 10\%$ also exhibit a floor that is approximately $20\log(10\%/4) = 32$ dB below the peak.

We should remark that the four simulations leading to the plots in Figs. 7(a)-(d) include dither and cover a wide variety of cases [4]: (a) a third-order single-stage modulator with a noise variance of $0.4T_{VCO}^2$, (b) a third-order MASH 1-2 modulator with a noise variance of $0.94T_{VCO}^2$, (c) a second-order MASH 1-1 modulator with a noise variance of $0.29T_{VCO}^2$, and (d) a fourth-order MASH 1-1-1-1 modulator with a noise variance of $1.66T_{VCO}^2$. We observe that, for such a range of modulator orders and noise variances, our prediction incurs only about 1 dB of error.

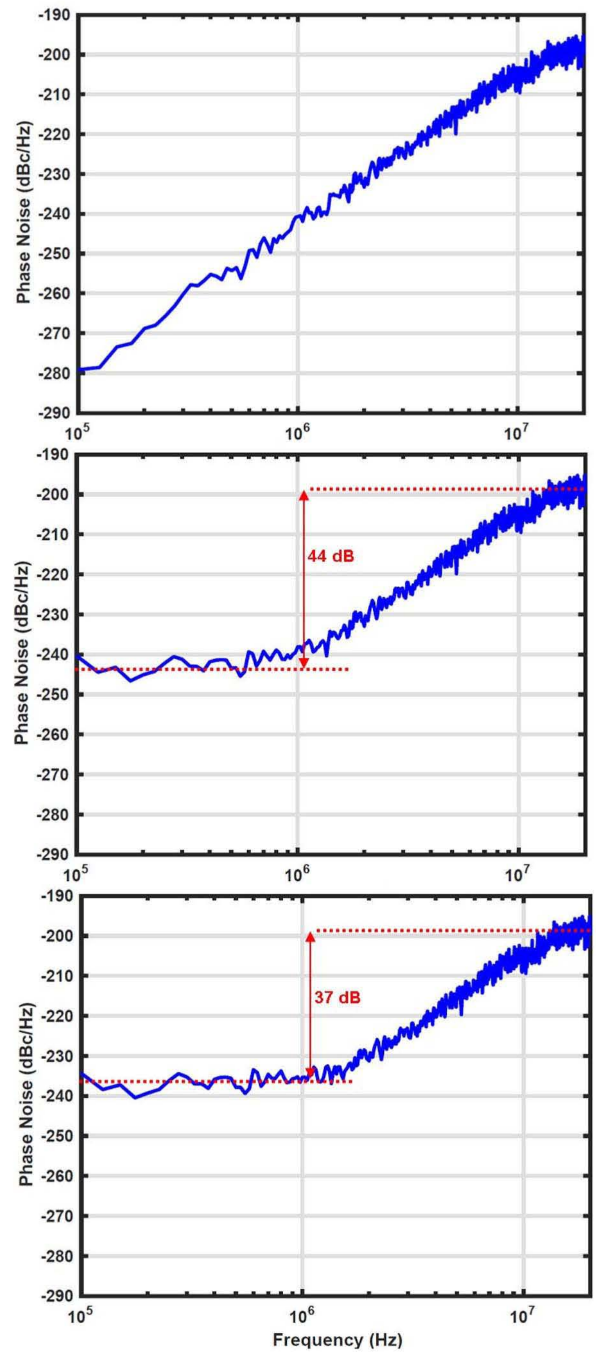

Fig. 6. Charge pump output spectra for no mismatch (top), (b) 2.5% mismatch (middle), and (c) 5% mismatch (bottom).

Our analysis has made two approximations: (1) only the first term in Eq. (1) is included, and (2) the phase noise power in Fig. 5 has been represented by only two impulses. Allowing us to arrive at the simple result $K = \Delta I/(4I_P)$, this approach still provides a reasonable accuracy. Nevertheless,

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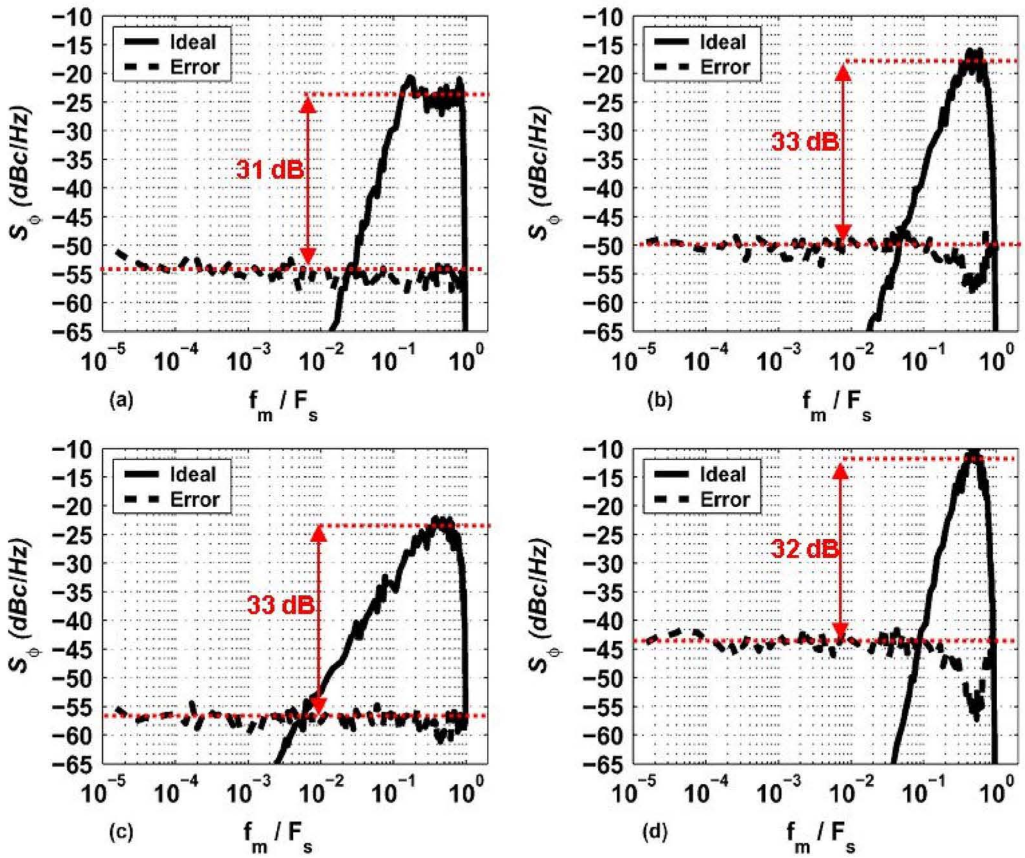

Fig. 7. Simulation results from [4] for a CP mismatch of 10% (annotation in red added).

it is possible to further improve the approximations: the fit in Fig. 4 can include more terms in the Taylor series, and the spectrum in Fig. 5 can be modeled by N impulses evenly distributed from 0 to $f_{REF}/2$.

V. Conclusion

A simple, intuitive analysis of charge pump mismatches in $\Sigma\Delta$ fractional-N synthesizers reveals that the ratio of the folded noise floor to the peak of the quantization noise spectrum is given by $K = \Delta I/(4 I_P)$. The key result here is that this universal ratio is relatively independent of the synthesizer design parameters. The designer can then readily decide how much mismatch is tolerable for a given application.

Acknowlegment

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