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Mutual Injection Pulling Between Oscillators [1]

Behzad Razavi
Electrical Engineering Department
University of California, Los Angeles, CA 90095

Abstract

This paper proposes a theory for the behavior of free-running or phase-locked oscillators that experience mutual injection pulling. The time- and frequency-domain responses are derived for each case and the profile of the resulting sidebands is calculated analytically. Experimental results obtained for two 1-GHz CMOS PLLs that are resistively coupled on-chip are presented.

I. Introduction

The phenomenon of pulling has been studied extensively for a single oscillator under injection of an independent sinusoid [1, 2, 3]. However, in some applications, coupling through the supply and the substrate may lead to *mutual* pulling between two oscillators. In broadband data transceivers, for example, the transmit phase-locked loop (PLL) and the receive clock and data recovery circuit may operate at slightly different frequencies (because the latter is locked to the incoming data, i.e., a crystal frequency at the far end), thus pulling each other. Furthermore, emerging wireless systems such as ultra wide-band transceivers may incorporate multiple PLLs [4] and must deal with unwanted mutual pulling.

This paper analyzes the mutual injection pulling between two free-running or phase-locked oscillators in both time and frequency domains. Section II describes the effect of pulling in response to a modulated sinusoid and Section III applies the results to pulling between two free-running oscillators. Section IV extends the study to phase-locked oscillators and Section V presents experimental results.

II. Pulling by a Modulated Sinusoid

While pulling each other, two oscillators experience output phase modulation. It is therefore necessary to first determine the response of a single oscillator to an independent phase-modulated input. Consider the conceptual representation shown in Fig. 1, where the oscillator tank resonates at ω_0, θ_{inj} denotes the input phase modulation, and the output is expressed in terms of the *input* frequency and some phase modulation, θ_{out}. We wish to determine the behavior of θ_{out}.

In a manner similar to the analysis in [3], we express V_X as a single sinusoid having a time-dependent phase:

$$V_X(t) \approx V_{osc}\cos(\omega_{inj}t + \psi), \quad (1)$$

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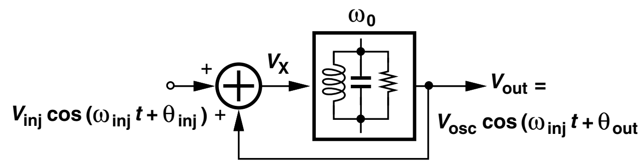

Fig. 1. Conceptual feedback oscillator with modulated input.

where the injection level is assumed small and

$$\psi = \tan^{-1}\frac{V_{inj}\sin\theta_{inj} + V_{osc}\sin\theta_{out}}{V_{inj}\cos\theta_{inj} + V_{osc}\cos\theta_{out}}. \quad (2)$$

After experiencing the phase shift of the LC tank in the oscillator, V_X emerges as V_{out} and hence:

$$\psi + \tan^{-1}\left[\frac{2Q}{\omega_0}\left(\omega_0 - \omega_{inj} - \frac{d\psi}{dt}\right)\right] = \theta_{out}, \quad (3)$$

where the $\tan^{-1}$ term on the left-hand side represents the phase shift introduced by the tank. For small injection levels, Eq. (2) yields

$$\tan(\theta_{out} - \psi) = \frac{V_{inj}\sin(\theta_{out} - \theta_{inj})}{V_{osc} + V_{inj}\cos(\theta_{out} - \theta_{inj})} \quad (4)$$

and also, $d\psi/dt \approx d\theta_{out}/dt$. Using these results in (3), we obtain

$$\frac{d\theta_{out}}{dt} = \omega_0 - \omega_{inj} - \frac{\omega_0}{2Q}\frac{V_{inj}}{V_{osc}}\sin(\theta_{out} - \theta_{inj}). \quad (5)$$

This result serves as an extension of Adler's equation [1] for low-level, phase-modulated inputs.

III. Mutual Pulling between Free-Running Oscillators

Two oscillators experiencing bilateral coupling can be modeled as shown in Fig. 2, where ω_1 and ω_2 denote resonance frequencies of the tanks ($\omega_1 \approx \omega_2 = \omega_0$) and α the coupling factor. Note that, to simplify the algebra, both outputs are expressed in terms of a single frequency, $\hat{\omega}$. The derivations below naturally lead to an acceptable value for $\hat{\omega}$.

Writing Eq. (5) for the two oscillators gives

$$\frac{d\theta_1}{dt} = \omega_1 - \hat{\omega} - \frac{\omega_1}{2Q}\frac{\alpha V_{osc}}{V_{osc}}\sin(\theta_1 - \theta_2) \quad (6)$$

$$\frac{d\theta_2}{dt} = \omega_2 - \hat{\omega} - \frac{\omega_2}{2Q}\frac{\alpha V_{osc}}{V_{osc}}\sin(\theta_2 - \theta_1). \quad (7)$$

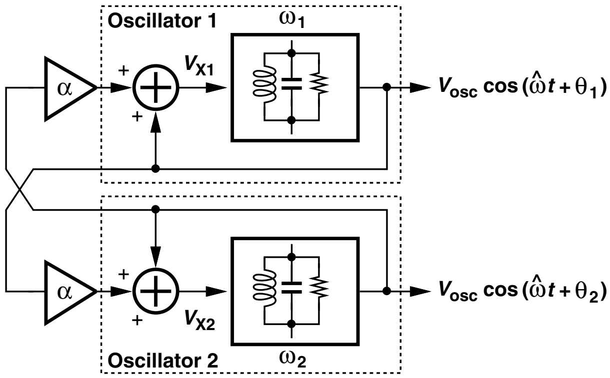

Fig. 2. Two oscillators under mutual pulling.

Adding the two sides of these equations and assuming $\omega_1 \approx \omega_2 = \omega_0$, we have

$$\frac{d(\theta_1 + \theta_2)}{dt} = \omega_1 + \omega_2 - 2\hat{\omega}, \tag{8}$$

and hence

$$\theta_2 = (\omega_1 + \omega_2 - \hat{\omega})t - \theta_1. \tag{9}$$

Substituting for θ_2 in (6) yields

$$\frac{du}{dt} = \omega_1 - \omega_2 - 2\frac{\alpha\omega_0}{2Q}\sin u, \tag{10}$$

where $u = 2\theta_1 - (\omega_1 + \omega_2 - 2\hat{\omega})t$. In analogy with Adler's original equation [1], we conclude from (10) that u varies periodically with a frequency of

$$\omega_b = \sqrt{(\omega_1 - \omega_2)^2 - \left(2\alpha\frac{\omega_0}{2Q}\right)^2}. \tag{11}$$

Furthermore, we observe that if $\hat{\omega}$ is defined as $(\omega_1 + \omega_2)/2$, then $2\theta_1$ tracks u with no frequency offset. Thus, the output $V_{osc}\cos(\hat{\omega}t + \theta_1)$ exhibits sidebands at $n\omega_b$ around $(\omega_1 + \omega_2)/2$. Note from (9) that this choice of $\hat{\omega}$ also translates to $\theta_2 = -\theta_1$.

With the aid of the graphical analysis proposed in [3], we can plot u, θ_1, and θ_2 as a function of time (Fig. 3). The key observation here is that θ_1 varies with a beat frequency of ω_b and remains around 45° for part of the time. Similarly, θ_2 remains around $-45°$ for part of the period. In other words, the two oscillators are almost injection-locked with a phase difference of 90° for part of the period, and rapidly go through a 360° phase rotation at the end. Simulations indicate that Eq. (11) predicts the beat frequency with reasonable accuracy for coupling factors less that -30 dB. The output spectra of the two oscillators thus appear as depicted in Fig. 4.

To compute the pulled frequencies, ω_{1p} and ω_{2p}, we write from Fig. 4: $\omega_{2p} - \omega_{1p} = \omega_b$ and hence $(\omega_1 + \omega_2)/2 - \omega_{1p} = \omega_b/2$. It follows that

$$\omega_{1p} = \frac{\omega_1 + \omega_2 - \omega_b}{2} \tag{12}$$

$$\omega_{2p} = \frac{\omega_1 + \omega_2 + \omega_b}{2} \tag{13}$$

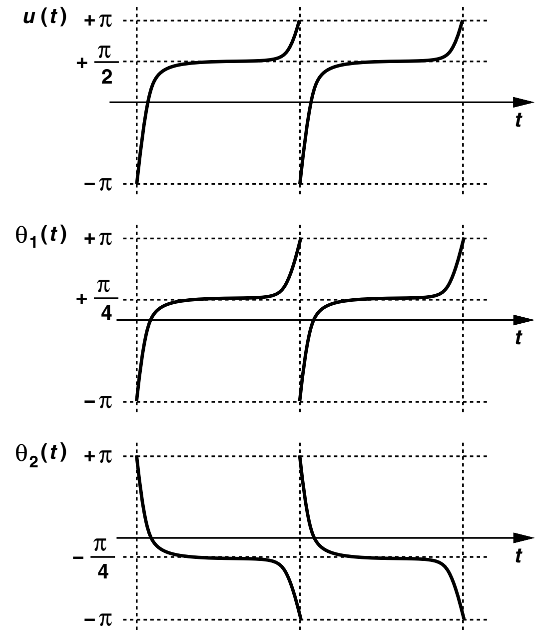

Fig. 3. Time-domain behavior of output phases of two oscillators.

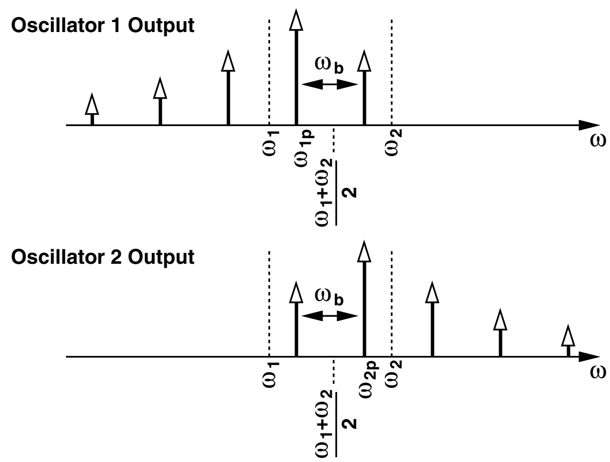

Fig. 4. Output spectra under mutual pulling condition.

It is also possible to determine the minimum amount of coupling that guarantees injection locking of the two oscillators. Setting $d\theta_1/dt = 0$ in (6) gives

$$\alpha = Q\frac{\omega_1 - \omega_2}{\omega_0}\frac{1}{\sin(\theta_1 - \theta_2)}. \tag{14}$$

Thus, the minimum α occurs if $\theta_1 - \theta_2 = 90°$:

$$\alpha_{min} = Q\frac{\omega_1 - \omega_2}{\omega_0}. \tag{15}$$

IV. Mutual Pulling between PLLs

For the derivations in this section, we must first compute the response of a voltage-controlled oscillator (VCO) to phase-modulated injection. Assuming that the control voltage contains only a small perturbation, one can prove that (5) must be modified to:

$$\frac{d\theta_{out}}{dt} = \omega_0 + K_{VCO}V_{cont} - \omega_{inj} - \frac{\omega_0}{2Q}\frac{V_{inj}}{V_{osc}}\sin(\theta_{out} - \theta_{inj}). \tag{16}$$

The injection-pulled, phase-locked oscillator model described in [3] can now be extended to two oscillators as shown in Fig. 5. Here, the oscillators are phase-locked so as to operate at $\omega_1 = \omega_{REF1}$ and $\omega_2 = \omega_{REF2}$ but experience mutual pulling and hence phase modulation. For VCO_1, we have from (16):

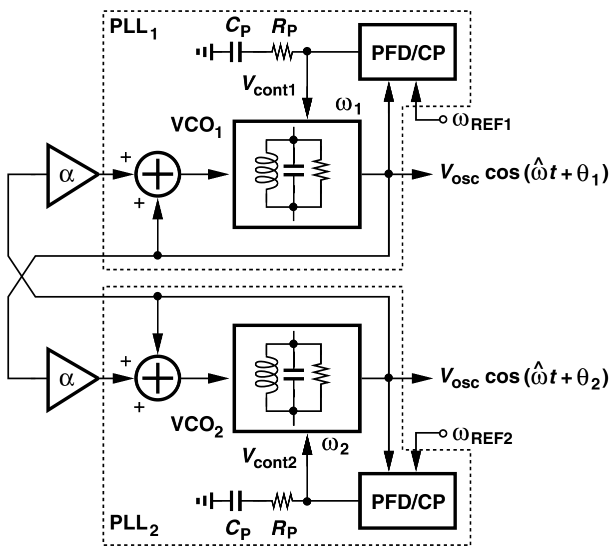

Fig. 5. Two phase-locked oscillators under mutual pulling.

$$\frac{d\theta_1}{dt} = \omega_1 + K_{VCO}V_{cont1} - \hat{\omega} - \frac{\alpha\omega_1}{2Q}\sin(\theta_1 - \theta_2). \tag{17}$$

At this point, it is convenient to define a new parameter ϕ_1 such that $\hat{\omega}t + \theta_1 = \omega_1 t + \phi_1$. Thus, the output of the phase/frequency detector (PFD) becomes proportional to $\omega_{REF}t - (\omega_1 t + \phi_1) = -\phi_1$.[2] It follows that

$$V_{cont1} = -\frac{I_P R_P}{2\pi}\phi_1(t) - \frac{I_P}{2\pi C_P}\int \phi_1(t)dt, \tag{18}$$

where I_P denotes the charge pump current. Also, (17) reduces to

$$\begin{aligned}\frac{d\phi_1}{dt} &= K_{VCO}V_{cont1} - \frac{\alpha\omega_1}{2Q}\sin[(\omega_1 - \omega_2)t + \phi_1 - \phi_2] \\ &\approx K_{VCO}V_{cont1} + \frac{\alpha\omega_1}{2Q}\sin(\omega_2 - \omega_1)t,\end{aligned} \tag{19}$$

where it is assumed that ϕ_1, ϕ_2, and hence $\phi_1 - \phi_2$ are much less than 1 rad (i.e., the corruption is relatively small).

Combining (18) and (19), differentiating both sides of the result with respect to time, and defining

$$\zeta = \frac{R_P}{2}\sqrt{\frac{I_P K_{VCO} C_P}{2\pi}} \tag{20}$$

$$\omega_n = \sqrt{\frac{I_P K_{VCO}}{2\pi C_P}}, \tag{21}$$

we have

$$\frac{d^2\phi_1}{dt^2} + 2\zeta\omega_n\frac{d\phi_1}{dt} + \omega_n^2\phi_1 = \frac{\alpha\omega_1}{2Q}(\omega_2 - \omega_1)\cos(\omega_2 - \omega_1)t. \tag{22}$$

It follows that

$$\begin{aligned}\phi_1(t) &= \frac{\alpha}{2Q}\frac{\omega_1(\omega_2 - \omega_1)}{\sqrt{[(\omega_2 - \omega_1)^2 - \omega_n^2]^2 + 4\zeta^2\omega_n^2(\omega_2 - \omega_1)^2}} \\ &\quad \times \cos[(\omega_2 - \omega_1)t - \beta],\end{aligned} \tag{23}$$

where β is a constant phase. In a manner similar to a single PLL under sinusoidal injection [3], the output phase varies sinusoidally, creating symmetric sidebands at $\omega_1 - (\omega_2 - \omega_1) = 2\omega_1 - \omega_2$ and $\omega_1 + (\omega_2 - \omega_1) = \omega_2$. Also, the magnitude of the sidebands falls as $|\omega_2 - \omega_1|$ becomes very small or very large. With similar observations applied to θ_2 in Fig. 5, the output spectra of the two PLLs appear as shown in Fig. 6.

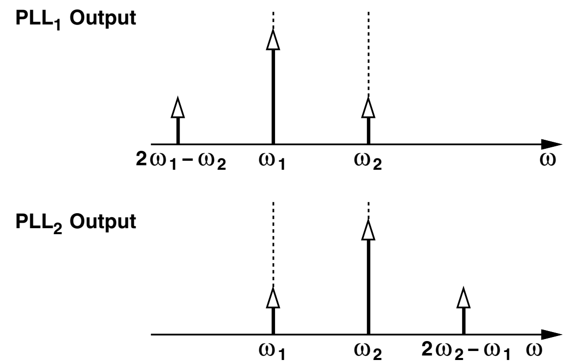

Fig. 6. Output spectra of two phase-locked oscillators under mutual pulling.

V. Experimental Results

A test chip containing two nominally identical PLLs and other characterization circuits has been fabricated in 0.35-μm CMOS technology. Employing a 1-GHz LC VCO, each PLL operates with an independent reference frequency and a feedback divide ratio of 4. Figure 7 shows the die photograph of the two PLLs.

Illustrated in Fig. 8 is the coupling mechanism between the two oscillators. The value of R_1 is chosen high enough so as not to degrade the Q of the tanks.

Since the oscillators incorporate inductors, they may experience coupling through the substrate, an effect that can introduce error in the controlled environment described above. This issue is mitigated by allowing a spacing of about 200 μm and placing a grounded guard band between the two PLLs. Also, each oscillator employs two asymmetric inductors (rather than one symmetric inductor) to minimize the effect of magnetic coupling.

The greatest challenge in this quantitative study arises from various sources of uncertainty in the parameter values and

[2] If we retain the form $\hat{\omega}t + \theta_1$, then the output of the PFD is proportional to $(\omega_{REF} - \hat{\omega})t - \theta_1$, complicating the following algebra.

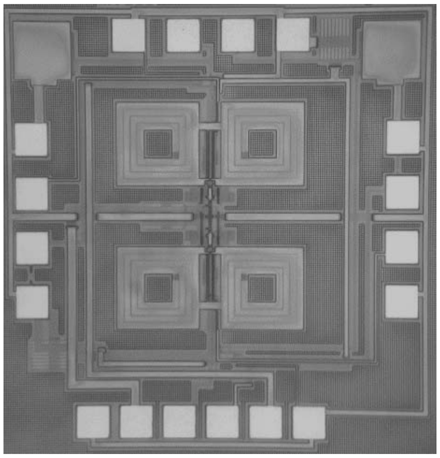

Fig. 7. Die photograph of dual PLLs.

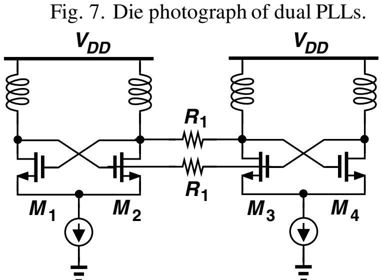

Fig. 8. Resistive coupling between oscillators.

hence the need for calibration. Specifically, the Q of the inductors and varactors is not known accurately, and the coupling factor is difficult to measure at 1 GHz. For these reasons, several other versions of the prototype are included that allow the measurement of the coupling factor, the tank Q, the charge pump current, and the resistor in the loop filter. For example, one version utilizes one PLL as a signal source to injection-lock the other free-running oscillator. Since the measured lock range is proportional to the injection level (and hence the coupling factor) and $1/Q$, the ratio α/Q can be obtained.

Figure 9 shows the measured spectra of the two PLLs when one operates at 1.02 GHz and the other at 1.04 GHz. Figure 10 plots the measured magnitude of the sidebands along with the prediction provided by Eq. 23. The discrepancy around 8-MHz frequency offset is attributed to possible coupling paths and resonances in the probe station on which the testing is performed.

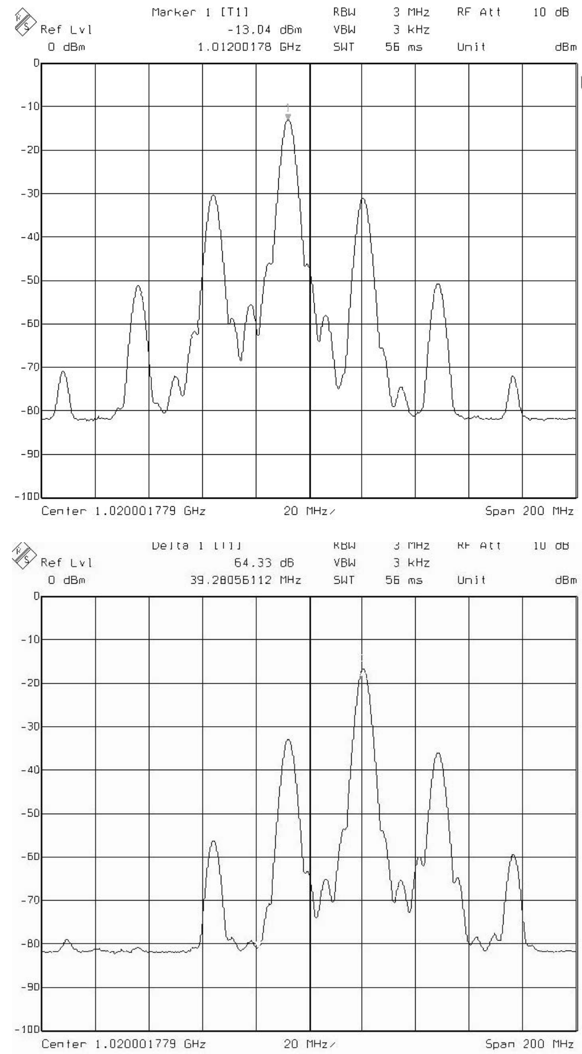

Fig. 9. Outputs of two PLLs under mutual injection pulling.

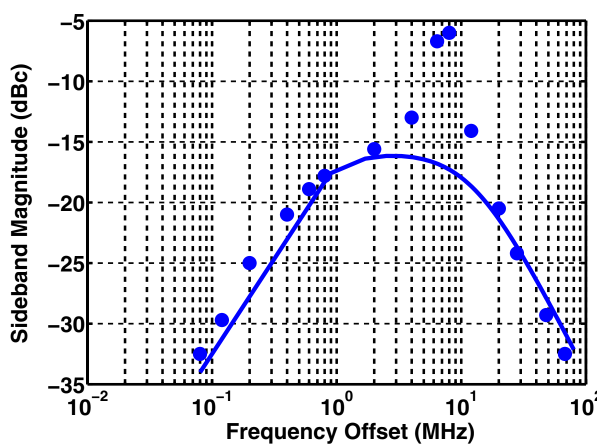

Fig. 10. Profile of sideband magnitudes of each PLL.

References

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[2] K. Kurokawa, "Injection Locking of Microwave Solid-State Oscillators," *Proc. of IEEE*, vol. 61, No. 10, pp. 1336-1410, Oct. 1973.

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