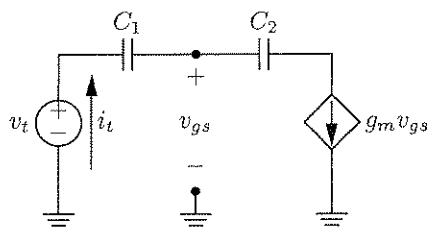
EE 215A
Eudean Sun

73/80

Homework 4
October 30, 2008

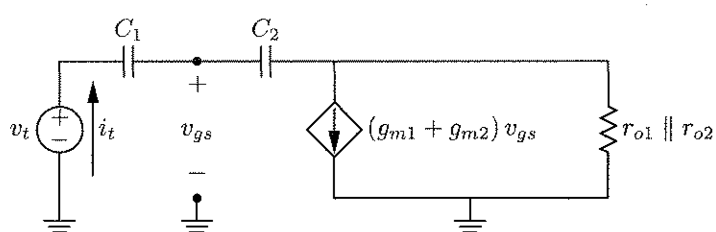
6.6 (a)



$$i_t = g_m v_{gs}$$
$$v_{gs} = v_t - \frac{i_t}{sC_1}$$
$$i_t = g_m \left(v_t - \frac{i_t}{sC_1} \right)$$
$$i_t \left(1 + \frac{g_m}{sC_1} \right) = g_m v_t$$
$$Z_{in} = \frac{v_t}{i_t} = \boxed{\frac{1}{g_m} + \frac{1}{sC_1}}$$

5/5

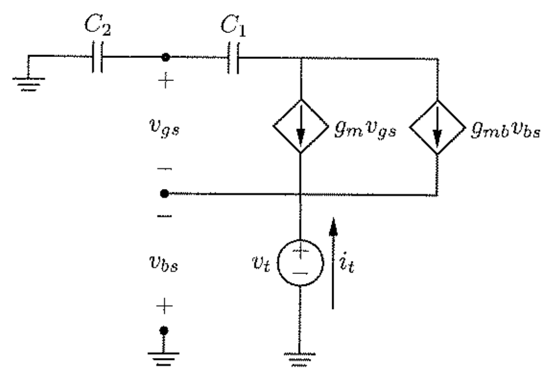
(b) Note that $v_{gs1} = v_{gs2} = v_{gs}$ in the small-signal model.



$$i_t = (g_{m1} + g_{m2})\, v_{gs} + \frac{v_t - \frac{i_t}{s(C_1+C_2)}}{r_{o1} \parallel r_{o2}}$$
$$v_{gs} = v_t - \frac{i_t}{sC_1}$$
$$i_t = (g_{m1} + g_{m2}) \left(v_t - \frac{i_t}{sC_1} \right) + \frac{v_t - \frac{i_t}{s(C_1+C_2)}}{r_{o1} \parallel r_{o2}}$$
$$i_t \left[1 + \frac{g_{m1} + g_{m2}}{sC_1} + \frac{1}{s\,(C_1 + C_2)\,(r_{o1} \parallel r_{o2})} \right] = v_t \left(g_{m1} + g_{m2} + \frac{1}{r_{o1} \parallel r_{o2}} \right)$$
$$Z_{in} = \frac{v_t}{i_t} = \boxed{\frac{1 + \frac{g_{m1}+g_{m2}}{sC_1} + \frac{1}{s(C_1+C_2)(r_{o1} \parallel r_{o2})}}{g_{m1} + g_{m2} + \frac{1}{r_{o1} \parallel r_{o2}}}}$$

5/5

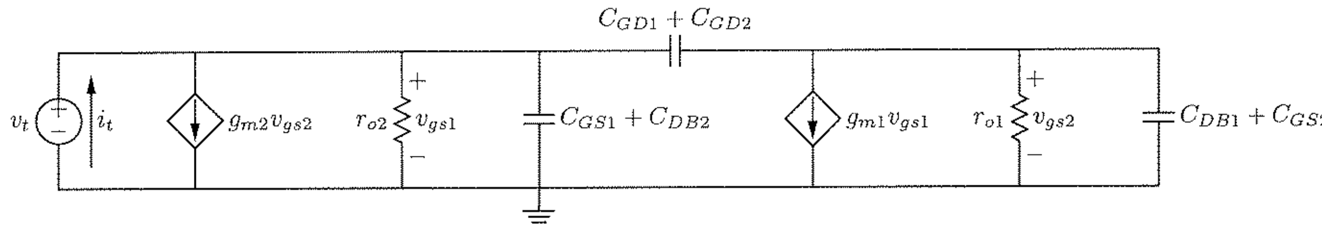
(c)



$$
\begin{aligned}
i_t &= -g_m v_{gs} - v_{mb} v_{bs} \\
&= -g_m v_{gs} + g_{mb} v_t \\
v_{gs} &= \frac{i_t}{sC_2} - v_t \\
i_t &= -g_m \left(\frac{i_t}{sC_2} - v_t \right) + g_{mb} v_t \\
i_t \left(1 + \frac{g_m}{sC_2} \right) &= v_t \left(g_m + g_{mb} \right) \\
Z_{in} = \frac{v_t}{i_t} &= \boxed{\frac{g_m + sC_2}{\left(g_m + g_{mb} \right) sC_2}}
\end{aligned}
$$

5/5

6.8 (b) First, the input impedance (excluding R_S):



$$i_t = v_t \left[g_{m1} + s\left(C_{GS1} + C_{DB2}\right) + \frac{1}{r_{o2}}\right] + v_{gs2}\left[g_{m2} + s\left(C_{DB1} + C_{GS2}\right) + \frac{1}{r_{o1}}\right]$$

$$v_{gs2} = \left[\left(v_t - v_{gs2}\right) s\left(C_{GD1} + C_{GD2}\right) - g_{m1}v_t\right]\left[r_{o1} \parallel \frac{1}{s\left(C_{DB1} + C_{GS2}\right)}\right]$$

$$v_{gs2}\left\{1 + s\left(C_{GD1} + C_{GD2}\right)\left[r_{o1} \parallel \frac{1}{s\left(C_{DB1} + C_{GS2}\right)}\right]\right\} = v_t\left[s\left(C_{GD1} + C_{GD2}\right) - g_{m1}\right]\left[r_{o1} \parallel \frac{1}{s\left(C_{DB1} + C_{GS2}\right)}\right]$$

$$v_{gs2} = v_t \frac{\left[s\left(C_{GD1} + C_{GD2}\right) - g_{m1}\right]\left[r_{o1} \parallel \frac{1}{s(C_{DB1}+C_{GS2})}\right]}{1 + s\left(C_{GD1} + C_{GD2}\right)\left[r_{o1} \parallel \frac{1}{s(C_{DB1}+C_{GS2})}\right]}$$

$$i_t = v_t\left[g_{m1} + s\left(C_{GS1} + C_{DB2}\right) + \frac{1}{r_{o2}}\right] + v_t \frac{\left[s\left(C_{GD1} + C_{GD2}\right) - g_{m1}\right]\left[r_{o1} \parallel \frac{1}{s(C_{DB1}+C_{GS2})}\right]}{1 + s\left(C_{GD1} + C_{GD2}\right)\left[r_{o1} \parallel \frac{1}{s(C_{DB1}+C_{GS2})}\right]}\left[g_{m2} + s\left(C_{DB1} + C_{GS2}\right) + \frac{1}{r_{o1}}\right]$$

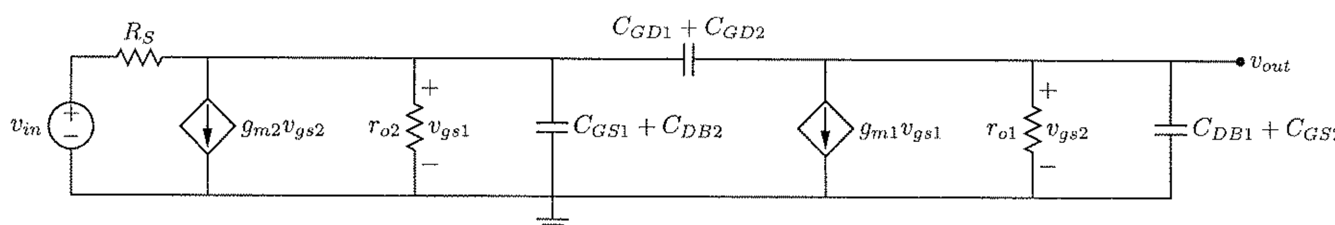
$$= v_t\left[g_{m1} + s\left(C_{GS1} + C_{DB2}\right) + \frac{1}{r_{o2}} + \frac{\left[s\left(C_{GD1} + C_{GD2}\right) - g_{m1}\right]\left[r_{o1} \parallel \frac{1}{s(C_{DB1}+C_{GS2})}\right]\left[g_{m2} + s\left(C_{DB1} + C_{GS2}\right) + \frac{1}{r_{o1}}\right]}{1 + s\left(C_{GD1} + C_{GD2}\right)\left[r_{o1} \parallel \frac{1}{s(C_{DB1}+C_{GS2})}\right]}\right]$$

$$Z_{in} = \frac{v_t}{i_t} = \left[g_{m1} + s\left(C_{GS1} + C_{DB2}\right) + \frac{1}{r_{o2}} + \frac{\left[s\left(C_{GD1} + C_{GD2}\right) - g_{m1}\right]\left[r_{o1} \parallel \frac{1}{s(C_{DB1}+C_{GS2})}\right]\left[g_{m2} + s\left(C_{DB1} + C_{GS2}\right) + \frac{1}{r_{o1}}\right]}{1 + s\left(C_{GD1} + C_{GD2}\right)\left[r_{o1} \parallel \frac{1}{s(C_{DB1}+C_{GS2})}\right]}\right]^{-1}$$

$$= \left[g_{m1} + s\left(C_{GS1} + C_{DB2}\right) + \frac{1}{r_{o2}} + \frac{\left[s\left(C_{GD1} + C_{GD2}\right) - g_{m1}\right]\left[g_{m2} + s\left(C_{DB1} + C_{GS2}\right) + \frac{1}{r_{o1}}\right]}{\frac{1}{r_{o1}} + s\left(C_{DB1} + C_{GS2} + C_{GD1} + C_{GD2}\right)}\right]^{-1}$$

$$= \boxed{\left(\frac{1}{g_{m1}} \parallel \frac{1}{s\left(C_{GS1} + C_{DB2}\right)} \parallel r_{o2} \parallel \frac{\frac{1}{r_{o1}} + s\left(C_{DB1} + C_{GS2} + C_{GD1} + C_{GD2}\right)}{\left[s\left(C_{GD1} + C_{GD2}\right) - g_{m1}\right]\left[g_{m2} + s\left(C_{DB1} + C_{GS2}\right) + \frac{1}{r_{o1}}\right]}\right)}$$

Now for the transfer function. Note that the following small signal model is very difficult to analyze. However, we can make an observation that simplifies the problem greatly. Since we've already found Z_{in}, we know that $v_{gs1} = \frac{Z_{in}}{Z_{in}+R_S}v_{in}$. Using this, we only need to find $\frac{v_{out}}{v_{gs1}}$ in order to find the total gain (since we can just multiply this result by $\frac{v_{gs1}}{v_{in}}$).



$$v_{out} = \left[(v_{gs1} - v_{out})\, s\,(C_{GD1} + C_{GD2}) - g_{m1} v_{gs1}\right]\left[r_{o1} \parallel \frac{1}{s\,(C_{DB1} + C_{GS2})}\right]$$

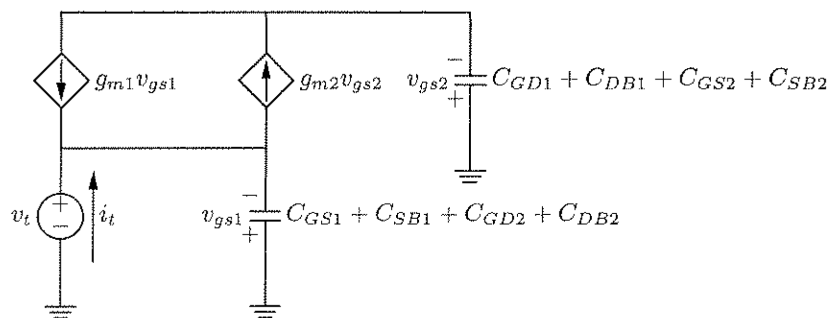
$$v_{out}\left\{1 + s\,(C_{GD1} + C_{GD2})\left[r_{o1} \parallel \frac{1}{s\,(C_{DB1} + C_{GS2})}\right]\right\} = v_{gs1}\left[s\,(C_{GD1} + C_{GD2}) - g_{m1}\right]\left[r_{o1} \parallel \frac{1}{s\,(C_{DB1} + C_{GS2})}\right]$$

$$\frac{v_{out}}{v_{gs1}} = \frac{\left[s\,(C_{GD1} + C_{GD2}) - g_{m1}\right]\left[r_{o1} \parallel \frac{1}{s(C_{DB1}+C_{GS2})}\right]}{1 + s\,(C_{GD1} + C_{GD2})\left[r_{o1} \parallel \frac{1}{s(C_{DB1}+C_{GS2})}\right]}$$

5/5

$$\boxed{\frac{v_{out}}{v_{in}} = \frac{Z_{in}}{Z_{in} + R_S} \cdot \frac{\left[s\,(C_{GD1} + C_{GD2}) - g_{m1}\right]\left[r_{o1} \parallel \frac{1}{s(C_{DB1}+C_{GS2})}\right]}{1 + s\,(C_{GD1} + C_{GD2})\left[r_{o1} \parallel \frac{1}{s(C_{DB1}+C_{GS2})}\right]}}$$

(e) First, the input impedance (excluding R_S):



$$i_t = g_{m1} v_t + g_{m2} v_{gs2} + v_t s\,(C_{GS1} + C_{SB1} + C_{GD2} + C_{DB2})$$

$$v_{gs2} = -\frac{g_{m2} v_{gs2} + g_{m1} v_t}{s\,(C_{GD1} + C_{DB1} + C_{GS2} + C_{SB2})}$$

$$v_{gs2}\left[1 + \frac{g_{m2}}{s\,(C_{GD1} + C_{DB1} + C_{GS2} + C_{SB2})}\right] = -\frac{g_{m1} v_t}{s\,(C_{GD1} + C_{DB1} + C_{GS2} + C_{SB2})}$$

$$\begin{aligned} v_{gs2} &= -\frac{g_{m1} v_t}{s\,(C_{GD1} + C_{DB1} + C_{GS2} + C_{SB2})} \cdot \frac{s\,(C_{GD1} + C_{DB1} + C_{GS2} + C_{SB2})}{g_{m2} + s\,(C_{GD1} + C_{DB1} + C_{GS2} + C_{SB2})} \\ &= -\frac{g_{m1} v_t}{g_{m2} + s\,(C_{GD1} + C_{DB1} + C_{GS2} + C_{SB2})} \\ i_t &= v_t\left[g_{m1} + s\,(C_{GS1} + C_{SB1} + C_{GD2} + C_{DB2}) - \frac{g_{m1} g_{m2}}{g_{m2} + s\,(C_{GD1} + C_{DB1} + C_{GS2} + C_{SB2})}\right] \\ Z_{in} &= \frac{v_t}{i_t} = \left[g_{m1} + s\,(C_{GS1} + C_{SB1} + C_{GD2} + C_{DB2}) - \frac{g_{m1} g_{m2}}{g_{m2} + s\,(C_{GD1} + C_{DB1} + C_{GS2} + C_{SB2})}\right]^{-1} \\ &= \boxed{\frac{g_{m2} + s\,(C_{GD1} + C_{DB1} + C_{GS2} + C_{SB2})}{\left[g_{m1} + s\,(C_{GS1} + C_{SB1} + C_{GD2} + C_{DB2})\right]\left[g_{m2} + s\,(C_{GD1} + C_{DB1} + C_{GS2} + C_{SB2})\right] - g_{m1} g_{m2}}} \end{aligned}$$

We can employ the same method used in part (b) to find the transfer function (in this case finding $\frac{v_{out}}{v_{s1}} = -\frac{v_{out}}{v_{gs1}}$).

4/5

7.5

$$A_v = -\left(g_{m1} + g_{m2}\right)\left(r_{o1} \parallel r_{o2}\right)$$

$$\overline{V_{n,out}^2} = \frac{8}{3}kT\left(g_{m1} + g_{m2}\right)\left(r_{o1} \parallel r_{o2}\right)^2$$

$$\overline{V_n^2} = \frac{\frac{8}{3}kT\left(g_{m1} + g_{m2}\right)\left(r_{o1} \parallel r_{o2}\right)^2}{A_v^2}$$

$$= \boxed{\frac{\frac{8}{3}kT}{g_{m1} + g_{m2}}}$$

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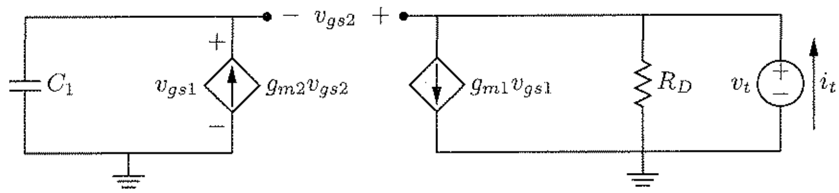
Comparing to Eq. (7.59), we can see that originally we wanted to maximize g_{m1} and minimize g_{m2} in order to minimize the noise. However, in this case, we want to minimize both g_{m1} and g_{m2} in order to minimize the noise. This is because in the circuit in Fig. 7.35, g_{m2} did not contribute to the gain, whereas in this case it does.

5/5

(b) Here is the small-signal model:



$$i_t = g_{m1}v_{gs1} + \frac{v_t}{R_D}$$

$$v_{gs1} = \frac{g_{m2}v_{gs2}}{sC_1} = \frac{g_{m2}\left(v_t - v_{gs1}\right)}{sC_1}$$

$$v_{gs1} = \frac{g_{m2}v_t}{sC_1\left(1+\frac{g_{m2}}{sC_1}\right)} = \frac{g_{m2}}{g_{m2}+sC_1}v_t$$

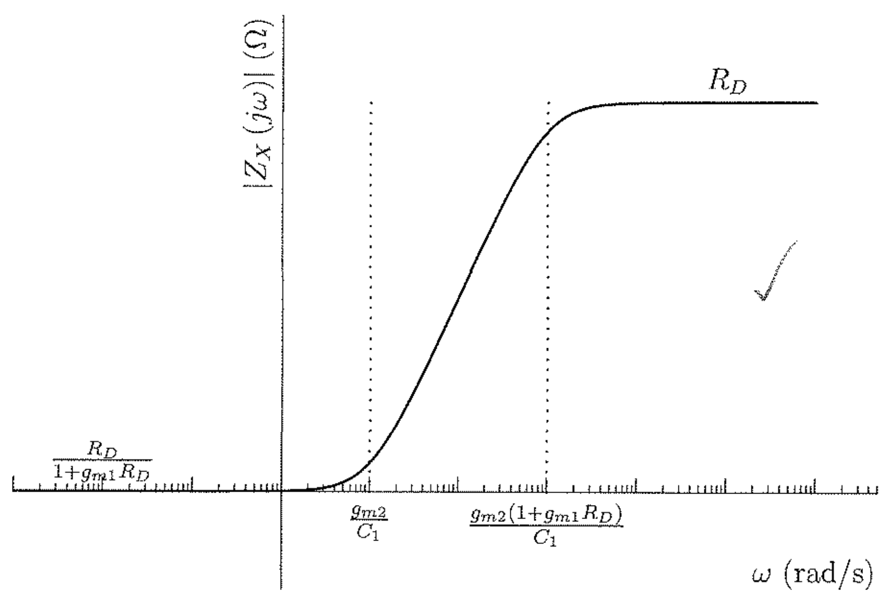
$$i_t = v_t\left(\frac{g_{m1}g_{m2}}{g_{m2}+sC_1} + \frac{1}{R_D}\right)$$

$$Z_X = \frac{v_t}{i_t} = \boxed{R_D \parallel \frac{g_{m2}+sC_1}{g_{m1}g_{m2}}}$$

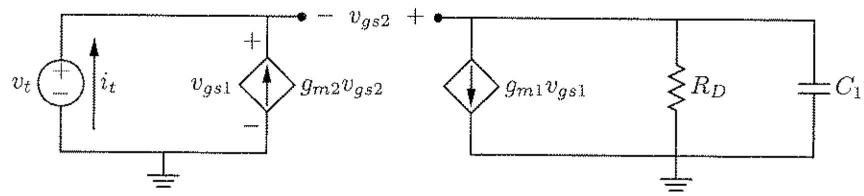
$$= \frac{\left(g_{m2}+sC_1\right)R_D}{g_{m2}+sC_1+g_{m1}g_{m2}R_D}$$

$$\left|Z_X\left(j\omega\right)\right| = \frac{\sqrt{\left(g_{m2}R_D\right)^2 + \left(\omega C_1 R_D\right)^2}}{\sqrt{g_{m2}^2\left(1+g_{m1}R_D\right)^2 + \left(\omega C_1\right)^2}}$$

Here's the amplitude plotted on a log-log scale.

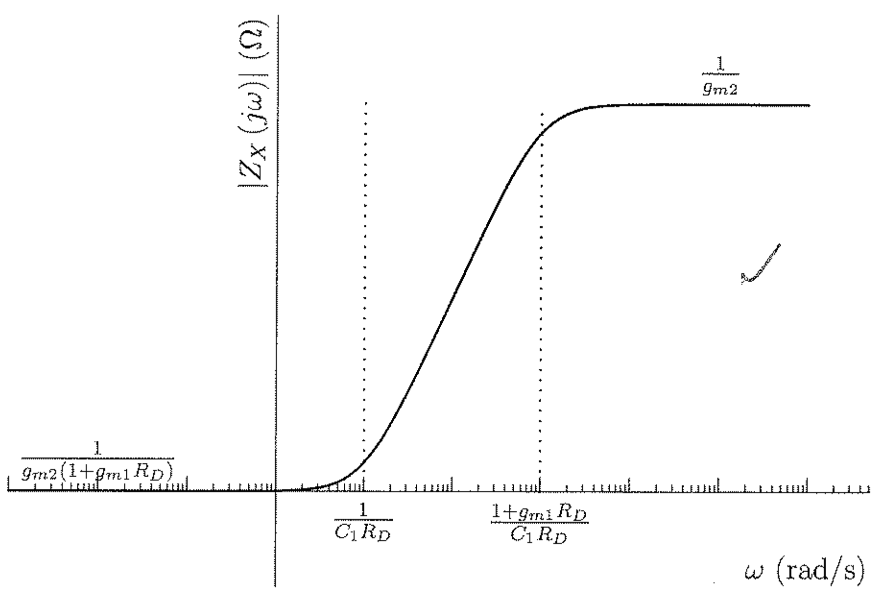


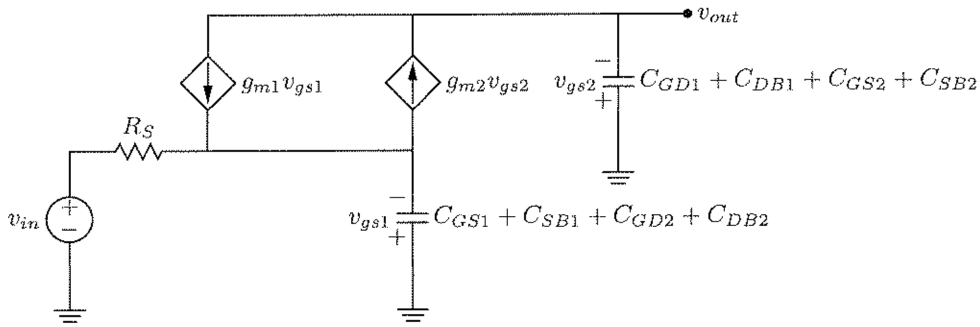
6.12 (a) Here is the small-signal model:



$$
\begin{aligned}
i_t &= -g_{m2}v_{gs2} \\
v_{gs2} &= -g_{m1}v_t\left(R_D \parallel \frac{1}{sC_1}\right) - v_t \\
i_t &= g_{m2}v_t\left[g_{m1}\left(R_D \parallel \frac{1}{sC_1}\right) + 1\right] \\
Z_X &= \frac{v_t}{i_t} = \frac{1}{g_{m2}\left[g_{m1}\left(R_D \parallel \frac{1}{sC_1}\right) + 1\right]} \\
&= \boxed{\frac{1 + sC_1R_D}{g_{m1}g_{m2}R_D + g_{m2}\left(1 + sC_1R_D\right)}} \\
\left|Z_X\left(j\omega\right)\right| &= \frac{\sqrt{1 + \left(\omega C_1R_D\right)^2}}{\sqrt{g_{m2}^2\left(g_{m1}R_D + 1\right)^2 + \left(\omega g_{m2}C_1R_D\right)^2}}
\end{aligned}
$$

Here's the amplitude plotted on a log-log scale.





$$v_{out} = -\frac{g_{m2}v_{out} + g_{m1}v_{gs1}}{s\left(C_{GD1}+C_{DB1}+C_{GS2}+C_{SB2}\right)}$$

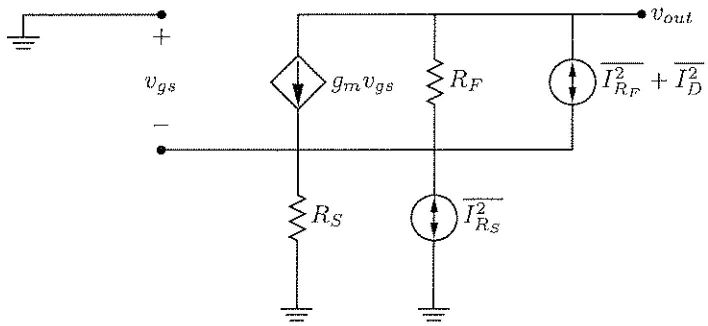
$$v_{out}\left[1 + \frac{g_{m2}}{s\left(C_{GD1}+C_{DB1}+C_{GS2}+C_{SB2}\right)}\right] = -\frac{g_{m1}v_{gs1}}{s\left(C_{GD1}+C_{DB1}+C_{GS2}+C_{SB2}\right)}$$

$$-\frac{v_{out}}{v_{gs1}} = \frac{g_{m1}}{s\left(C_{GD1}+C_{DB1}+C_{GS2}+C_{SB2}\right)} \cdot \frac{s\left(C_{GD1}+C_{DB1}+C_{GS2}+C_{SB2}\right)}{g_{m2}+s\left(C_{GD1}+C_{DB1}+C_{GS2}+C_{SB2}\right)}$$

$$= \frac{g_{m1}}{g_{m2}+s\left(C_{GD1}+C_{DB1}+C_{GS2}+C_{SB2}\right)}$$

$$\frac{v_{out}}{v_{in}} = \boxed{\frac{Z_{in}}{Z_{in}+R_S} \cdot \frac{g_{m1}}{g_{m2}+s\left(C_{GD1}+C_{DB1}+C_{GS2}+C_{SB2}\right)}}$$

7.6 (c) Here's the small-signal model with noise sources drawn separately:



Since the noise sources are uncorrelated, we can use superposition of the noise powers at the output. First, consider when $\overline{I_{R_S}^2}$ is off and $\overline{I_D^2}$ and $\overline{I_{R_F}^2}$ are on.

$$\begin{aligned}
\overline{I_{R_F}^2} &= \frac{4kT}{R_F} \\
\overline{I_D^2} &= 4kT\frac{2}{3}g_m \\
\overline{V_{n,out}^2} &= \left(\overline{I_{R_F}^2} + \overline{I_D^2}\right) R_F^2 \\
&= 4kT\left(\frac{1}{R_F} + \frac{2}{3}g_m\right) R_F^2
\end{aligned}$$

Now, consider when $\overline{I_{R_S}^2}$ is on and $\overline{I_D^2}$ and $\overline{I_{R_F}^2}$ are off (note that since polarity doesn't matter for noise analysis, I'm treating v_{gs} as positive for convenience):

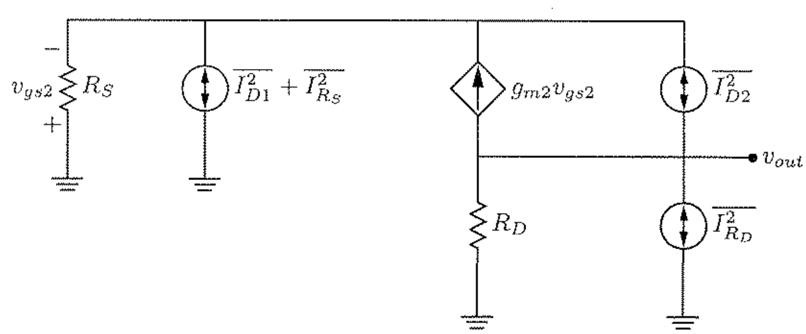
$$\begin{aligned}
\overline{I_{R_S}^2} &= \frac{4kT}{R_S} \\
v_{gs} &= \sqrt{\overline{I_{R_S}^2}}R_S \\
&= \sqrt{4kTR_S} \\
v_{out} &= g_m v_{gs} R_F + v_{gs} \\
&= v_{gs}\left(1 + g_m R_F\right) \\
\overline{V_{n,out}^2} &= 4kTR_S\left(1 + g_m R_F\right)^2
\end{aligned}$$

Combining the contributions of these noise sources gives:

$$\begin{aligned}
\overline{V_{n,out}^2} &= 4kT\left[\left(\frac{2}{3}g_m + \frac{1}{R_F}\right) R_F^2 + R_S\left(1 + g_m R_F\right)^2\right] \\
A_v &= -g_m R_F \text{ from Eq. (3.71) with } R_D \to \infty \\
\overline{V_{n,in}^2} &= \frac{\overline{V_{n,out}^2}}{A_v^2} \\
&= \boxed{4kT\left[\frac{2}{3}\frac{1}{g_m} + \frac{1}{g_m^2 R_F} + R_S\left(1 + \frac{1}{g_m R_F}\right)^2\right]}
\end{aligned}$$

✓ 5/5

(f) First, note that since $v_{gs1} = 0$ always (since we ground the input to find the output noise), we can ignore M_1's transconductance term. Here is the resulting small-signal model with noise sources drawn separately:



Once again, let's use superposition. First, assume $\overline{I_{D1}^2}$, $\overline{I_{R_S}^2}$, and $\overline{I_{D2}^2}$ are off and $\overline{I_{R_D}^2}$ is on.

$$g_{m2}v_{gs2} = -v_{gs2} \Rightarrow v_{gs2} = 0$$
$$\overline{V_{n,out}^2} = \overline{I_{R_D}^2} R_D^2$$

Now, assume $\overline{I_{D1}^2}$, $\overline{I_{R_S}^2}$, and $\overline{I_{R_D}^2}$ are off and $\overline{I_{D2}^2}$ is on.

$$\begin{aligned}
-v_{gs2} &= \left(g_{m2}v_{gs2} + \sqrt{\overline{I_{D2}^2}} \right) R_S \\
v_{gs2}\left(1 + g_{m2}R_S\right) &= \sqrt{\overline{I_{D2}^2}} R_S \\
v_{out} &= \left(-g_{m2}v_{gs2} + \sqrt{\overline{I_{D2}^2}} \right) R_D \\
&= \left(1 - \frac{g_{m2}R_S}{1 + g_{m2}R_S} \right) \sqrt{\overline{I_{D2}^2}} R_D \\
&= \left(\frac{1}{1 + g_{m2}R_S} \right) \sqrt{\overline{I_{D2}^2}} R_D \\
\overline{V_{n,out}^2} &= \left(\frac{1}{1 + g_{m2}R_S} \right)^2 \overline{I_{D2}^2} R_D^2
\end{aligned}$$

Now, assume $\overline{I_{D2}^2}$ and $\overline{I_{R_D}^2}$ are off and $\overline{I_{D1}^2}$ and $\overline{I_{R_S}^2}$ are on.

$$\begin{aligned}
-v_{gs2} &= \left(\sqrt{\overline{I_{D1}^2} + \overline{I_{R_S}^2}} + g_{m2}v_{gs2} \right) R_S \\
v_{gs2}\left(1 + g_{m2}R_S\right) &= \left(\sqrt{\overline{I_{D1}^2} + \overline{I_{R_S}^2}} \right) R_S \\
\overline{v_{gs2}^2} &= \frac{\left(\overline{I_{D1}^2} + \overline{I_{R_S}^2} \right) R_S^2}{\left(1 + g_{m2}R_S\right)^2} \\
\overline{V_{n,out}^2} &= g_{m2}^2 \overline{v_{gs2}^2} R_D^2 \\
&= \frac{\left(\overline{I_{D1}^2} + \overline{I_{R_S}^2} \right) g_{m2}^2 R_S^2 R_D^2}{\left(1 + g_{m2}R_S\right)^2}
\end{aligned}$$

Combining these results, we get:

$$\overline{V_{n,out}^2} = \overline{I_{R_D}^2}R_D^2 + \left(\frac{1}{1+g_{m2}R_S}\right)^2 \overline{I_{D2}^2}R_D^2 + \frac{\left(\overline{I_{D1}^2} + \overline{I_{R_S}^2}\right) g_{m2}^2 R_S^2 R_D^2}{(1+g_{m2}R_S)^2}$$

$$\overline{I_{R_D}^2} = \frac{4kT}{R_D}$$

$$\overline{I_{R_S}^2} = \frac{4kT}{R_S}$$

$$\overline{I_{D1}^2} = 4kT\frac{2}{3}g_{m1}$$

$$\overline{I_{D2}^2} = 4kT\frac{2}{3}g_{m2}$$

$$\overline{V_{n,out}^2} = 4kTR_D + 4kT\frac{2}{3}g_{m2}R_D^2\left(\frac{1}{1+g_{m2}R_S}\right)^2 + \left(\frac{g_{m2}R_SR_D}{1+g_{m2}R_S}\right)^2\left(4kT\frac{2}{3}g_{m1} + \frac{4kT}{R_S}\right)$$

$$= 4kTR_D^2\left[\frac{1}{R_D} + \frac{2}{3}g_{m2}\left(\frac{1}{1+g_{m2}R_S}\right)^2 + \left(\frac{g_{m2}R_S}{1+g_{m2}R_S}\right)^2\left(\frac{2}{3}g_{m1} + \frac{1}{R_S}\right)\right]$$

$$A_v = -g_{m1}g_{m2}\left(R_S \parallel \frac{1}{g_{m2}}\right)R_D$$

$$= -g_{m1}g_{m2}\frac{R_S}{1+g_{m2}R_S}R_D$$

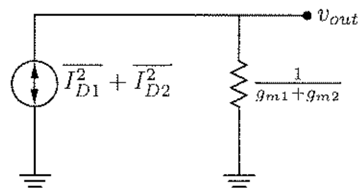
$$= -\frac{g_{m1}g_{m2}R_SR_D}{1+g_{m2}R_S}$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{A_v^2}$$

$$= \boxed{4kT\left[\left(\frac{1+g_{m2}R_S}{g_{m1}g_{m2}R_S}\right)^2\frac{1}{R_D} + \frac{2}{3}\frac{1}{g_{m2}}\left(\frac{1}{g_{m1}R_S}\right)^2 + \frac{2}{3}\frac{1}{g_{m1}} + \frac{1}{g_{m1}^2R_S}\right]}$$

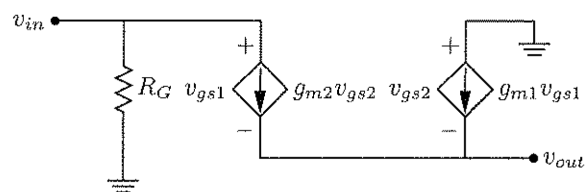
√ 5/5

7.9 (e) First, note that when we short the input to ground, we will eliminate R_G from the circuit and cause M_1 and M_2 to be diode-connected in the small-signal model, reducing them to simple equivalent resistances (which are in parallel, since M_1 and M_2 share source, drain, and gate connections in the small-signal model). Thus, the small-signal model for calculating the noise is as follows:



$$\overline{V^2_{n,out}} = 4kT\frac{2}{3}(g_{m1} + g_{m2})\left(\frac{1}{g_{m1} + g_{m2}}\right)^2$$
$$= 4kT\frac{2}{3}\left(\frac{1}{g_{m1} + g_{m2}}\right)$$

Here's the small-signal model for finding the gain:



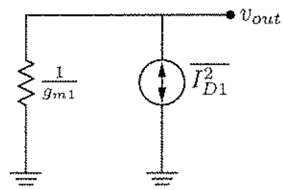
$$g_{m1}v_{gs1} = -g_{m2}v_{gs2}$$
$$g_{m1}(v_{in} - v_{out}) = g_{m2}v_{out}$$
$$v_{out}(g_{m1} + g_{m2}) = g_{m1}v_{in}$$
$$A_v = \frac{v_{out}}{v_{in}} = \frac{g_{m1}}{g_{m1} + g_{m2}}$$

Thus, the input-referred thermal noise is:

$$\overline{V^2_{n,in}} = \frac{\overline{V^2_{n,out}}}{A_v^2}$$
$$= \boxed{4kT\frac{2}{3}\frac{1}{g_{m1}^2}(g_{m1} + g_{m2})}$$

✓ Correct? 3/3

(f) Note that when we short the input to ground, M_2's drain and source are both shorted to ground, eliminating it (and its noise) from the circuit. Also observe that shorting the input to ground causes M_1 to become diode-connected in the small-signal model, reducing it to an equivalent resistance of $1/g_{m1}$. Thus, the small-signal model for finding the noise is:



$$\overline{V_{n,out}^2} = \overline{I_{D1}^2}\frac{1}{g_{m1}^2}$$

By inspection, we can see that the gain of the circuit is 1, since I_1 fixes v_{gs1} since it is an ideal current source. Thus, we have:

$$\begin{aligned} A_v &= 1 \\ \overline{V_{n,in}^2} &= \frac{\overline{V_{n,out}^2}}{A_v^2} \\ &= 4kT\frac{2}{3}g_{m1}\frac{1}{g_{m1}^2} \\ &= \boxed{4kT\frac{2}{3}\frac{1}{g_{m1}}} \end{aligned}$$

Current? 3/5

4. (a) In equilibrium, M_1, M_2, and M_3 will each carry half of I_{SS} (neglecting channel length modulation). Using a series of I_D vs. V_{DS} plots, we can find the widths to be:

$$W_1 = W_2 = W_3 = \boxed{8.5675\ \mu\text{m}}$$ ✓

(b)

$$V_{b2,max} = V_{DD} - |V_{ov5}| - (|V_{ov4}| + |V_{TH4}|)$$
$$V_{b1,min} = V_{in,CM} - V_{TH2} + V_{ov3} + V_{TH3} = V_{in,CM} + V_{ov3}$$
$$|V_{ov4}| = |V_{ov5}| = \sqrt{\frac{I_{SS}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_p}} = 102\ \text{mV}$$
$$V_{THn} = 0.53\ \text{V}$$
$$V_{THp} = -0.50\ \text{V}$$
$$V_{b2,max} = \boxed{1.10\ \text{V}}$$
$$V_{b1,min} = \boxed{1.05\ \text{V}}$$

The output voltage swing can be found as follows:

$$V_{b1,min} - V_{TH3} < V_{out} < V_{b2,max} + |V_{TH4}|$$
$$\boxed{0.52\ \text{V} < V_{out} < 1.6\ \text{V}}$$ ✓

(c) We can use SPICE to model the capacitance and resistance seen at the mirror node (the drain of M_1) to estimate the mirror pole. Note that in order to keep all devices saturated, I used $V_{b1} = V_{b2} = 1.1$ V in the simulation.

$$R_{mirror} = 420.72\ \Omega$$
$$C_{mirror} = 812.61\ \text{pF}$$
$$\omega_{p,mirror} = 2.925 \times 10^6\ \text{rad/s}$$
$$f_{p,mirror} = \frac{\omega_{p,mirror}}{2\pi} = \boxed{465.5\ \text{kHz}}$$ ✗ 3/5

(d) To approximate the gain, we can use the equation from the textbook as follows:

$$A_v \approx g_{m2}\left\{[r_{o3} + (1 + (g_{m3} + g_{mb3}) r_{o3}) r_{o2}] \parallel [r_{o5} + (1 + (g_{m4} + g_{mb4}) r_{o4}) r_{o5}]\right\}$$

We can obtain the transconductances and output resistances of each transistor from SPICE. Doing so, we have:

$$
\begin{aligned}
g_{m2} &= 1.5273 \text{ mS} \\
g_{m3} &= 1.6336 \text{ mS} \\
g_{m4} &= 1.5988 \text{ mS} \\
g_{m5} &= 1.3872 \text{ mS} \\
g_{mb3} &= 0.3181730 \text{ mS} \\
g_{mb4} &= 0.4650890 \text{ mS} \\
\lambda_2 &\approx 0.994 \text{ V}^{-1} \\
\lambda_3 &\approx 0.497 \text{ V}^{-1} \\
\lambda_4 &\approx 0.696 \text{ V}^{-1} \\
\lambda_5 &\approx 4.076 \text{ V}^{-1} \\
r_{o2} &= 10.00 \text{ k}\Omega \\
r_{o3} &= 20.00 \text{ k}\Omega \\
r_{o4} &= 14.28 \text{ k}\Omega \\
r_{o5} &= 2.44 \text{ k}\Omega \\
A_v &= \boxed{99.2}
\end{aligned}
$$

To find the unity-gain bandwidth, let's first find the dominant pole (at the output). We know R_{out} from the above calculation ($R_{out} = \frac{A_v}{-g_{m2}}$), and we can find the total output capacitance with SPICE:

$$
\begin{aligned}
R_{out} &= 66.1 \text{ k}\Omega \\
C_{out} &= 827.619 \text{ pF} \\
\omega_{p,out} &= 18.3 \times 10^3 \text{ rad/s} \\
f_{p,out} &= \boxed{2.91 \text{ kHz}}
\end{aligned}
$$

Assuming that the other poles fall beyond the unity-gain bandwidth and that the gain will fall by 20 dB/dec after the dominant pole, we can perform a simple calculation to find the unity-gain bandwidth:

$$
\begin{aligned}
20 \log |A_v| - 20x &= 0 \\
x &= 2
\end{aligned}
$$

This means that the unity-gain bandwidth will occur 2 decades past the dominant pole, or at $f = \boxed{291 \text{ kHz}}$.

Here is the SPICE netlist for simulating the gain and unity-gain bandwidth:

```
* HW4

.inc '215a.sp'
.param WN=8.5675u
.param WP=40u
.param LMIN=0.18u
.param ASN=WN*0.6u
.param ADN=ASN
.param ASP=WP*0.6u
.param ADP=ASP
.param PSN=2*WN+1.2u
.param PDN=PSN
```

```
.param PSP=2*WP+1.2u
.param PDP=PSP

ISS vs1 gnd 250uA

M1 vd1 vinp vs1 gnd CMOSN W=WN L=LMIN AS=ASN AD=ADN PS=PSN PD=PDN
M2 vd2 vinn vs1 gnd CMOSN W=WN L=LMIN AS=ASN AD=ADN PS=PSN PD=PDN
M3 vout vb1 vd2 gnd CMOSN W=WN L=LMIN AS=ASN AD=ADN PS=PSN PD=PDN
M4 vout vb2 vd5 vdd CMOSP W=WP L=LMIN AS=ASP AD=ADP PS=PSP PD=PDP
M5 vd5 vd1 vdd vdd CMOSP W=WP L=LMIN AS=ASP AD=ADP PS=PSP PD=PDP
M6 vd1 vd1 vdd vdd CMOSP W=WP L=LMIN AS=ASP AD=ADP PS=PSP PD=PDP
CL vout gnd 0.8pF

vcm vcm gnd 0.9V
vin vin gnd AC 1
einp vinp vcm vin gnd 0.5
einn vinn vcm vin gnd -0.5

vdd vdd gnd 1.8V
vb1 vb1 gnd 1.1V
vb2 vb2 gnd 1.1V

.op
.ac dec 500 10 1T
.tf v(vout) vin
.measure ac funity when vdb(vout)=0
.option post=2 nomod
.end
```

This netlist gives the following results for the gain and unity-gain bandwidth:

$$A_v = 100.1475 \quad \checkmark$$
$$f_{unity} = 203.43 \text{ kHz} \quad \checkmark$$

We can see that the gain is very close to the value we calculated, which makes sense since we extracted the device parameters accurately with SPICE.

The unity-gain bandwidth is also similar to the value calculated by hand. It is slightly off, however, since the dominant pole and gain we calculated are both a bit off (according to a SPICE pole-zero analysis, the dominant pole is at 2.3680 kHz).

5/5