HW#3
EE 215A

Name: Jin Hwan Jeon

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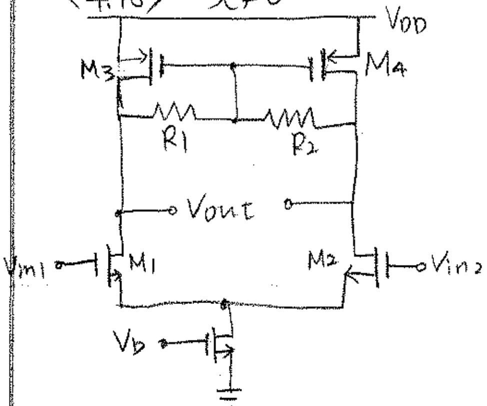
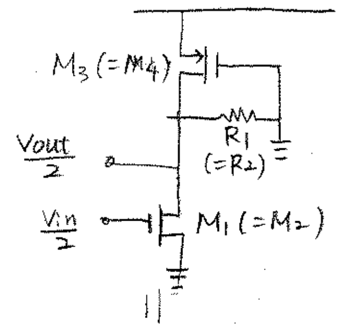
#1.

<4.18> $\lambda \neq 0$





<4.38(d)>

Assumption: All transistors ($M_1 = M_2$, $M_3 = M_4$) (and resistors ($R_1 = R_2$)) are identical.
That is, no process variation is considered.

$$\boxed{A_v = -g_{m1}(r_{o1} \parallel r_{o3} \parallel R_1)}$$

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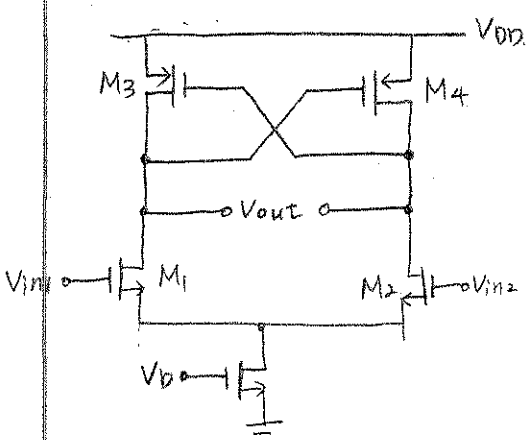
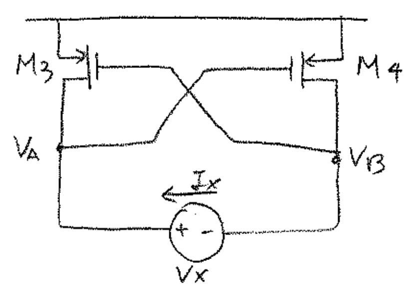


<4.39(c)>



Find impedance (looked up from two output node) $(\lambda = 0)$

$V_x = V_A - V_B$

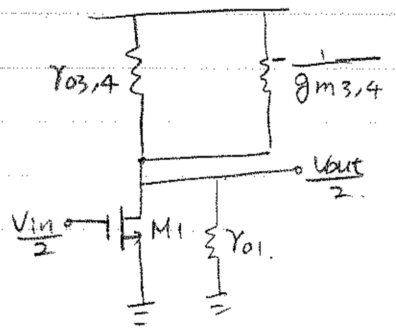
$\left.\begin{array}{l} I_x = g_{m3} V_B \\ I_x = -g_{m4} V_A \end{array}\right) \Rightarrow 2I_x = g_{m3} V_B - g_{m4} V_A \quad \text{①}$

Assume $g_{m3} = g_{m4}$

① becomes, $2I_x = -g_{m3,4}(V_A - V_B)$

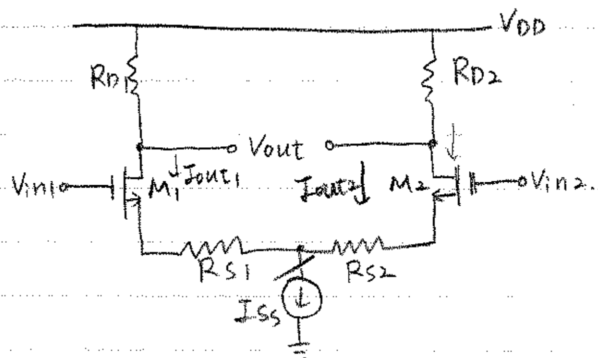
$= g_{m3,4} V_x$

$\therefore \dfrac{V_x}{I_x} = \dfrac{-2}{g_{m3,4}}$



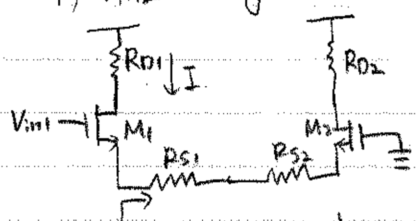
$$\boxed{A_v = -g_{m1}\left(r_{o1} \,\|\, r_{o3,4} \,\|\, -\frac{1}{g_{m3,4}}\right)}$$

#2. ⟨4.21⟩ $\lambda = 0$, $\gamma \neq 0$



Using superposition

i) V_{in2} is grounded.



$$R_{eq} = R_{S1} + R_{S2} + \frac{1}{g_{m2} + g_{mb2}}$$

$$G_{m1} = \frac{g_{m1}}{1 + (g_{m1} + g_{mb1})\left(R_{S1} + R_{S2} + \frac{1}{g_{m2} + g_{mb2}}\right)}$$

ii) V_{in1} is grounded, G_{m2} is exactly same as G_{m1} except $g_{m1} \to g_{m2}$, $g_{m2} \to g_{m1}$

$$\therefore G_{m2} = \frac{g_{m2}}{1 + (g_{m2} + g_{mb2})\left(R_{S1} + R_{S2} + \frac{1}{g_{m1} + g_{mb1}}\right)}$$

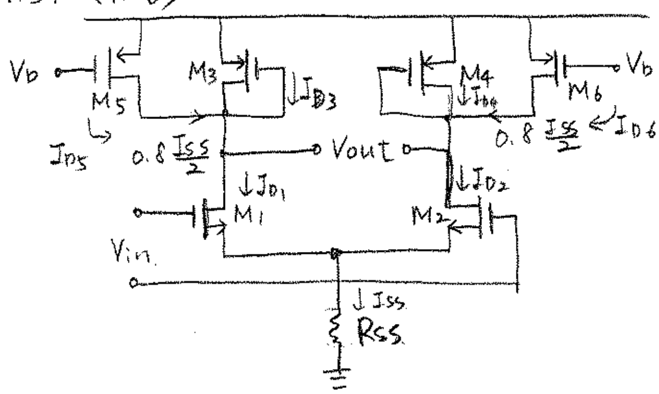
$\Rightarrow I_{out1} = I_1 - I_2 \qquad I_{out2} = I_2 - I_1$

$V_{out} = -R_{D1} I_{out1} + R_{D2} I_{out2} = -R_{D1}(I_1 - I_2) - R_{D2}(I_1 - I_2)$

$= -(R_{D1} + R_{D2})(I_1 - I_2)$

$$\boxed{\therefore V_{out} = -(R_{D1} + R_{D2})(G_{m1} V_{in1} - G_{m2} V_{in2})}$$

#3. <4.26>



$$CMRR = \left| \frac{A_{DM}}{A_{CM-DM}} \right|$$

$$R_D = \frac{1}{g_{m4}} \| r_{o1} \| r_{o3} \| r_{o5} \approx \frac{1}{g_{m4}} \quad (\because r_o \text{ is usually large})$$

$$I_{D5} + I_{D3} = \frac{I_{SS}}{2} = I_{D4} + I_{D6}$$

$$I_{D6} = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_6 \left(|V_{GS6}| - |V_{TH,6}|\right)^2$$

$$\Rightarrow \frac{\partial I_{D6}}{\partial V_{TH,p}} = -\mu_p C_{ox} \left(\frac{W}{L}\right)_6 \left(|V_{GS6}| - |V_{TH,p}|\right)$$

$$\Rightarrow \partial I_{D6} = -\mu_p C_{ox} \left(\frac{W}{L}\right)_6 \left(V_{DD} - V_b - |V_{TH,p}|\right) \cdot \partial V_{TH,p}$$

$$\Rightarrow \Delta I_{D6} = \mu_p C_{ox} \left(\frac{W}{L}\right)_6 \left(V_{DD} - V_b - |V_{TH,p}|\right) \cdot \Delta V_{TH,p}$$

$$g_{m4} = \sqrt{2 \mu_p C_{ox} \left(\frac{W}{L}\right)_4 I_{D4}}$$

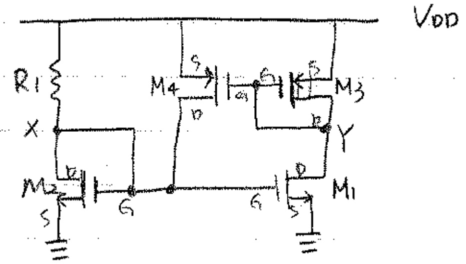
$$\frac{\partial g_{m4}}{\partial I_{D4}} = \sqrt{\frac{\mu_p C_{ox} \left(\frac{W}{L}\right)_4}{2 I_{D4}}} \Rightarrow \Delta g_{m4} = \sqrt{\frac{\mu_p C_{ox} \left(\frac{W}{L}\right)_4}{2 I_{D4}}} \, \Delta I_{D4}$$

$$\frac{\partial R_D}{\partial g_{m4}} = -\frac{1}{g_{m4}} \qquad \frac{\Delta R_D}{R_D} = -\frac{\Delta g_{m4}}{g_{m4}} \quad \left(\because R_D = \frac{1}{g_{m4}}\right)$$

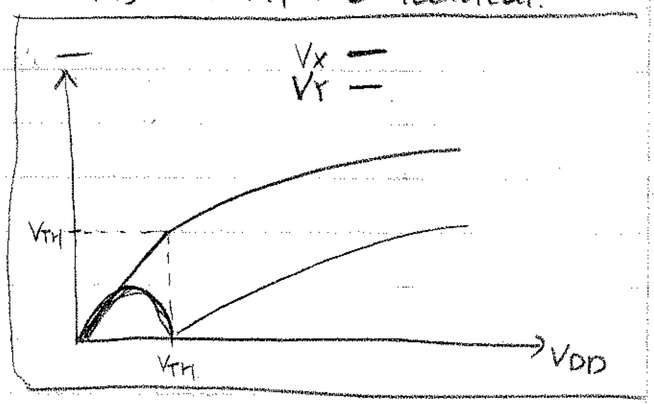
$$\frac{\Delta R}{R_D} = \overbrace{\left(\frac{I_{D6}}{I_{D4}}\right)}^{4} \cdot \frac{V_{TH,p}}{V_{DD} - V_b - |V_{TH,p}|} \cdot \frac{\Delta V_{TH,p}}{V_{TH,p}}$$

$$CMRR = \frac{1 + 2 g_{m1} R_{SS}}{\Delta R_D / R_D} = \boxed{\frac{1 + 2 g_{m1} R_{SS}}{4} \cdot \frac{(V_{DD} - V_b - |V_{TH,p}|)}{V_{TH,p}} \cdot \frac{V_{TH,p}}{\Delta V_{TH,p}}}$$

#4 ⟨5.10(a)⟩



Assume M_1 and M_2 are identical
M_3 and M_4 are identical.



i) $V_{DD} < V_{TH1,2}$

All transistors are off.

$\therefore V_X = V_{DD}$

V_Y is floating

ii) $V_{DD} > V_{TH1,2}$

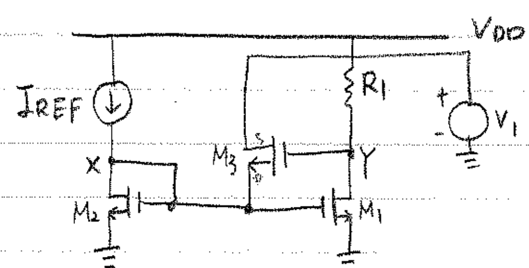
M_2 turns on, so M_1 turns on.
(saturation) (triode)

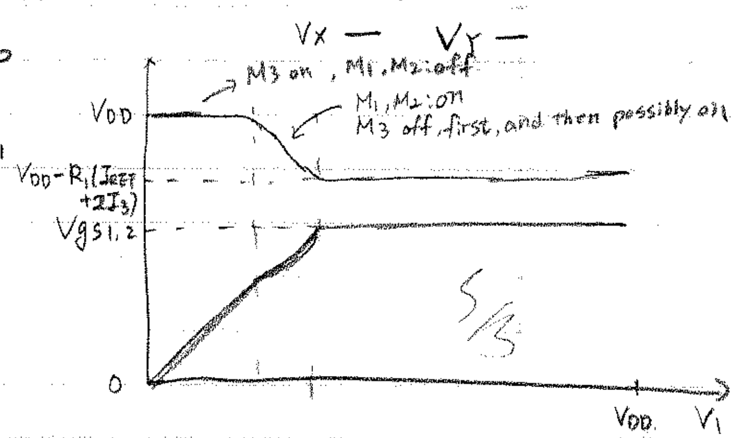
M_3 : saturation, M_4: triode region.

$V_X = V_{gs2}$ $\qquad V_Y = V_{DD} - |V_{gs3}| \approx V_{DD} - V_{gs2}$ (due to $I_{D2} = I_{D1} = |I_{D3}|$)

$V_X > V_Y$

#5 ⟨5.12(b)⟩ V_X and V_Y as a function of V_1 for $0 < V_1 < V_{DD}$





i) $V_1 = 0$

M_1 and M_2 turn off.

$V_X = 0\,V$ $\qquad V_Y = V_{DD}$ $\qquad V_{DS3} = I_{REF} \times \frac{1}{g_{m3}}$

At low value of V_1, roles of drain and source of M_3 are switched.

ii)

As V_{in} increases, V_{DS3} (That is, V_X) increases.
When it reaches $V_X - V_{th1,2}$, M_1 and M_2 turn on. V_Y starts to decrease.

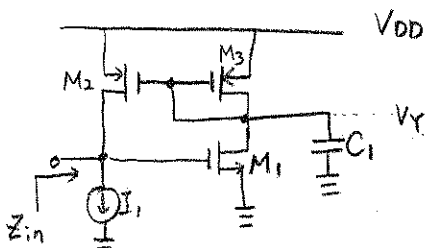
iii) When M_3 is off.

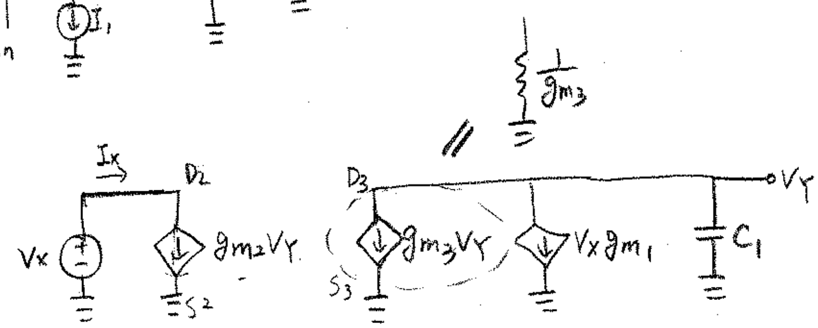
$V_X = V_{gs2}$

$V_Y = V_{DD} - I_{REF} R_1$

If $V_{th3} < V_Y - V_X$, M_3 turns on, to increase I_2. As a result I_1 also increases. Thus, $V_Y = V_{DD} - R_1(I_{REF} + 2I_3)$

#6 <5.19>





$I_X = g_{m2} V_Y$ ..... ①

$V_Y = -V_X g_{m1} \cdot \left(\frac{1}{g_{m3}} \parallel \frac{1}{C_1 s}\right)$ ... ②

$$\frac{1}{g_{m3}} \parallel \frac{1}{C_1 s} = \frac{\frac{1}{g_{m3}} \cdot \frac{1}{C_1 s}}{\frac{1}{g_{m3}} + \frac{1}{C_1 s}} = \frac{\frac{1}{g_{m3} C_1 s}}{\frac{C_1 s + g_{m3}}{g_{m3} C_1 s}} = \frac{1}{C_1 s + g_{m3}}$$

plug ② into ①

$I_X = g_{m2}\left[-V_X g_{m1}\left(\frac{1}{g_{m3}} \parallel \frac{1}{C_1 s}\right)\right]$

$$\Leftrightarrow \frac{V_X}{I_X} = -\frac{1}{g_{m1} g_{m2}\left(\frac{1}{g_{m3}} \parallel \frac{1}{C_1 s}\right)}$$

$$= -\frac{g_{m3} + C_1 s}{g_{m1} g_{m2}} \approx \boxed{-\frac{C_1 s}{g_m^2}}\ \boxed{-\frac{1}{g_m}}$$ (If all transistors are identical)

→ resistive component

→ capacitive component

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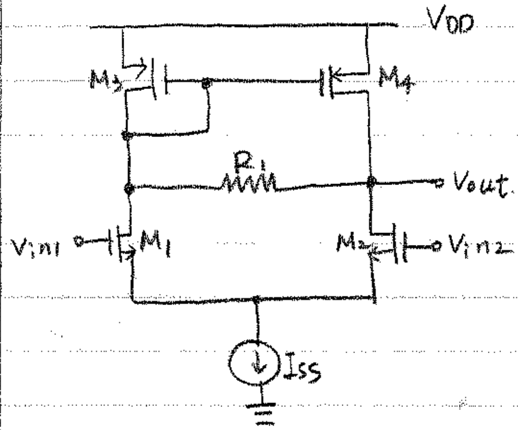
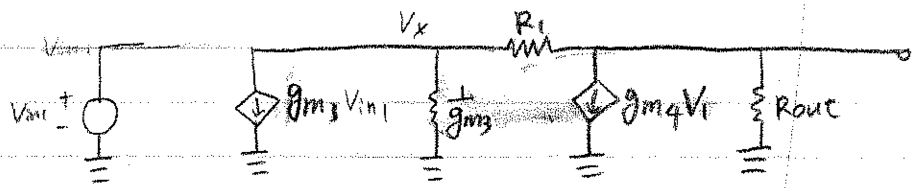
#7 <5.20(b)>



Using superposition,

1) $V_{in1} = +\frac{\Delta V}{2}$

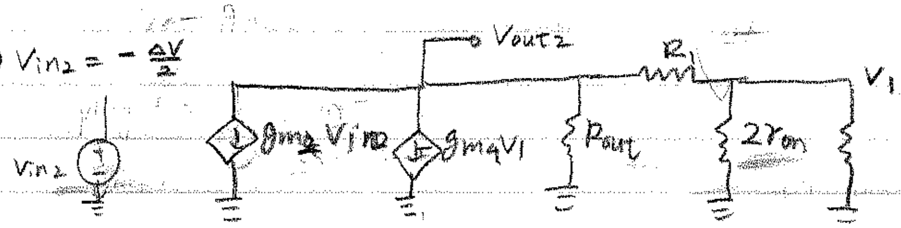


$$g_{m1} V_{in1} + g_{m3} V_x + \frac{V_x - V_{out1}}{R_1} = 0$$

$$g_{m4} V_1 + \frac{V_{out1}}{R_{out}} + \frac{V_{out1} - V_x}{R_1} = 0$$

$$\boxed{\frac{V_{out1}}{V_{in1}} = g_{m1} R_{out} \left[\frac{g_{m3} R - 1}{g_{m3} R + 2 R_{out} + 1} \right]}$$

2) $V_{in2} = -\frac{\Delta V}{2}$



$$V_1 = V_{out2} \cdot \frac{\frac{1}{g_{m3}} \| 2r_{on}}{R + 2r_{on} \| \frac{1}{g_{m3}}} = V_{out2} \cdot \frac{2r_{on}}{2r_{on} + 2g_{m3} r_{on} R_1 + R_1}$$

$$V_{out2} = \left(g_{m2} V_{in2} - g_{m4} V_{out2} \frac{2r_{on}}{2r_{on} + 2g_{m3} r_{on} R_1 + R_1} \right) \left(\left(R_1 + 2r_{on} \| \frac{1}{g_{m3}}\right) \| R_{out} \right)$$

$$\boxed{\frac{V_{out2}}{V_{in2}} = -g_{m2} \frac{\left[\left(R_1 + 2r_{on} \| \frac{1}{g_{m3}} \right) \right]}{1 + \frac{2g_{m4} r_{on}}{R_1 + r_{on} + 2g_{m3} r_{on} R_1} \cdot \left[\left(R_1 + 2r_{on} \| \frac{1}{g_{m3}}\right) \| R_{out} \right]}}$$

$$\therefore A_v = \frac{V_{out1} + V_{out2}}{V_{in1} - V_{in2}} = \frac{V_{out1} + V_{out2}}{\frac{\Delta V}{2} - \left(-\frac{\Delta V}{2}\right)} = \frac{V_{out1} + V_{out2}}{\Delta V}$$

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