EE 215A
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Homework 2
October 12, 2008

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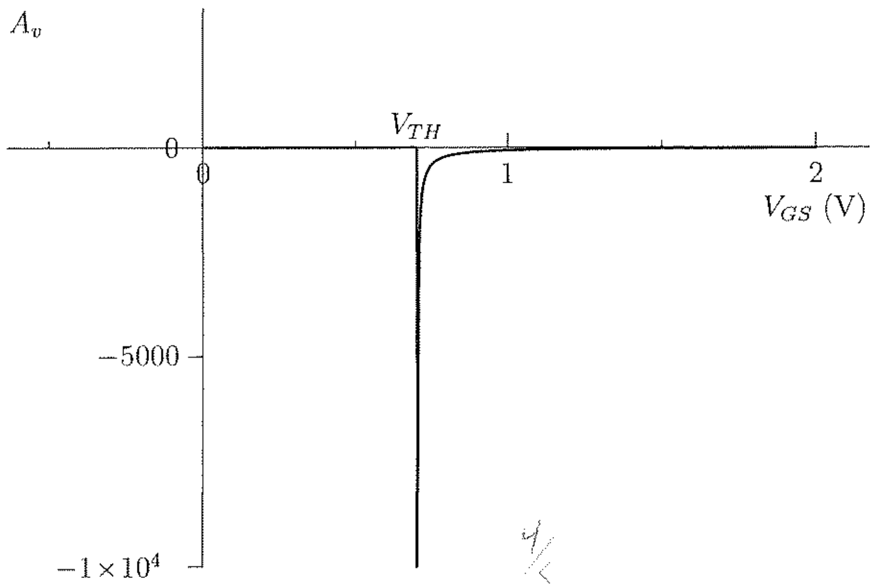
3.6 (a)

$$
\begin{aligned}
A_v &= -g_m r_o \\
&= -\frac{2I_D}{V_{GS} - V_{TH}} \frac{1}{\lambda I_D} \\
&= -\frac{2}{\lambda (V_{GS} - V_{TH})}
\end{aligned}
$$

Using the values from Table 2.1 ($\lambda = 0.1\ \text{V}^{-1}$, $V_{TH} = 0.7$ V), we get the following plot:



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(b)

$$
\begin{aligned}
A_v &= -g_m r_o \\
&= -\frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) \frac{1}{\lambda I_D} \\
&= -\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) \frac{2}{\lambda \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2} \\
&= -\frac{2}{\lambda (V_{GS} - V_{TH})}
\end{aligned}
$$

Note this is identical to part (a) since the intrinsic gain does not depend on either I_D or W/L (hence keeping one or the other constant makes no difference).

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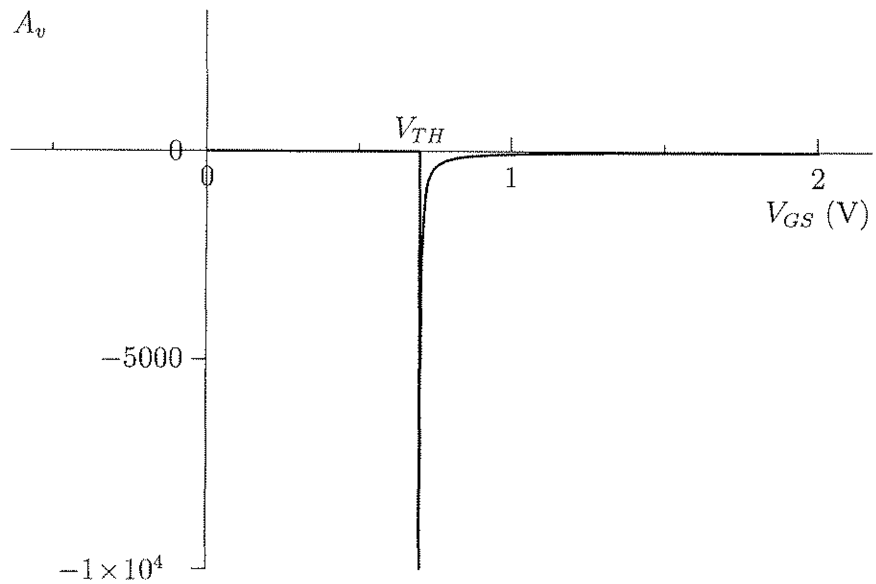
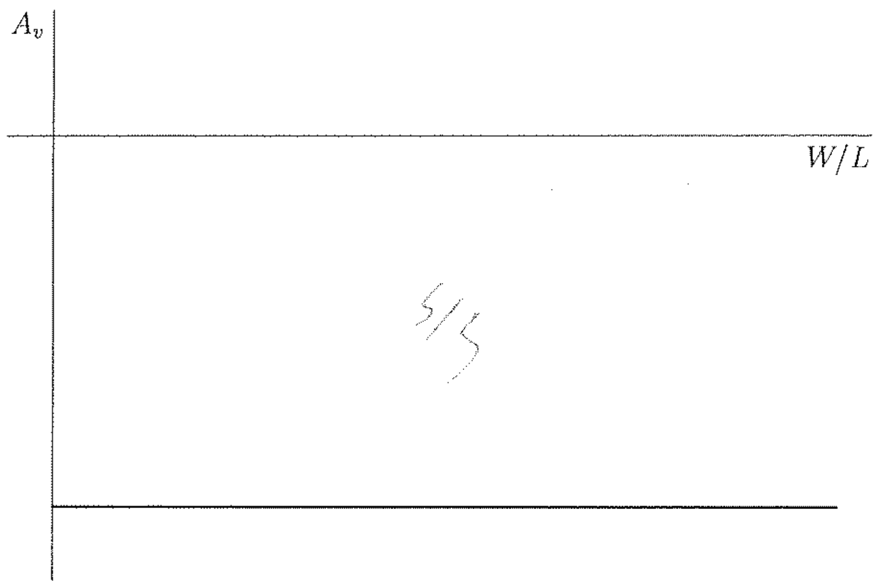
vs different I_D, $\frac{W}{L}$?


A_v
V_{TH}
0
0
1
2
V_{GS} (V)
−5000
-1×10^4

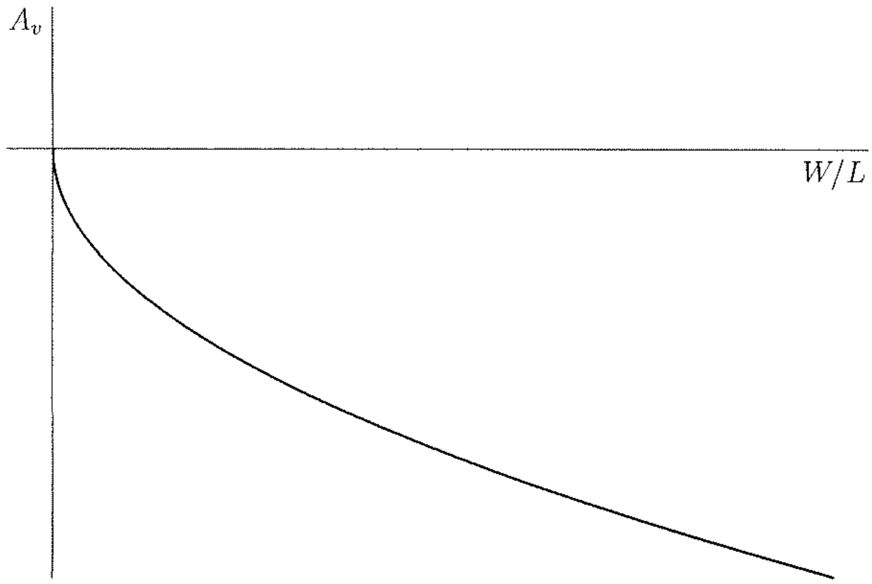
3.7 (a)

$$
\begin{aligned}
A_v &= -g_m r_o \\
&= -\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) \frac{1}{\lambda I_D} \\
&= -\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) \frac{2}{\lambda \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2} \\
&= -\frac{2}{\lambda (V_{GS} - V_{TH})}
\end{aligned}
$$



(b)

$$
\begin{aligned}
A_v &= -g_m r_o \\
&= -\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} \frac{1}{\lambda I_D} \\
&= -\frac{1}{\lambda} \sqrt{\frac{2\mu_n C_{ox} \frac{W}{L}}{I_D}}
\end{aligned}
$$



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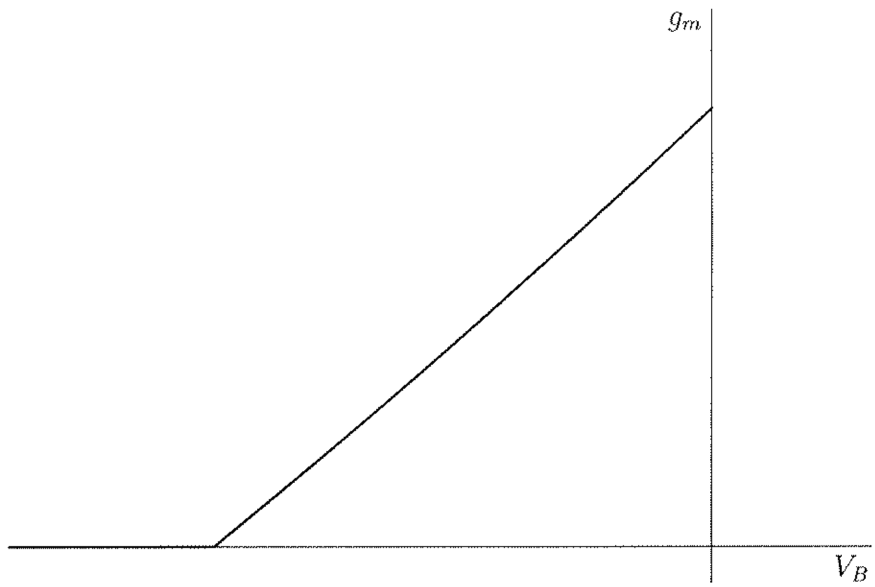
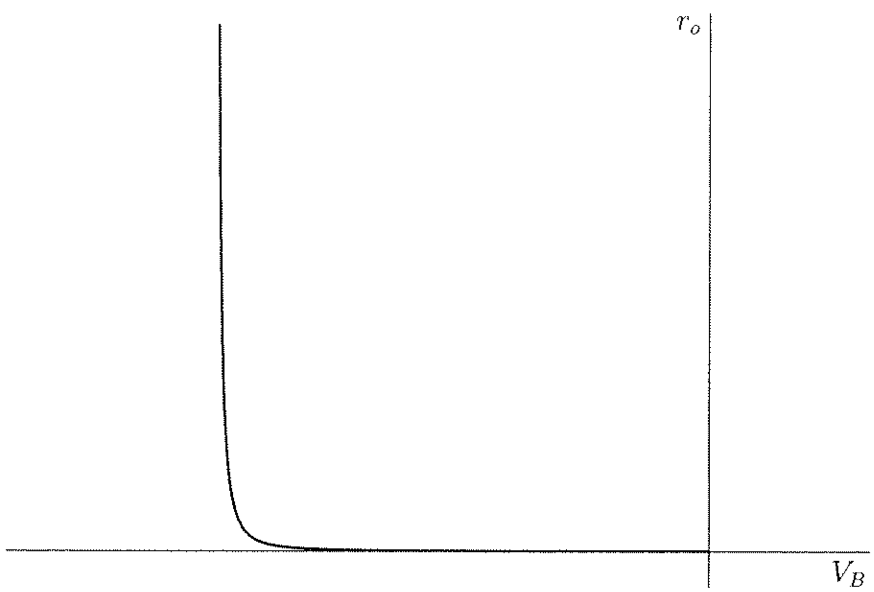
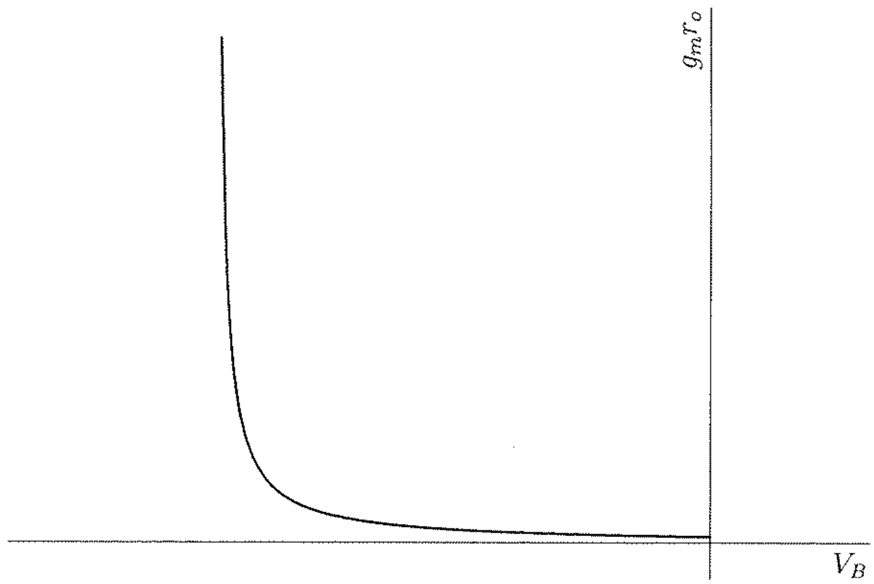
3.9

$$
\begin{aligned}
g_m &= \mu_n C_{ox} \frac{W}{L} \left[V_{GS} - V_{TH0} - \gamma \left(\sqrt{2\Phi_F + V_{SB}} - \sqrt{2\Phi_F} \right) \right] \\
r_o &= \frac{1}{\lambda I_D} \\
&= \frac{2}{\lambda \mu_n C_{ox} \frac{W}{L} \left[V_{GS} - V_{TH0} - \gamma \left(\sqrt{2\Phi_F + V_{SB}} - \sqrt{2\Phi_F} \right) \right]^2} \\
g_m r_o &= \frac{2}{\lambda \left[V_{GS} - V_{TH0} - \gamma \left(\sqrt{2\Phi_F + V_{SB}} - \sqrt{2\Phi_F} \right) \right]}
\end{aligned}
$$







Note that the sudden changes in each graph occur where V_{TH} increases to the point where $V_{GS} = V_{TH}$.

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3.16 (a)

$$V_{out} = V_{DD} - (I_{D1} - I_{D2})\, R_D$$

For $V_{in} < V_{TH1}$, $I_{D1} = I_{D2} = 0$ and $V_{out} = V_{DD}$. Once $V_{in} > V_{TH1}$, M_1 enters saturation and M_2 enters triode.

$$V_{out} = V_{DD} - \left\{ \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 - \mu_n C_{ox} \left(\frac{W}{L}\right)_2 \right.$$
$$\left. \left[2\,(V_{DD} - V_b - |V_{TH2}|)\,(V_{DD} - V_{out}) - (V_{DD} - V_{out})^2 \right] \right\} R_D$$

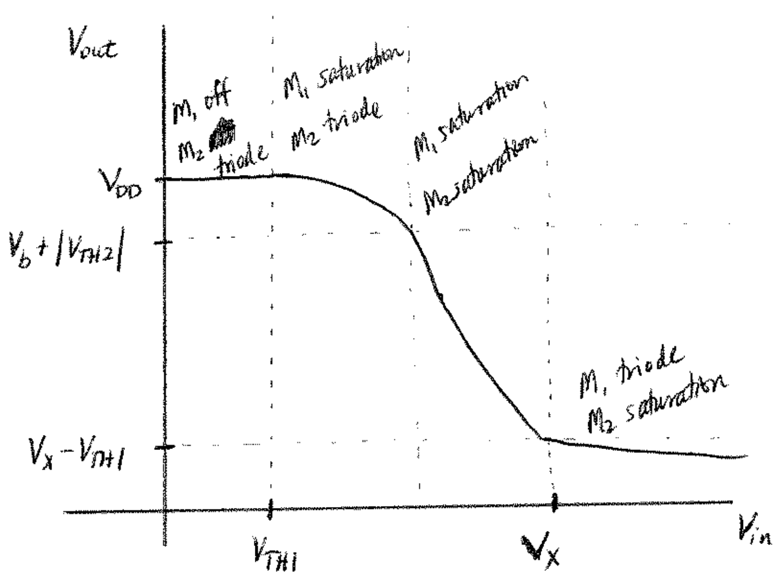
Once $V_{out} < V_b + |V_{TH2}|$, M_2 enters saturation.

$$V_{out} = V_{DD} - \left\{ \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 - \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - V_b - |V_{TH2}|)^2 \right\} R_D$$

Finally, once $V_{out} < V_{in} - V_{TH}$, M_1 enters triode.

$$V_{out} = V_{DD} - \left\{ \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[(V_{in} - V_{TH1})^2 V_{out} - V_{out}^2 \right] - \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - V_b - |V_{TH2}|)^2 \right\} R_D$$

Solving these equations analytically is impractical, so below is a qualitatively drawn plot based on the regions of operation of M_1 and M_2.



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3.18 (c)

$$
\begin{aligned}
V_{GS} &= V_b - V_X \\
V_{DS} &= V_{DD} - I_D R_D - V_X \\
V_{GS} &> V_{TH} \\
\Leftrightarrow V_b - V_X &> V_{TH} \\
\Leftrightarrow V_X &< V_b - V_{TH} \\
V_{DS} &> V_{GS} - V_{TH} \\
\Leftrightarrow V_{DD} - I_D R_D &> V_b - V_{TH} \\
\Leftrightarrow V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_b - V_X - V_{TH}\right)^2 R_D &> V_b - V_{TH} \\
\Leftrightarrow \sqrt{\frac{2\left(V_{DD} - V_b + V_{TH}\right)}{\mu_n C_{ox}\frac{W}{L} R_D}} &> V_b - V_X - V_{TH} \\
\Leftrightarrow V_X &> V_b - V_{TH} - \sqrt{\frac{2\left(V_{DD} - V_b + V_{TH}\right)}{\mu_n C_{ox}\frac{W}{L} R_D}} \\
I_X &= \frac{V_X}{R_S} - I_D
\end{aligned}
$$

$$
I_D = \begin{cases}
0 & V_X > V_b - V_{TH} \\
\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2 & V_b - V_{TH} - \sqrt{\frac{2(V_{DD} - V_b + V_{TH})}{\mu_n C_{ox}\frac{W}{L} R_D}} < V_X < V_b - V_{TH} \\
\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2\left(V_{GS} - V_{TH}\right)V_{DS} - V_{DS}^2\right] & V_X < V_b - V_{TH} - \sqrt{\frac{2(V_{DD} - V_b + V_{TH})}{\mu_n C_{ox}\frac{W}{L} R_D}}
\end{cases}
$$

$$
I_X = \begin{cases}
\frac{V_X}{R_S} & V_X > V_b - V_{TH} \\
\frac{V_X}{R_S} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2 & V_b - V_{TH} - \sqrt{\frac{2(V_{DD} - V_b + V_{TH})}{\mu_n C_{ox}\frac{W}{L} R_D}} < V_X < V_b - V_{TH} \\
\frac{V_X}{R_S} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2\left(V_{GS} - V_{TH}\right)V_{DS} - V_{DS}^2\right] & V_X < V_b - V_{TH} - \sqrt{\frac{2(V_{DD} - V_b + V_{TH})}{\mu_n C_{ox}\frac{W}{L} R_D}}
\end{cases}
$$

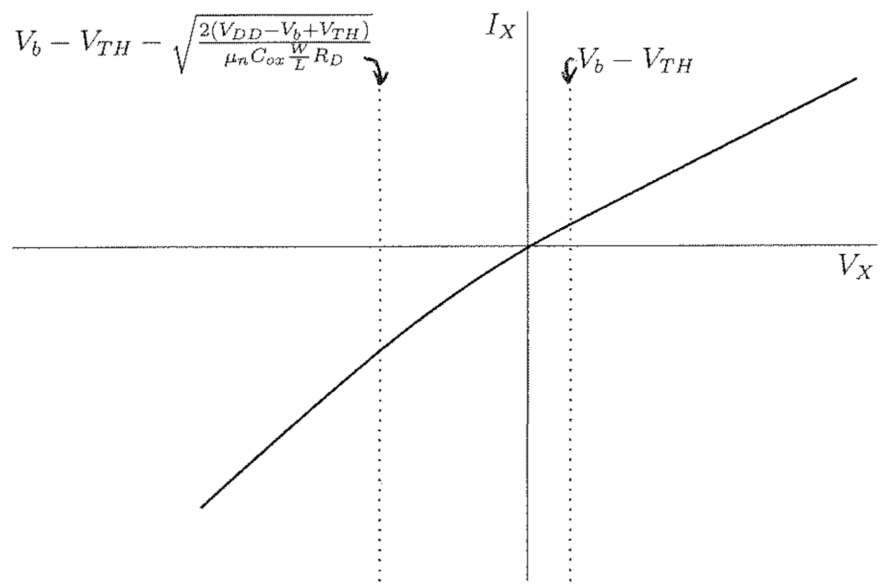
When in triode, I_X depends on $V_{DS} = V_{DD} - I_D R_D - V_X$, so we must re-arrange the triode expression to find I_X explicitly as a function of V_X.

$$
I_{D,tri} = \mu_n C_{ox}\frac{W}{L}\left[2\left(V_{GS} - V_{TH}\right)\left(V_{DD} - I_{D,tri}R_D - V_X\right) - \left(V_{DD} - I_{D,tri}R_D - V_X\right)^2\right]
$$

Let $K = \mu_n C_{ox}\frac{W}{L}$. Using Maxima, we have

$$
I_{D,tri} = \frac{\sqrt{(K R_D V_X)^2 + \left(2K^2 R_D^2 V_{TH} - 2K^2 R_D^2 V_b\right) V_X + K^2 R_D^2 V_{TH}^2 - \left(2K^2 R_D^2 V_b + 2K R_D\right) V_{TH} - 2K R_D V_{DD} + K^2 R_D^2 V_b^2 + 2K R_D V_b + 1}}{K R_D^2} +
$$

$$
+ \frac{K R_D V_{TH} + K R_D V_{DD} - K R_D V_b - 1}{K R_D^2}
$$

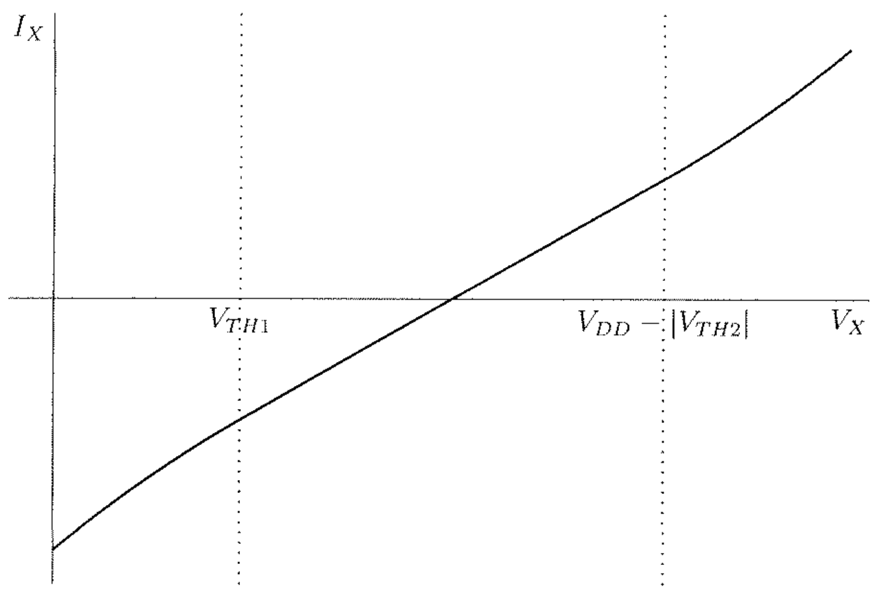

$V_b - V_{TH} - \sqrt{\frac{2(V_{DD}-V_b+V_{TH})}{\mu_n C_{ox}\frac{W}{L}R_D}}$
I_X
$V_b - V_{TH}$
V_X

3.19 (d)

$$\begin{aligned}
I_X &= I_{D1} - I_{D2} \\
V_{GS1} &= V_X \\
V_{GS1} &< V_{TH} \\
\Leftrightarrow V_X &< V_{TH1} \\
V_{SG2} &= V_{DD} - V_X \\
V_{SG2} &< |V_{TH2}| \\
\Leftrightarrow V_X &> V_{DD} - |V_{TH2}| \\
V_{DS1} &= V_X = V_{GS1} \\
V_{SD2} &= V_{DD} - V_X = V_{SG2}
\end{aligned}$$

Since $V_{DS1} = V_{GS1}$ and $V_{SD2} = V_{SG2}$, both M_1 and M_2 are always either in saturation or cutoff.

$$I_X = \begin{cases} -\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{SG2} - |V_{TH2}|)^2 & V_X < V_{TH1} \\ \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TH1})^2 - \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{SG2} - |V_{TH2}|)^2 & V_{TH1} < V_X < V_{DD} - |V_{TH2}| \\ \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TH1})^2 & V_X > V_{DD} - |V_{TH2}| \end{cases}$$



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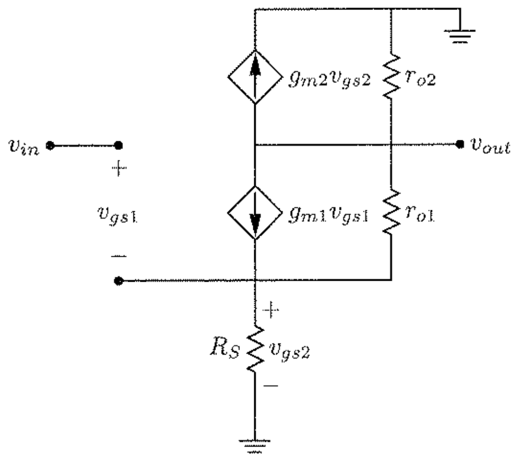
3.20 (e)



$$
\begin{aligned}
g_{m2}v_{gs2} + \frac{v_{out}}{r_{o2}} + g_{m1}v_{gs1} + \frac{v_{out} - v_{gs2}}{r_{o1}} &= 0 \\
-g_{m1}v_{gs1} + \frac{v_{gs2} - v_{out}}{r_{o1}} + \frac{v_{gs2}}{R_S} &= 0 \\
v_{gs2} &= v_{in} - v_{gs1} \\
g_{m2}\left(v_{in} - v_{gs1}\right) + \frac{v_{out}}{r_{o2}} + g_{m1}v_{gs1} + \frac{v_{out} - (v_{in} - v_{gs1})}{r_{o1}} &= 0 \\
-g_{m1}v_{gs1} + \frac{v_{in} - v_{gs1} - v_{out}}{r_{o1}} + \frac{v_{in} - v_{gs1}}{R_S} &= 0 \\
v_{gs1}\left(g_{m1} + \frac{1}{r_{o1}} + \frac{1}{R_S}\right) &= \frac{v_{in} - v_{out}}{r_{o1}} + \frac{v_{in}}{R_S} \\
v_{gs1} &= \left(\frac{v_{in} - v_{out}}{r_{o1}} + \frac{v_{in}}{R_S}\right)\left(\frac{1}{g_{m1}} \parallel r_{o1} \parallel R_S\right)
\end{aligned}
$$

$$
\begin{aligned}
v_{gs1}\left(g_{m2} - g_{m1} - \frac{1}{r_{o1}}\right) &= g_{m2}v_{in} + \frac{v_{out}}{r_{o2}} + \frac{v_{out} - v_{in}}{r_{o1}} \\
\left(\frac{v_{in} - v_{out}}{r_{o1}} + \frac{v_{in}}{R_S}\right)\left(\frac{1}{g_{m1}} \parallel r_{o1} \parallel R_S\right)\left(g_{m2} - g_{m1} - \frac{1}{r_{o1}}\right) &= g_{m2}v_{in} + \frac{v_{out}}{r_{o2}} + \frac{v_{out} - v_{in}}{r_{o1}}
\end{aligned}
$$

$$
v_{in}\left[\frac{1}{r_{o1}} - g_{m2} + \left(\frac{1}{r_{o1}} + \frac{1}{R_S}\right)\left(\frac{1}{g_{m1}} \parallel r_{o1} \parallel R_S\right)\left(g_{m2} - g_{m1} - \frac{1}{r_{o1}}\right)\right] = v_{out}\left\{\frac{1}{r_{o1}}\left[1 + \left(\frac{1}{g_{m1}} \parallel r_{o1} \parallel R_S\right)\left(g_{m2} - g_{m1} - \frac{1}{r_{o1}}\right)\right] + \frac{1}{r_{o2}}\right\}
$$

$$
\frac{v_{out}}{v_{in}} = \boxed{\frac{\frac{1}{r_{o1}} - g_{m2} + \left(\frac{1}{r_{o1}} + \frac{1}{R_S}\right)\left(\frac{1}{g_{m1}} \parallel r_{o1} \parallel R_S\right)\left(g_{m2} - g_{m1} - \frac{1}{r_{o1}}\right)}{\frac{1}{r_{o1}}\left[1 + \left(\frac{1}{g_{m1}} \parallel r_{o1} \parallel R_S\right)\left(g_{m2} - g_{m1} - \frac{1}{r_{o1}}\right)\right] + \frac{1}{r_{o2}}}}
$$

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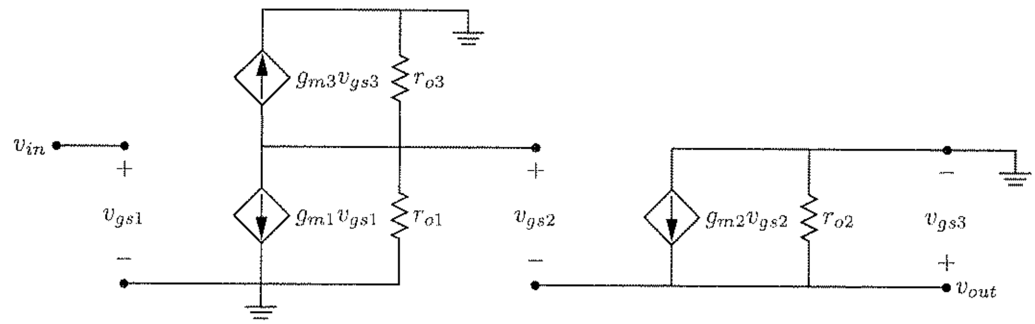
3.21 (h)



$$v_{gs1} = v_{in}$$

$$v_{gs3} = v_{out} = g_{m2}v_{gs2}r_{o2}$$

$$g_{m3}v_{gs3} + \frac{v_{gs2} + v_{gs3}}{r_{o3}} + \frac{v_{gs2} + v_{gs3}}{r_{o1}} + g_{m1}v_{gs1} = 0$$

$$g_{m3}v_{out} + \frac{v_{gs2} + v_{out}}{r_{o3}} + \frac{v_{gs2} + v_{out}}{r_{o1}} + g_{m1}v_{in} = 0$$

$$v_{gs2}\left(\frac{1}{r_{o1}} + \frac{1}{r_{o3}}\right) = -g_{m1}v_{in} - v_{out}\left(g_{m3} + \frac{1}{r_{o1}} + \frac{1}{r_{o3}}\right)$$

$$v_{gs2} = -\left[g_{m1}v_{in} + v_{out}\left(g_{m3} + \frac{1}{r_{o1}} + \frac{1}{r_{o3}}\right)\right](r_{o1} \parallel r_{o3})$$

$$= -\left[g_{m1}v_{in} + \frac{v_{out}}{\frac{1}{g_{m1}} \parallel r_{o1} \parallel r_{o3}}\right](r_{o1} \parallel r_{o3})$$

$$v_{out} = -g_{m2}r_{o2}\left[g_{m1}v_{in} + \frac{v_{out}}{\frac{1}{g_{m1}} \parallel r_{o1} \parallel r_{o3}}\right](r_{o1} \parallel r_{o3})$$

$$v_{out}\left[1 + \frac{g_{m2}r_{o2}(r_{o1} \parallel r_{o3})}{\frac{1}{g_{m1}} \parallel r_{o1} \parallel r_{o3}}\right] = -g_{m1}g_{m2}r_{o2}(r_{o1} \parallel r_{o3})\,v_{in}$$

$$\frac{v_{out}}{v_{in}} = \boxed{-\frac{g_{m1}g_{m2}r_{o2}(r_{o1} \parallel r_{o3})}{1 + \frac{g_{m2}r_{o2}(r_{o1} \parallel r_{o3})}{\frac{1}{g_{m1}} \parallel r_{o1} \parallel r_{o3}}}}$$

3.22 (b) Since M_1 and C_1 form a closed loop, no current can flow through M_2. Assuming $V_{b2} > V_{TH2}$, $V_X = 0$ always. This means that at time $t = 0$, $V_Y = V_{DD}$. Initially, M_1 is in saturation.

$$\begin{aligned}
I_{C_1} &= -C_1 \frac{dV_Y}{dt} \text{ (since } V_X = 0) \\
&= I_{D1} \\
&= \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2 \\
\int_{V_{DD}}^{V_Y} dV_Y &= -\int_0^t \frac{1}{C_1}\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2\, dt \\
V_Y - V_{DD} &= \frac{t}{C_1}\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2 \\
V_Y &= V_{DD} - \frac{t}{C_1}\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2
\end{aligned}$$

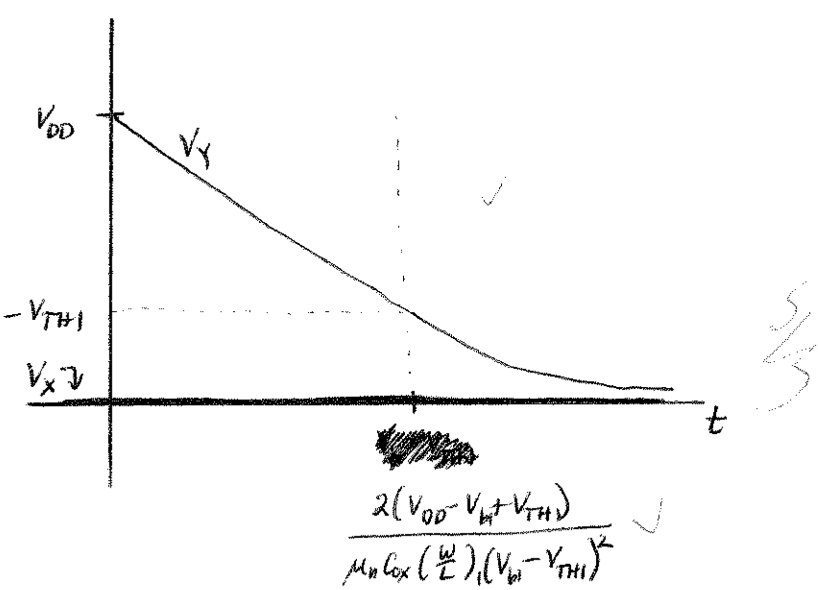
Once $V_Y = V_{b1} - V_{TH1}$, M_1 goes into triode. We can find the transition point as follows:

$$\begin{aligned}
V_Y &= V_{DD} - \frac{t}{C_1}\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2 = V_{b1} - V_{TH1} \\
t &= \frac{2\,(V_{DD} - V_{b1} + V_{TH1})}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2}
\end{aligned}$$

Now we have to solve the differential equation for this case:

$$-C_1 \frac{dV_Y}{dt} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2\,(V_{b1} - V_{TH})\,V_Y - V_Y^2\right]$$

Since this is again difficult to solve analytically, here is a qualitative plot.



2. (a) We can extract some parameters from the SPICE model. For simplicity, assume that M_2 and M_3 have their thresholds modified by 0.05 V due to the body effect.

$$
\begin{aligned}
V_{TH4} &= 0.480\text{ V}\\
\mu_n C_{ox} &= 261.2\ \mu\text{A/V}^2\\
V_{TH1} &= -0.500\text{ V}\\
\mu_p C_{ox} &= 108.0\ \mu\text{A/V}^2\\
\frac{W}{L} &= \frac{25}{0.18}\\
\lambda_n &= 0.323\text{ V}^{-1}\\
\lambda_p &= 0.324\text{ V}^{-1}
\end{aligned}
$$

$$
\begin{aligned}
V_{GS5} = V_{GS4} &= V_{TH5} + \sqrt{\frac{2I_{D5}}{\mu_n C_{ox}\frac{W}{L}\left(1+\lambda_n V_{DS5}\right)}}\\
V_{DS5} &= V_{GS5}\\
V_{GS4} &= 0.631\text{ V}\\
V_{DS4} &= V_{GS4} - V_{TH4}\ \text{(since } M_4 \text{ is at the edge of saturation)}\\
&= 0.151\text{ V}\\
I_{D4} = I_{D1} &= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS4}-V_{TH4}\right)^2\left(1+\lambda_n V_{DS4}\right)\\
&= 436\ \mu\text{A}\\
V_{SG1} &= |V_{TH1}| + \sqrt{\frac{2I_{D1}}{\mu_p C_{ox}\frac{W}{L}\left(1+\lambda_p V_{SD1}\right)}}\\
V_{SD1} &= V_{SG1} - |V_{TH1}|\ \text{(since } M_1 \text{ is at the edge of saturation)}\\
V_{SG1} &= |V_{TH1}| + \sqrt{\frac{2I_{D1}}{\mu_p C_{ox}\frac{W}{L}\left[1+\lambda_p\left(V_{SG1}-|V_{TH1}|\right)\right]}}\\
V_{SG1} &= 0.732\text{ V}\\
V_{SD1} &= 0.232\text{ V}\\
V_{out} &= V_{DD} - V_{SG1} = 1.068\text{ V}\\
V_{DS3} &= V_{out} - V_{DS4} = 0.916\text{ V}\\
V_{GS3} &= V_{TH3} + \sqrt{\frac{2I_{D3}}{\mu_n C_{ox}\frac{W}{L}\left(1+\lambda_n V_{DS3}\right)}}\\
V_{GS3} &= 0.666\text{ V}\\
V_{b2} &= V_{GS3} + V_{DS4} = \boxed{0.817\text{ V}}\\
V_{SD2} &= V_{DD} - V_{SD1} - V_{out}\\
&= 0.500\text{ V}\\
V_{SG2} &= |V_{TH2}| + \sqrt{\frac{2I_{D2}}{\mu_p C_{ox}\frac{W}{L}\left(1+\lambda_p V_{SD2}\right)}}\\
&= 0.774\text{ V}\\
V_{b1} &= V_{DD} - V_{SD1} - V_{SG2} = \boxed{0.794\text{ V}}
\end{aligned}
$$

(b) In order for all transistors to remain in saturation, we require $V_{out} > V_{b2} - V_{TH3}$ and $V_{out} < V_{b1} + |V_{TH2}|$. Thus, the swing is $\boxed{0.264\text{ V} < V_{out} < 1.367\text{ V}}$.

(c) The SPICE model does not provide a value of γ or η, so let's simply assume that $\eta = 0.3$ for M_2

and M_3 as an approximation.

$$\frac{V_{out}}{V_A} = -g_{m1}\left\{\left[r_{o1}+r_{o2}+(1+\eta)\,g_{m2}r_{o1}r_{o2}\right] \parallel \left[r_{o3}+r_{o4}+(1+\eta)\,g_{m3}r_{o3}r_{o4}\right]\right\}$$

$$g_{m1} = \mu_p C_{ox}\frac{W}{L}\left(V_{SG1}-|V_{TH1}|\right) = 3.48\ \text{mS}$$

$$r_{o1} = r_{o2} = \frac{1}{\lambda_p I_D} = 7.085\ \text{k}\Omega$$

$$g_{m2} = \mu_p C_{ox}\frac{W}{L}\left(V_{SG2}-|V_{TH2}|\right) = 3.36\ \text{mS}$$

$$g_{m3} = \mu_n C_{ox}\frac{W}{L}\left(V_{GS3}-V_{TH3}\right) = 4.93\ \text{mS}$$

$$g_{m4} = \mu_n C_{ox}\frac{W}{L}\left(V_{GS4}-V_{TH4}\right) = 5.48\ \text{mS}$$

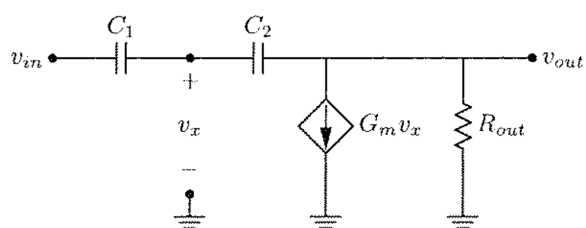
$$r_{o3} = r_{o4} = \frac{1}{\lambda_n I_D} = 7.107\ \text{k}\Omega$$

$$\frac{V_{out}}{V_A} = \boxed{-480}$$ ✓

$$G_m = 3.48\ \text{mS}$$

$$R_{out} = 137.9\ \text{k}\Omega$$

To find the closed-loop gain, we can model the open-loop amplifier using the values of G_m and R_{out} found above, then add the feedback network in separately.



$$v_x = v_{out} + (v_{out}-v_{in})\frac{C_1}{C_1+C_2}$$

$$G_m v_x + \frac{v_{out}}{R_{out}} + (v_{out}-v_{in})\,s\,(C_1 \parallel C_2) = 0$$

$$G_m v_{out} + G_m(v_{in}-v_{out})\frac{C_1}{C_1+C_2} + \frac{v_{out}}{R_{out}} + (v_{out}-v_{in})\,s\,(C_1 \parallel C_2) = 0$$

$$v_{out}\left[(sC_2-G_m)\frac{C_1}{C_1+C_2} + G_m + \frac{1}{R_{out}}\right] = v_{in}(sC_2-G_m)\frac{C_1}{C_1+C_2}$$

$$\frac{v_{out}}{v_{in}} = \frac{(sC_2-G_m)\frac{C_1}{C_1+C_2}}{(sC_2-G_m)\frac{C_1}{C_1+C_2}+G_m+\frac{1}{R_{out}}}$$

Let $s \to 0$ to get:

$$\frac{v_{out}}{v_{in}} = \frac{-G_m\frac{C_1}{C_1+C_2}}{-G_m\frac{C_1}{C_1+C_2}+G_m+\frac{1}{R_{out}}}$$

$$= \frac{-4/5G_m}{1/5G_m+\frac{1}{R_{out}}}$$

$$\approx \boxed{-4}$$

5/5 ✓

The approximation in the last step comes from assuming $1/5G_m \gg 1/R_{out}$, which is true for the values derived earlier.

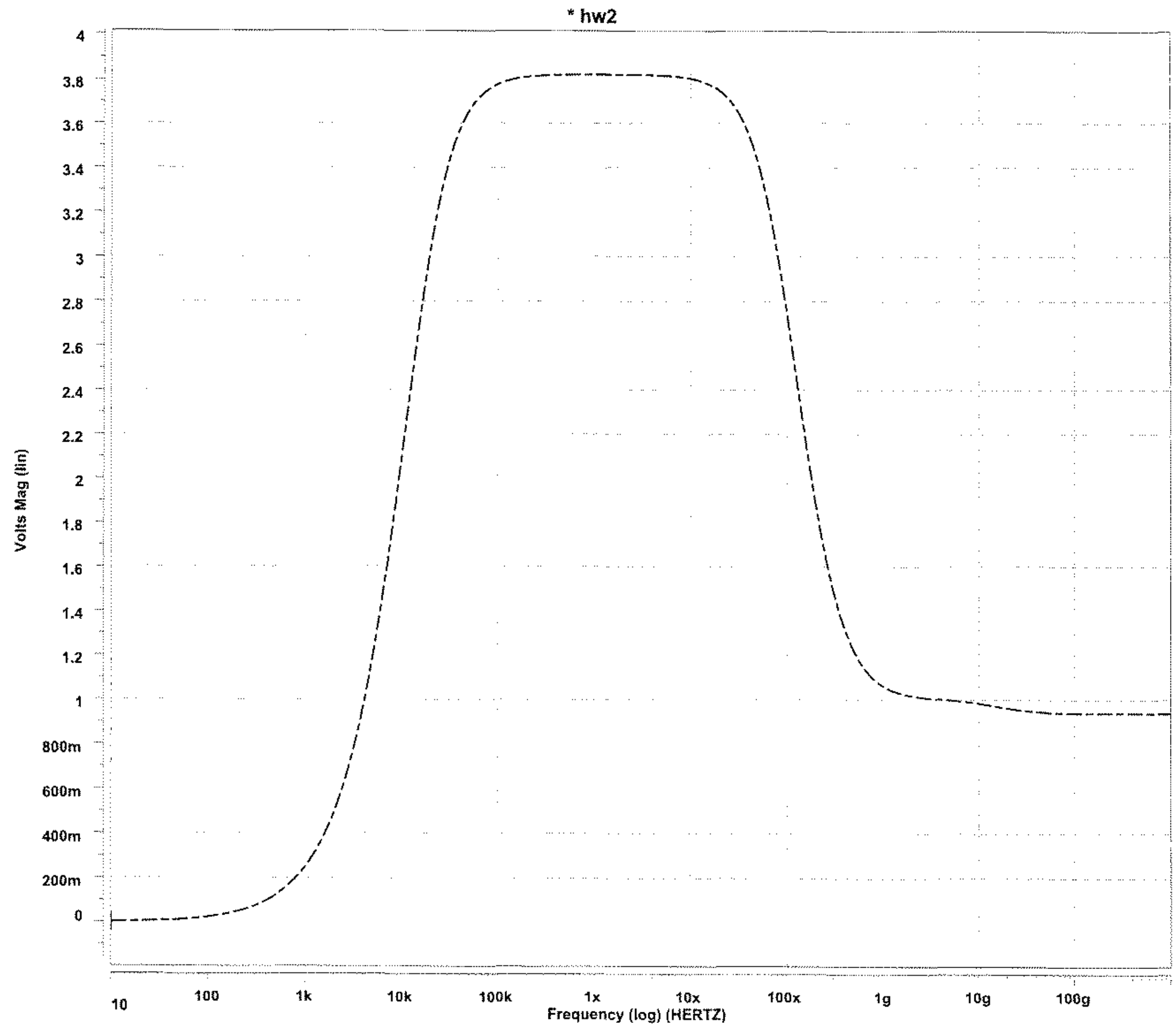
(d) Simulating the circuit in SPICE using V_{b1} and V_{b2} as derived, it turns out that M_1 and M_4 are just slightly in linear (which is likely a consequence of designing for them to be at the edge of saturation). In order to correct for this, I've lowered V_{b1} to 0.730 V and increased V_{b2} to 0.830 V, which puts all transistors in saturation. The resulting netlist is as follows:

```
* HW2

.inc 'ee215a.mod'

vin  vin   gnd  ac 1V
c1   vin   vg1  4pF
m1   vd1   vg1  vdd vdd CMOSP W=25u L=0.18u
m2  vout   vb1  vd1 vdd CMOSP W=25u L=0.18u
c2   vg1   vout 1pF
m3  vout   vb2  vd4 gnd CMOSN W=25u L=0.18u
m4   vd4   vg4  gnd gnd CMOSN W=25u L=0.18u
m5   vg4   vg4  gnd gnd CMOSN W=25u L=0.18u
vdd  vdd   gnd  1.8V
i1   vdd   vg4  500uA
r1   vg1   vout 10MEG
vb1  vb1   gnd  0.730V
vb2  vb2   gnd  0.830V

.op
.ac dec 500 10 1T
.option post=2 nomod

.end
```

The output from the operating point analysis shows that all transistors are saturated with a bias current of 394 μA, which is close to the calculated value of 436 μA:

```
element  0:m1        0:m2        0:m3        0:m4        0:m5
model    0:cmosp     0:cmosp     0:cmosn     0:cmosn     0:cmosn
region     Saturati    Saturati    Saturati    Saturati    Saturati
id       -393.6548u -393.6548u  393.6548u  393.6548u  500.0000u
```

The gain plot shows the magnitude of the midband gain is 3.8, which is very close to the calculated value of 4.


* hw2
Volts Mag (lin)
4
3.8
3.6
3.4
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
800m
600m
400m
200m
0
10
100
1k
10k
100k
1x
10x
100x
1g
10g
100g
Frequency (log) (HERTZ)