EE 215C Winter 2009
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58/60

Homework 1
January 11, 2009

1.

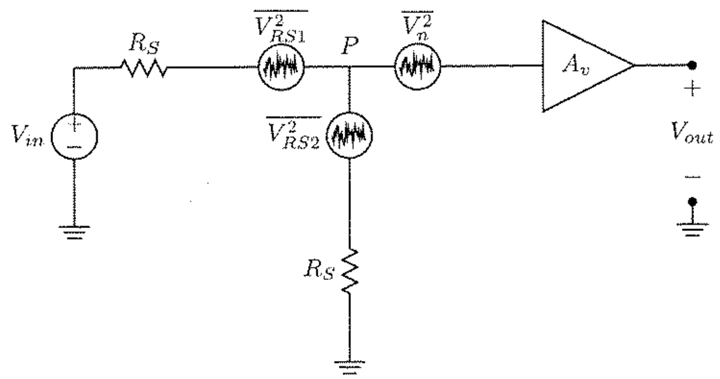

Figure 1: LNA driven by a source with impedance R_S and impedance matched with a resistor to ground

(a) Since the LNA has a high input impedance, we can model the input-referred noise with just a voltage source $\overline{V_n^2}$, as shown in Fig. 1. From Eq. (2.81), we know that NF_1 (the noise figure of the LNA itself with respect to R_S) is:

$$NF_1 = 1 + \frac{\overline{V_n^2}}{4kTR_S}$$

From Eq. (2.84), we can find the noise figure of the overall circuit by finding $V_{n,out}^2$. First, note that $\overline{V_{RS1}^2}$ and $\overline{V_{RS2}^2}$ see the same voltage gain $\alpha = 1/2$ between the input port and node P.

$$\begin{aligned}
V_{n,out}^2 &= \alpha^2 A_v^2 \overline{V_{RS1}^2} + \alpha^2 A_v^2 \overline{V_{RS2}^2} + A_v^2 \overline{V_n^2} \\
&= \alpha^2 A_v^2 4kTR_S + \alpha^2 A_v^2 4kTR_S + A_v^2 \overline{V_n^2} \\
&= A_v^2 \left(\alpha^2 8kTR_S + \overline{V_n^2}\right) \\
NF &= \frac{V_{n,out}^2}{A^2} \frac{1}{4kTR_S} \text{ (Eq. 2.84)} \\
&= \frac{A_v^2 \left(\alpha^2 8kTR_S + \overline{V_n^2}\right)}{\alpha^2 A_v^2} \frac{1}{4kTR_S} \\
&= 2 + \frac{V_n^2}{kTR_S} \\
&= \boxed{4NF_1 - 2}
\end{aligned}$$

✓ (5)

(b) Let $NF_0 = 2$ be the noise figure of the resistor to ground at node P (in general, this is $1 + R_S/R_P$ where R_P is the resistor value, but since $R_S = R_P$, this reduces to 2).
We can see that we'll need to reference the noise figure of the LNA to the output impedance of the previous stage (let's call this result NF_1'). We can see that the impedance driving this stage is $R_S/2$, so we'll reference NF_1' to a source impedance of $R_S/2$.
Finally, we need to find the available power gain of R_P. From Eq. (2.104), we have:

$$A_P = \left(\frac{R_{in}}{R_S + R_{in}}\right)^2 A_v^2 \frac{R_S}{R_{out}}$$

We can see that $R_{in} = R_S$, $A_v = 1$, and $R_{out} = R_S/2$. Thus, we get $A_P = 1/2$.

$$\begin{aligned} NF &= NF_0 + \frac{NF_1' - 1}{A_P} \\ NF_0 &= 2 \\ NF_1' &= 1 + \frac{\overline{V_n^2}}{2kTR_S} = 2NF_1 - 1 \\ A_P &= 1/2 \\ NF &= 2 + \frac{2NF_1 - 1 - 1}{1/2} \\ &= \boxed{4NF_1 - 2} \end{aligned}$$

✓ (5)

This matches the noise figure calculated in part (a).

2. The SPICE netlist is listed at the end of this problem.

(5) (a) From the SPICE model, we have:

$$\begin{aligned} \mu_n &= 310.235\ \text{cm}^2/\text{V}\cdot\text{s} \\ t_{ox} &= 4.1\ \text{nm} \\ C_{ox} &= 0.8418\ \mu\text{F/cm}^2 \\ \mu_n C_{ox} &= 261.2\ \mu\text{A/V}^2 \end{aligned}$$

From the current mirror that biases M_1, we see that:

$$I_{D1} = I_{D3} = 5I_{D4} = 2.5\ \text{mA}$$

Since the SPICE model doesn't provide ϕ_F or γ, we'll have to assume a reasonable value for η for this calculation, then adjust the width in SPICE until $g_{m1} + g_{mb1} = (50\ \Omega)^{-1}$. Let's assume $\eta = 0.1$.

$$\begin{aligned} g_{m1}(1+\eta) = (50\ \Omega)^{-1} &= \sqrt{2\frac{W_1}{L_1}\mu_n C_{ox} I_{D1}}\,(1+\eta) \\ W_1 &= \boxed{45.56\ \mu\text{m}} \end{aligned}$$

✓

Using $W_1 = 45.56\ \mu$m in the SPICE simulation gives

$$\begin{aligned} g_{m1} &= 16.7837\ \text{mS} \\ g_{mb1} &= 3.0493\ \text{mS} \end{aligned}$$

Thus, we have $g_{m1} + g_{mb1} = (50.42\ \Omega)^{-1}$, which matches well with the calculations.

(b) We want the inductor to resonate with the capacitance at the output at 5.2 GHz. SPICE reports the total capacitance at the output due to transistors M_1 and M_2, and we can calculate the

(5)

parasitic capacitance due to L_1 based on the inductor model provided.

$$
\begin{aligned}
C_{D1,tot} &= 71.8427 \text{ fF} \\
C_{G2,tot} &= 36.8818 \text{ fF} \\
C_P &= L_1 \times 10^9 \times 10^{-14} = 10^{-5} L_1 \\
f = 5.2 \text{ GHz} &= \frac{1}{2\pi\sqrt{L_1\left(C_{D1,tot} + C_{G2,tot} + C_P\right)}} \\
&= \frac{1}{2\pi\sqrt{L_1\left(C_{D1} + C_{G2} + 10^{-5}L_1\right)}} \\
L_1 &= \boxed{5.665 \text{ nH}} \\
C_P &= 56.65 \text{ fF} \\
Q &= \frac{R_P}{\omega L_1} \\
&= \frac{R_P}{2\pi f L_1} \\
R_P &= 2\pi f L_1 Q \\
&= 740.36\ \Omega
\end{aligned}
$$

Using these values in the SPICE simulation and performing an AC analysis produces the gain vs. frequency plot shown in Fig. 2. The plot shows a peak right around $f = 5.2$ GHz.

(c) L_1 will present a very different impedance at 5.2 GHz than it does at DC, meaning that looking only at $(g_m + g_{mb})^{-1}$ as the input resistance neglects the inductor's impedance at 5.2 GHz. Using SPICE, we find that $W_1 = 71$ µm gives $R_{in} = 49.80\ \Omega$ at 5.2 GHz. However, since we changed W_1, $C_{D1,tot}$ will also change. This causes the resonance frequency to shift as well, meaning we need to recalculate L_1 (causing R_{in} to change, and so on). Iterating over these calculations a few times produces:

$$
\begin{aligned}
W_1 &= \boxed{83.0\ \mu\text{m}} \\
L_1 &= 4.42 \text{ nH} \\
C_P &= 44.2 \text{ fF} \\
R_P &= 577.65\ \Omega \\
R_{in} &= 49.75\ \Omega
\end{aligned}
$$

The new AC response is shown in Fig. 3.

(d) Performing an FFT analysis in SPICE (using a 8192-point FFT and a Blackman window for weighting) gives the output spectrum shown in Fig. 4. The two primary tones are located at 5.2 GHz and 5.3 GHz, and the IM_3 products are located at 5.1 GHz and 5.4 GHz. The amplitude of the input tones was chosen to be 10 mV to ensure we do not experience any gain compression. From this plot, we can find IIP_3 as follows:

$$
\begin{aligned}
IIP_3|_{\text{dBm}} &= \frac{\Delta P|_{\text{dB}}}{2} + P_{in}|_{\text{dBm}} \\
P_{in} &= \frac{V_{in,rms}^2}{4R_S} = \frac{V_{in}^2}{8R_S} \\
V_{in} &= 10 \text{ mV} \\
P_{in} &= 0.25\ \mu\text{W} = -36.0206 \text{ dBm} \\
\Delta P &= 77.0807 \text{ dB} \\
IIP_3 &= \boxed{2.51975 \text{ dBm}}
\end{aligned}
$$

We can read the gain off of the AC response in Fig. 3.

$$A_v = 5.4785 = \boxed{14.77\ \text{dB}}$$

(e) First, we need to calculate L_2 for resonance at 5.2 GHz.

$$\begin{aligned} C_{D2,tot} &= 23.8292\ \text{fF} \\ f = 5.2\ \text{GHz} &= \frac{1}{2\pi\sqrt{L_2\left(C_{D2,tot} + 60\ \text{fF} + 10^{-5}L_2\right)}} \\ L_2 &= 6.3558\ \text{nH} \end{aligned}$$

Plotting the AC response of the circuit in SPICE shows this value of L_2 to be a little too large (i.e., the resonance occurs slightly below 5.2 GHz). Using $L_2 = 6.3$ nH puts the resonance at 5.2 GHz. Fig. 5 shows the AC response of the circuit, from which we can obtain the gain.

$$A_v = 4.1442 = \boxed{12.35\ \text{dB}}$$

Fig. 6 shows the output spectrum for a two-tone test, which allows us to calculate IIP_3.

$$\begin{aligned} IIP3|_{\text{dBm}} &= \frac{\Delta P|_{\text{dB}}}{2} + P_{in}|_{\text{dBm}} \\ P_{in} &= \frac{V_{in,rms}^2}{4R_S} = \frac{V_{in}^2}{8R_S} \\ V_{in} &= 10\ \text{mV} \\ P_{in} &= 0.25\ \mu\text{W} = -36.0206\ \text{dBm} \\ \Delta P &= 81.5554\ \text{dB} \\ IIP_3 &= \boxed{4.7571\ \text{dBm}} \end{aligned}$$

(f) When we cascade the two stages, the capacitance at the gate of M_2 will change (it will increase due to the Miller effect acting on C_{GD2}), meaning the resonance frequency of the first stage will change. We must adjust L_1 to compensate for this increased capacitance in order to retain a resonance frequency of 5.2 GHz. Using $L_1 = 3.3$ nH maintains the appropriate resonance frequency when the stages are cascaded. Fig. 7 shows the AC response of the overall LNA.

The overall voltage gain is $\boxed{16.423}$. This is substantially less than the product of the voltage gains measured in parts (d) and (e). However, this is expected, since cascading the stages will decrease the gain of the first stage due to loading (and due to the smaller value of L_1, which forces a smaller R_P in order to maintain the same Q). The loaded gain of the first stage is 4.2973, which gives an overall gain of 17.81 when multiplied by the gain of the second stage, giving relatively good agreement with the measured overall gain.

The overall IIP_3 can be found from the spectrum of the output (shown in Fig. 8) as in previous parts of the problem. Doing so gives $IIP_3 = \boxed{-7.626\ \text{dBm}}$. We can calculate an expected IIP_3 based on the result from (d) and (e) as follows (let subscript A indicate the first stage and subscript B indicate the second stage):

$$\begin{aligned} IIP_{3,A} &= 2.51975\ \text{dBm} = 0.845313\ \text{V} \\ IIP_{3,B} &= 4.7571\ \text{dBm} = 1.09367\ \text{V} \\ A_{v,A} &= 4.2973 \\ \frac{1}{IIP_3^2} &= \frac{1}{IIP_{3,A}^2} + \frac{A_{v,A}^2}{IIP_{3,B}^2} \\ IIP_3 &= 0.244\ \text{V} = -8.28\ \text{dBm} \end{aligned}$$

This agrees relatively well with the value measured directly from the LNA.

(g) When we cascade the two stages, the input resistance drops to $R_{in} = 43.67\,\Omega$. This is due to the increased capacitance at the output of the first stage and the decreased value of L_1 (which decreases R_P). The impedance at the drain of M_1 is lowered due to these effects, causing the input resistance to decrease.

(h) The second stage limits the IIP_3 of the LNA. If we look at the two terms in the equation for the IIP_3 of a cascade, we can see that in this case:

$$\frac{1}{IIP_{3,A}^2} = 1.39947$$
$$\frac{A_{v,A}^2}{IIP_{3,B}^2} = 15.439$$

Clearly, the second term dominates. Qualitatively, we can see that the IIP_3 of each individual stage is similar, but the moderate gain of the first stage causes the IIP_3 of the second stage to dominate.

Here is the SPICE netlist used in this problem (the first and second stage were simulated independently before being combined in this netlist):

```
* EE215C HW1 Problem 2

.inc '215a.sp'
.option post accurate nomod

.param W1=83e-6
.param L1=3.3e-9
.param CP='L1*1e-5'
.param Q=4
.param PI=3.14159265358979
.param FREQ=5.2e9
.param RP='2*PI*FREQ*L1*Q'
.param L2=6.3e-9
.param CP2='L2*1e-5'
.param RP2='2*PI*L2*FREQ*Q'

vdd   vdd   gnd   1.8

* Used for AC analysis
** vin   vin   gnd   AC   1

* Used for FFT
vin1  vin1  gnd   sin(0 10m 5.2G 0 0 0)
vin2  vin   vin1  sin(0 10m 5.3G 0 0 0)

* Comment out r1 when finding R_in
r1    vin   1     50
c1    1     vd3   1uF

m3    vd3   vd4   gnd   gnd   CMOSN W=25u L=0.25u
+PS='2*25e-6+1.2e-6' PD='2*25e-6+1.2e-6' AS='25e-6*0.6e-6' AD='25e-6*0.6e-6'
m4    vd4   vd4   gnd   gnd   CMOSN W=5u  L=0.25u
+PS='2*5e-6+1.2e-6' PD='2*5e-6+1.2e-6' AS='5e-6*0.6e-6' AD='5e-6*0.6e-6'
c2    vd4   gnd   0.2p
i1    vdd   vd4   0.5m
```

```
m1    vout1 vdd   vd3   gnd   CMOSN W=W1 L=0.18u
+PS='2*W1+1.2e-6' PD='2*W1+1.2e-6' AS='W1*0.6e-6' AD='W1*0.6e-6'
l1    vdd   vout1 L1
rp    vdd   vout1 RP
cp1   vdd   gnd   CP
cp2   vout1 gnd   CP

m2   vout   vout1 vs2   gnd   CMOSN W=15u L=0.18u
+PS='2*15e-6+1.2e-6' PD='2*15e-6+1.2e-6' AS='15e-6*0.6e-6' AD='15e-6*0.6e-6'
i2   vs2    gnd   2m
c3   vs2    gnd   1p
l2   vdd    vout  L2
c4   vout   gnd   60f
cp21 vout   gnd   CP2
cp22 vdd    gnd   CP2
rp2  vdd    vout  RP2

* Used when finding resonance frequency, gain, and R_in
.ac lin 1000 1G 10G

* Used to measure gain
** .measure ac gain find vm(vout) at = 5.2e9

* Used to measure R_in
** .measure ac itest find ir(vin) at = 5.2e9
** .measure ac rin param = '-1/itest'

* Used to plot the spectrum of the output to find IIP3
* Use a 8192-point FFT with a Blackman window
.tran 0.1p 100n
.fft v(vout) fmin=4.75G fmax=5.75G np=8192 format=unorm window=black

.end
```

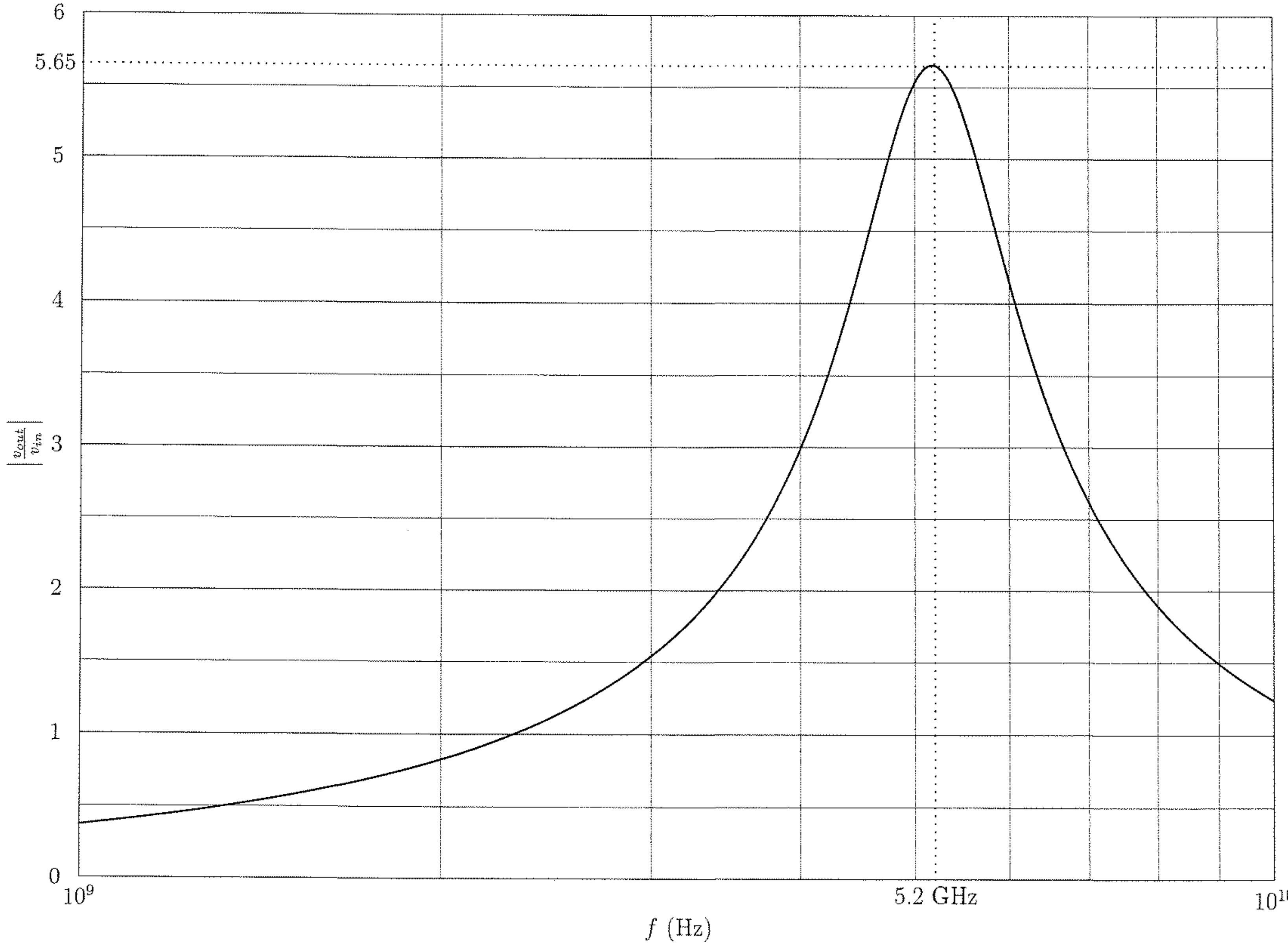

Figure 2: AC response of the LNA showing resonance at 5.2 GHz

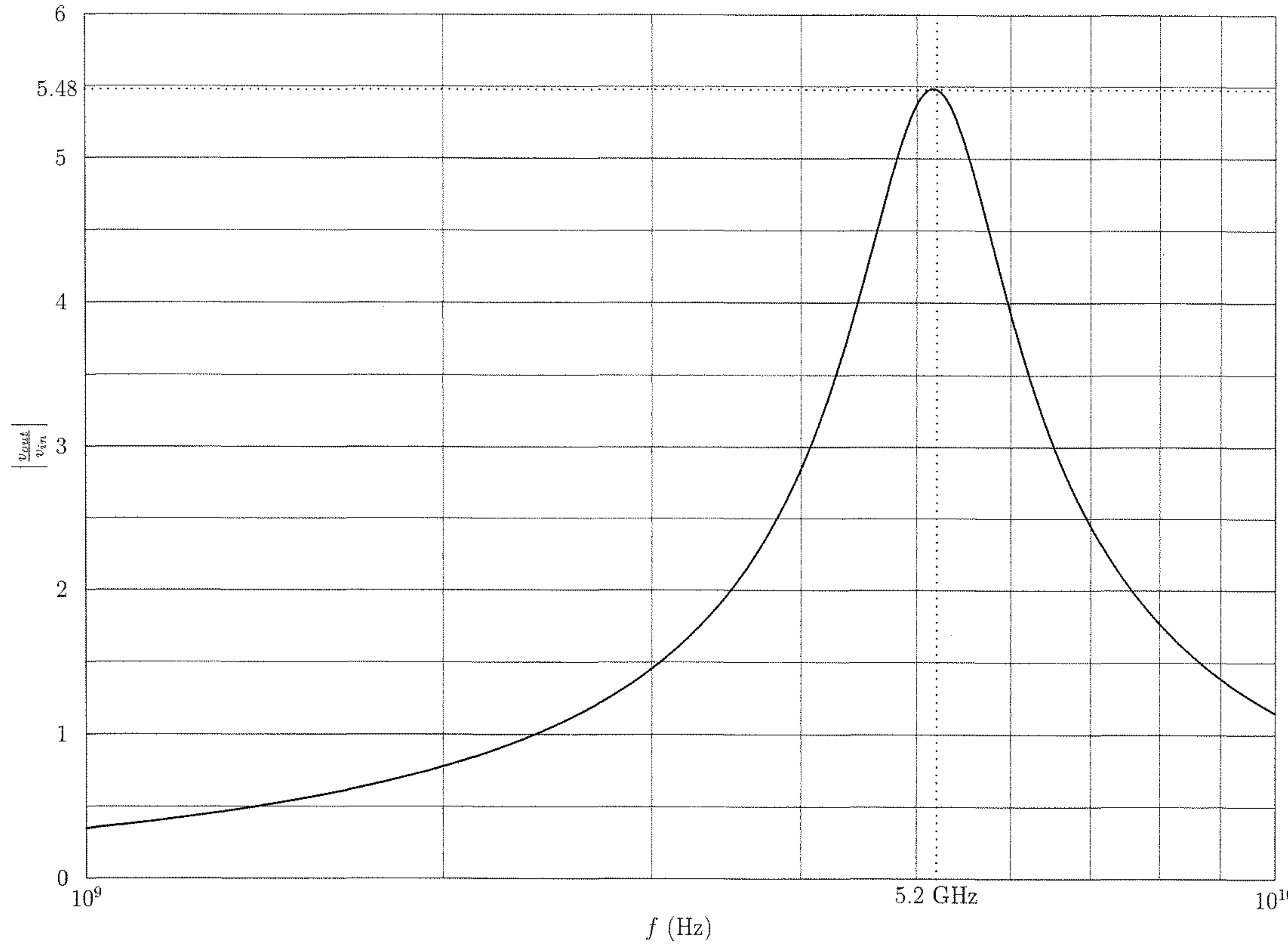

Figure 3: AC response of the LNA showing resonance at 5.2 GHz with an impedance match at the input

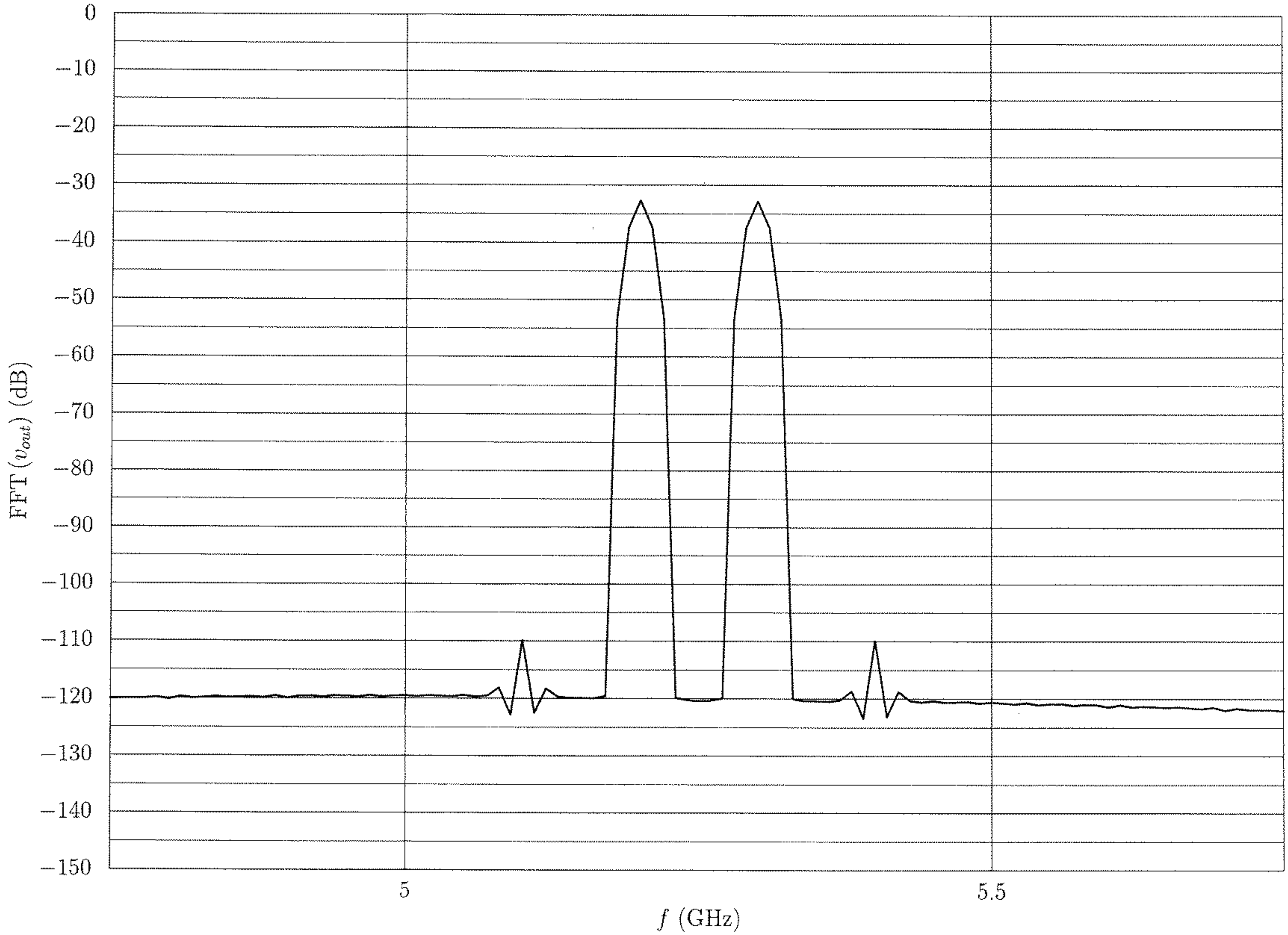

Figure 4: Output spectrum of the LNA with a two-tone input

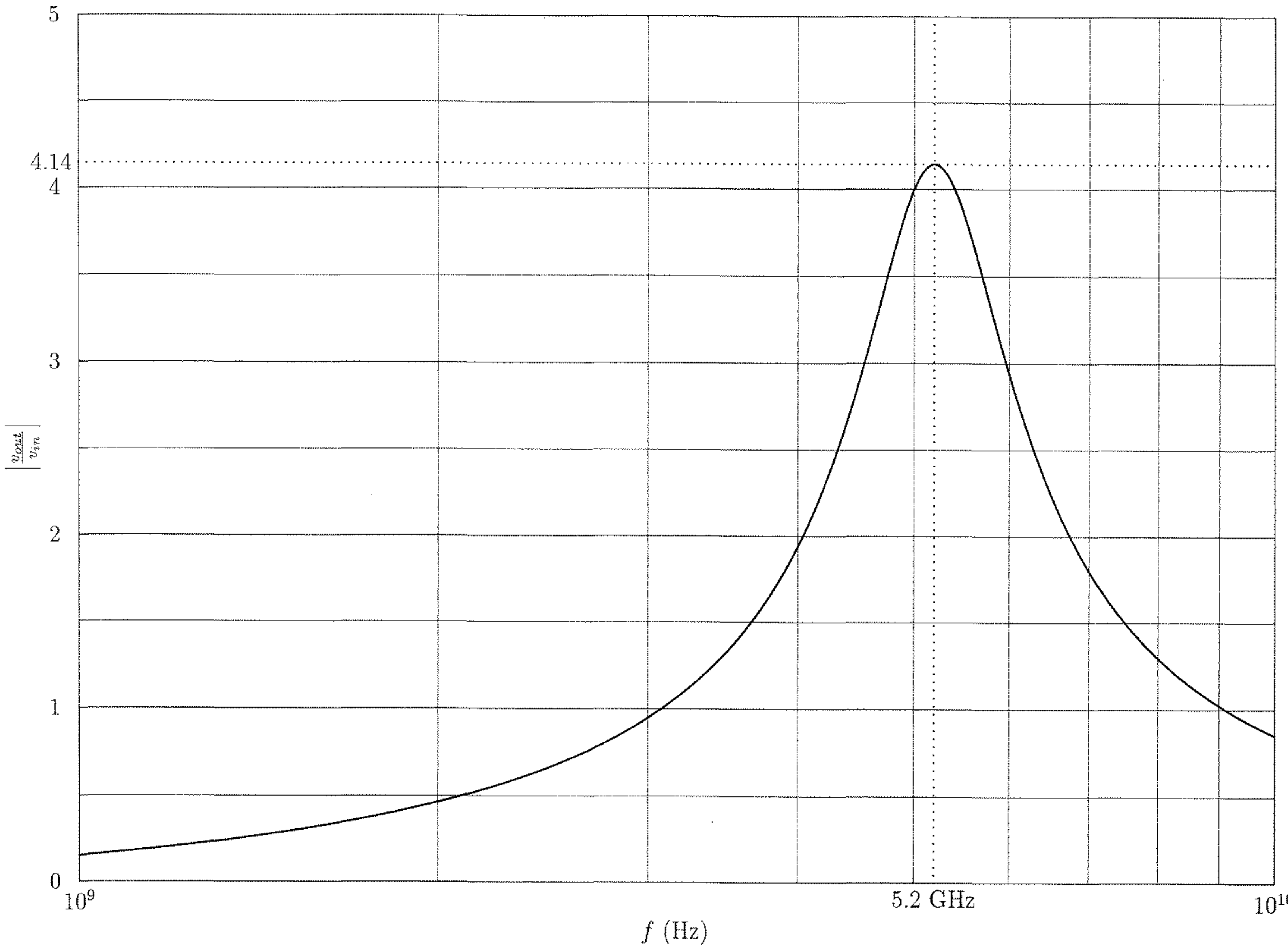

Figure 5: AC response of the second stage of the LNA showing resonance at 5.2 GHz

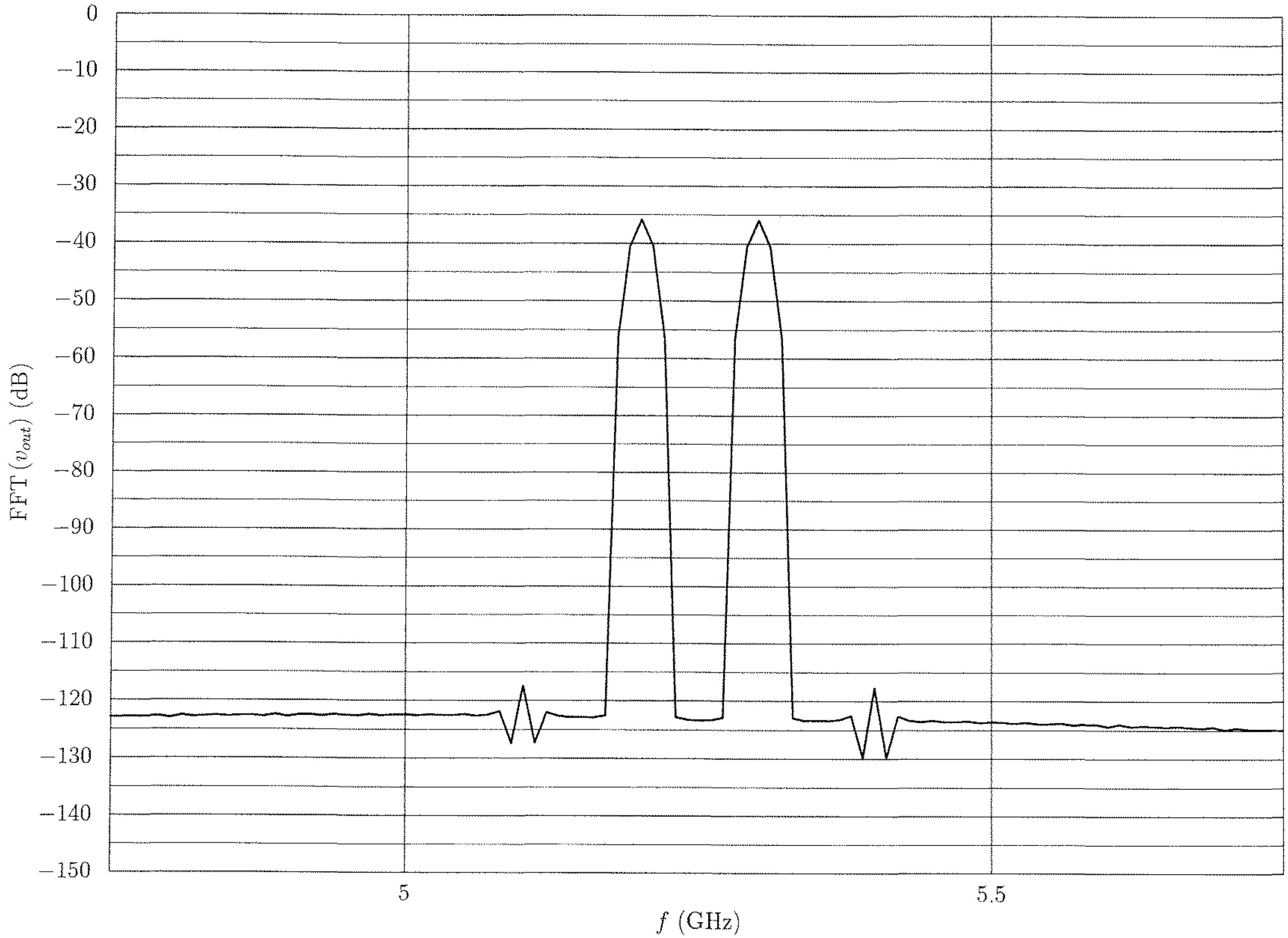

Figure 6: Output spectrum of the second stage of the LNA with a two-tone input

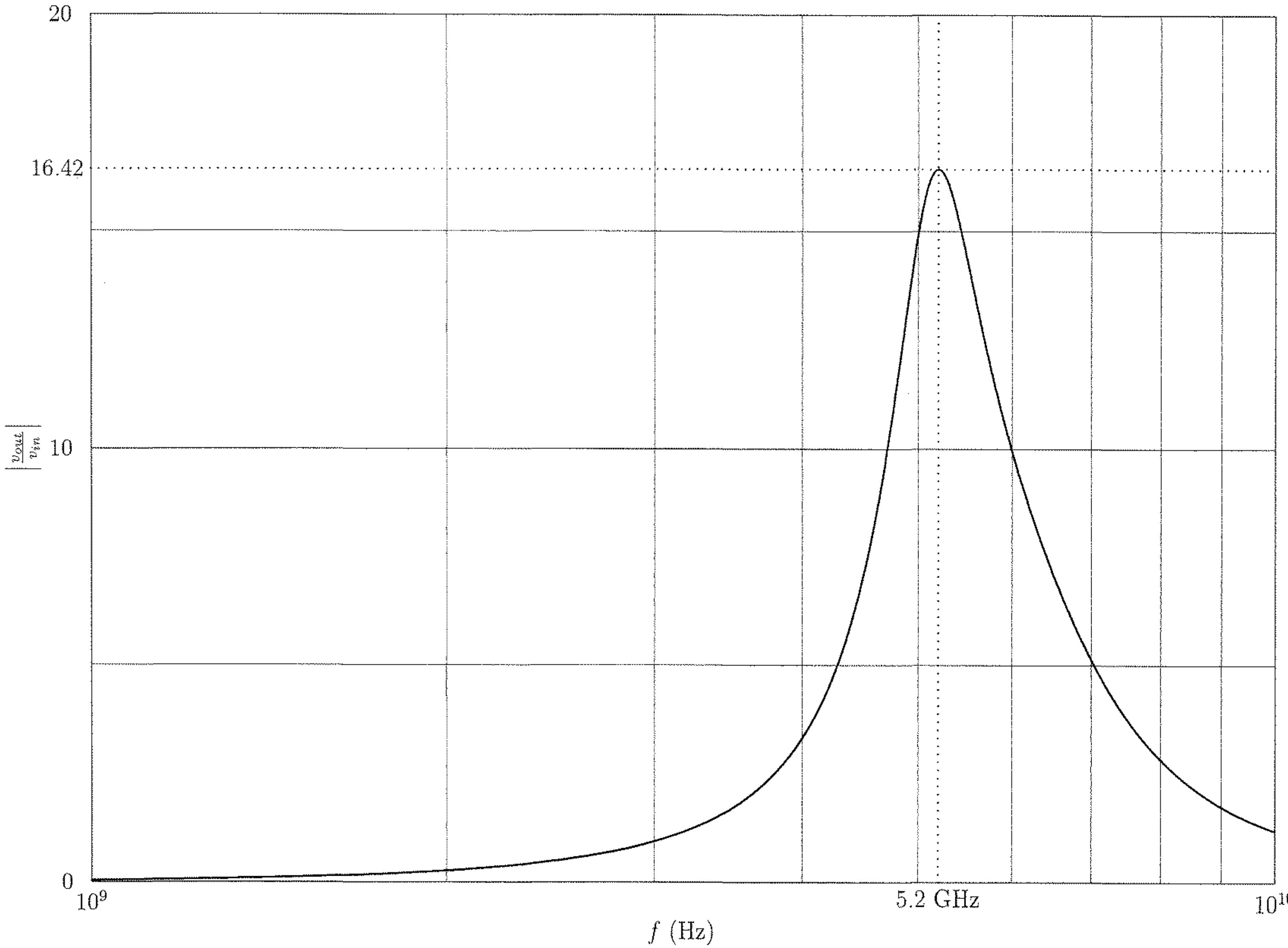

Figure 7: AC response of the overall LNA

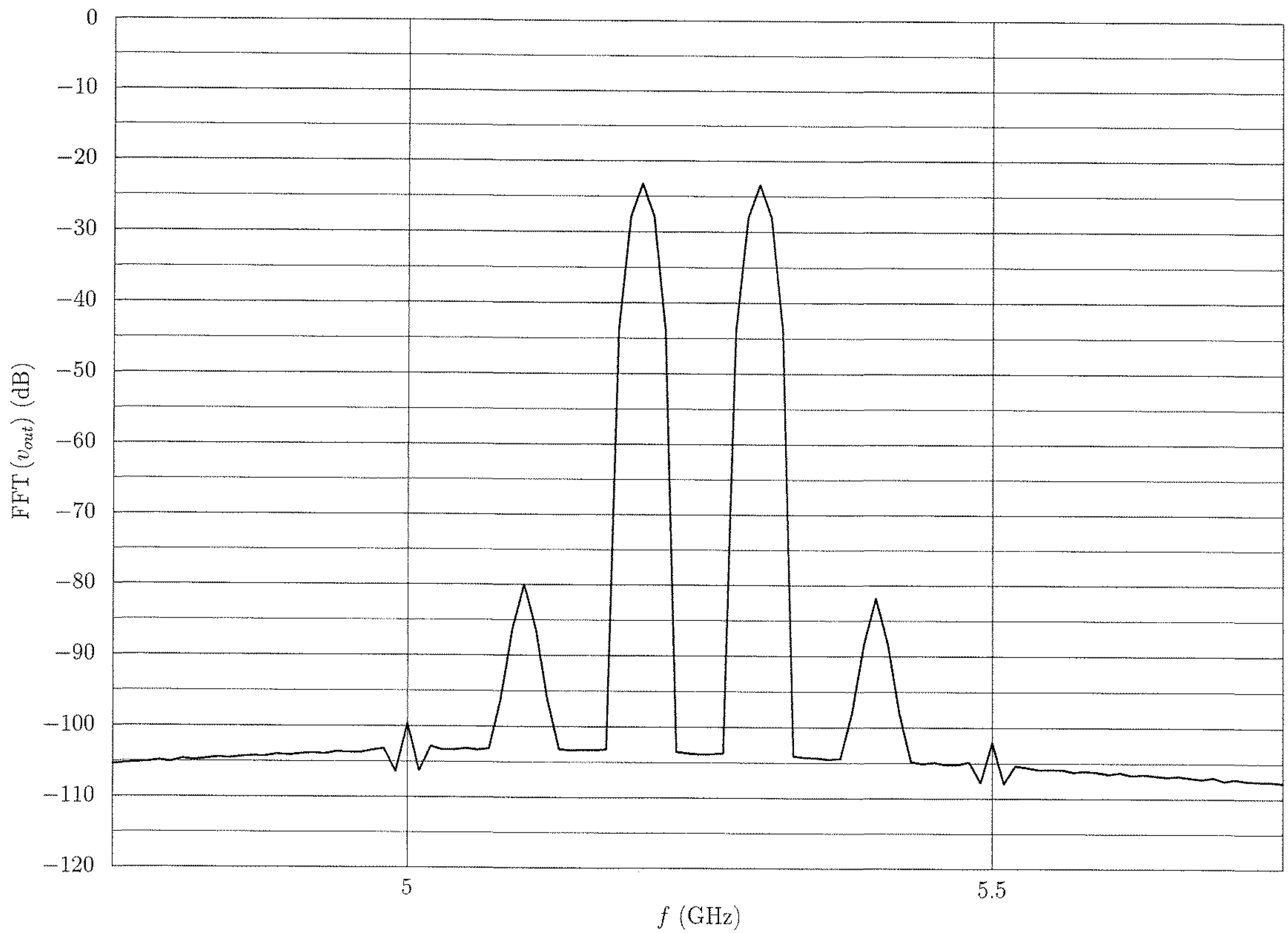

Figure 8: Output spectrum of the overall LNA with a two-tone input