215 A Fall 2011 UID

HW#3

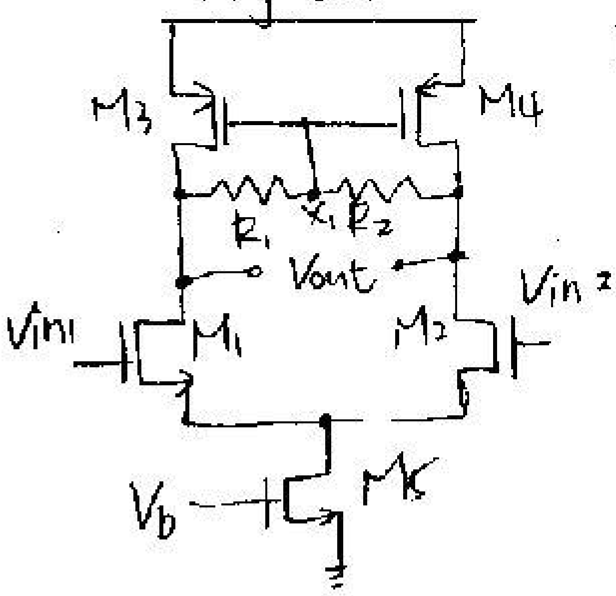
Chen, Hsinhung

40/40 + 40/40 = 80/80

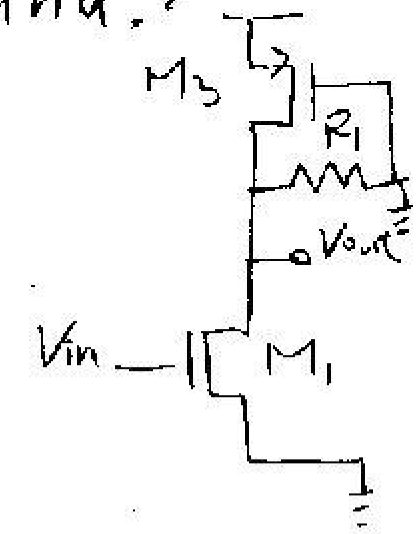
P1

4.18 (d)

Fig 4.38



circuit is fully differentian so node X_1 can be treated as virtual ground. The half circuit is shown right.



$$\frac{V_{out}}{V_{in}} = g_{m1} \times (r_{o1} \parallel r_{o3} \parallel R_1)$$

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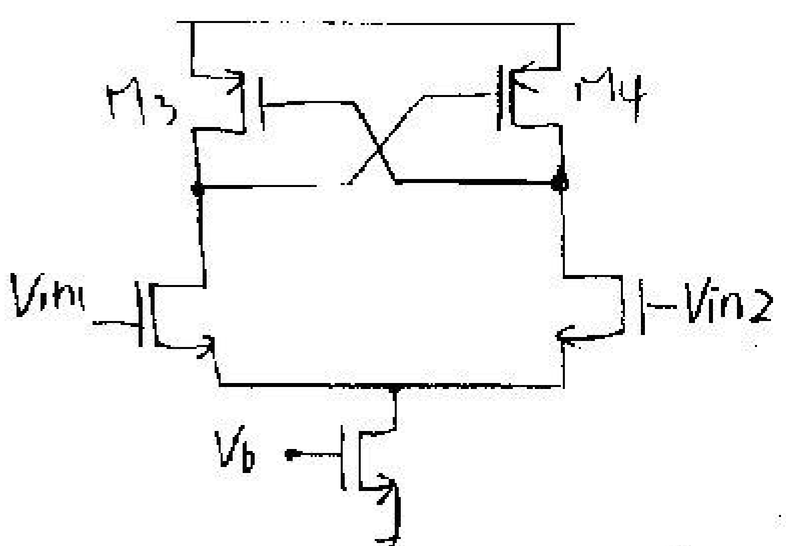
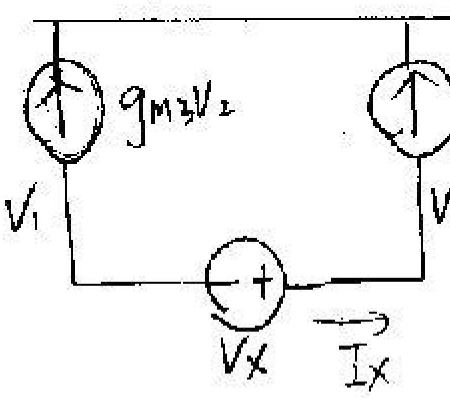
Fig 4.39 (c)



M3, M4 is a cross-coupled load, and can be analyzed first with λ neglected first.

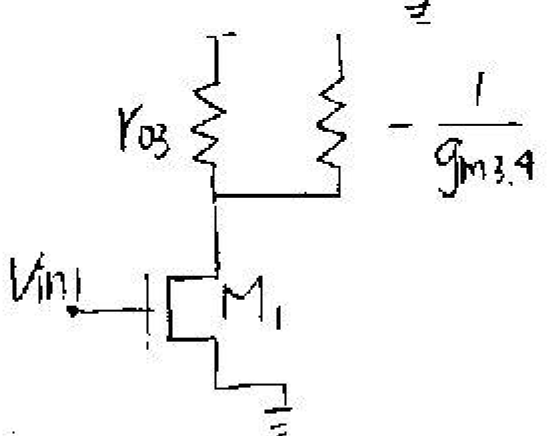


$$I_X = g_{m4} V_1$$

$$-I_X = g_{m3} V_2$$

assume $g_{m3} = g_{m4} = g_{m3,4}$

$$\frac{V_X}{I_X} = \frac{V_2 - V_1}{I_X} = \frac{-\frac{I_X}{g_{m3}} - \frac{I_X}{g_{m4}}}{I_X} = -\frac{2}{g_{m3,4}}$$



Then we can separate the cross-coupled load into half circuit. The overall half circuit is shown left.

$$A_V = \frac{V_{out}}{V_{in}} = g_{m1} \times \left[r_{o1} \parallel r_{o3} \parallel \left(-\frac{1}{g_{m3,4}}\right)\right]$$

$$= -g_{m1} \times \frac{1}{r_{o1}^{-1} + r_{o3}^{-1} - g_{m3,4}}$$

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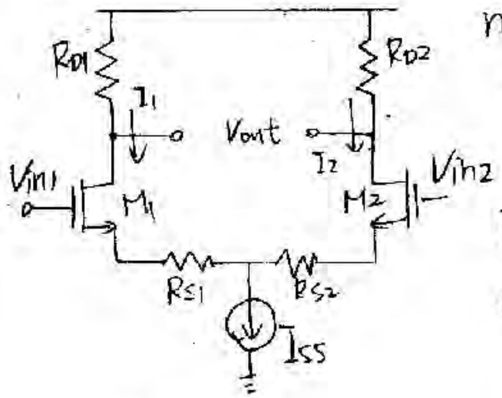
Here the cross-coupled structure induce a negative resistance, $-\frac{1}{g_{m3,4}}$ which must be smaller than $\frac{1}{r_{o1}} + \frac{1}{r_{o3}}$ in order not to induce a positive feedback and instability.

4.21

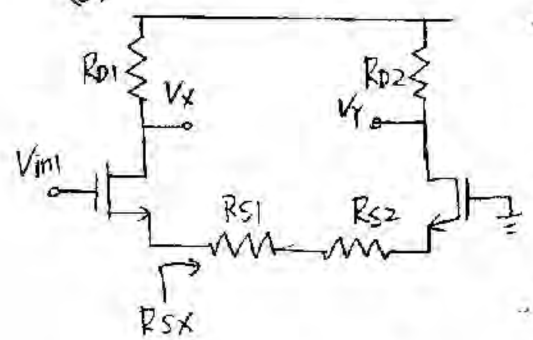


If no symmetry is assumed, half circuit is not valid, therefore we can use superposition to calculate the impact of input on each output.

First calculate V_{in1} and let $V_{in2}=0$, the circuit can be expressed as figure (a).

$$R_{SX} = R_{S1} + R_{S2} + \frac{1}{g_{m2}+g_{mb2}}$$

(a)



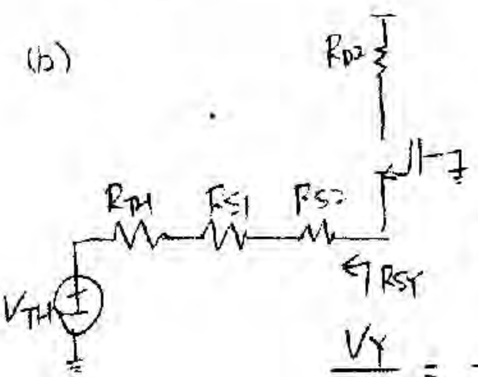
Therefore R_{D1}, M_1, R_{SX} form a CS with source degeneration.

$$\frac{V_X}{V_{in1}} = \frac{g_{m1} \cdot R_{D1}}{1+(g_{m1}+g_{mb1})R_{SX}}$$

To calculate the contribution of V_{in1} to V_Y, the circuit can be re-drawn through Thevenin equivalen circuit as figure (b), where

$$V_{TH} = V_{in1}, \quad R_{TH} = \frac{1}{g_{m1}+g_{mb1}}.$$

(b)



The circuit is the same as CG amplifier

$$\frac{V_Y}{V_{in1}} = \frac{R_{D2}}{R_{SY}+\frac{1}{g_{m2}+g_{mb2}}} = \frac{(g_{m2}+g_{mb2})R_{D2}}{1+(g_{m2}+g_{mb2})R_{SY}}, \quad R_{SY} = \frac{1}{g_{m1}+g_{mb1}} + R_{S1}+R_{S2}$$

$$\therefore (V_X - V_Y)\Big|_{\text{due to } V_{in1}} = \left[\frac{g_{m1} R_{D1}}{1+(g_{m1}+g_{mb1})R_{SX}} - \frac{(g_{m2}+g_{mb2})R_{D2}}{1+(g_{m2}+g_{mb2})R_{SY}}\right] \cdot V_{in1} \quad -(1)$$

Similarly,

$$(V_X - V_Y)\Big|_{\text{due to } V_{in2}} = \left[-\frac{g_{m2} R_{D2}}{1+(g_{m2}+g_{mb2})R_{SY}} + \frac{(g_{m1}+g_{mb1})R_{D1}}{1+(g_{m1}+g_{mb1})R_{SX}}\right] \cdot V_{in2} \quad -(2)$$

$$V_{out} = V_X - V_Y = (1) + (2)$$

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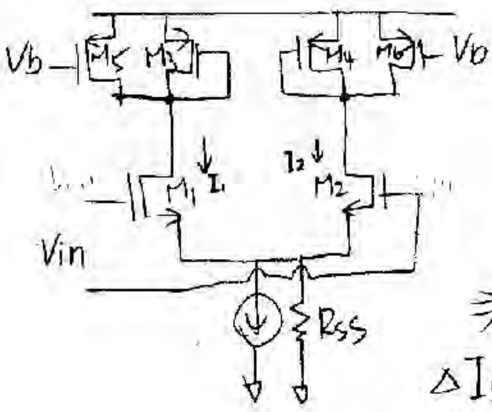
4.26



assume $\lambda = 0 \Rightarrow R_{D3} \simeq \frac{1}{g_{m3}}$.

The mismatch of threshold voltage between M5, M6 will cause a ΔI between I_5 and I_6.

However, $I_5 + I_3 = \frac{1}{2} I_{SS} = I_4 + I_6$

$\Rightarrow \Delta I_5 + \Delta I_3 = 0 \quad \Rightarrow \Delta I_5 = -\Delta I_3$

$$\Delta I_5 = \frac{\partial I_5}{\partial V_{TH}} \times \Delta V_{TH} = -\mu_p C_{ox} \left(\frac{W}{L}\right)_5 (V_{GS} - V_{TH}) \cdot \Delta V_{TH}$$

$$= -2 I_{D5} \frac{\Delta V_{TH}}{V_{DD} - V_b - |V_{TH,P}|} \Rightarrow \Delta I_3 = 2 I_{D5} \frac{\Delta V_{TH}}{V_{DD} - V_b - |V_{TH,P}|}$$

$$\Delta g_{m3} = \frac{\partial g_{m3}}{\partial I_{D3}} \cdot \Delta I_3$$

$$= \sqrt{\frac{\mu_p C_{ox} \left(\frac{W}{L}\right)_3}{2 I_{D3}}} \times \Delta I_3 \Rightarrow \frac{\Delta g_{m3}}{g_{m3}} = \frac{\Delta I_3}{2 I_3} = \frac{I_5}{I_3} \times \frac{\Delta V_{TH}}{V_{DD} - V_b - |V_{TH,P}|} = 4 \frac{\Delta V_{TH}}{V_{DD} - V_b - |V_{TH,P}|} \quad \text{—(1)}$$

$$A_{CM \to DM} = -\frac{g_{m1}}{1 + g_{m1} \cdot 2R_{SS}} \times \Delta R_D \quad , \quad A_{DM \to DM} = -g_{m1} R_D$$

$$\Rightarrow CMRR = \frac{A_{DM \to DM}}{A_{CM \to DM}} = \frac{1 + g_{m1} \cdot 2R_{SS}}{\frac{\Delta R_D}{R_D}} \quad \text{—(2)}$$

$$R_D = \frac{1}{g_{m3}} \quad \Delta R_D = \frac{\partial R_D}{\partial g_{m3}} \cdot \Delta g_{m3} = -\frac{1}{g_{m3}^2} \Delta g_{m3} \Rightarrow \frac{\Delta R_D}{R_D} = -\frac{1}{g_{m3}} \cdot \Delta g_{m3} \quad \text{—(3)}$$

Substitute $\frac{\Delta R_D}{R_D}$ in (2) with (1) and (3), and neglect the minus sign,

$$CMRR = \frac{1 + g_{m1} \cdot 2R_{SS}}{4 \frac{\Delta V_{TH}}{V_{DD} - V_b - |V_{TH,P}|}}$$

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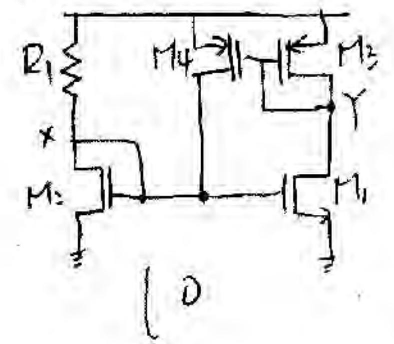
5.10 (a)



When $V_{DD} < V_{TH1,2}$, all transistors are off, and $V_X = V_{DD}$, V_Y = floating

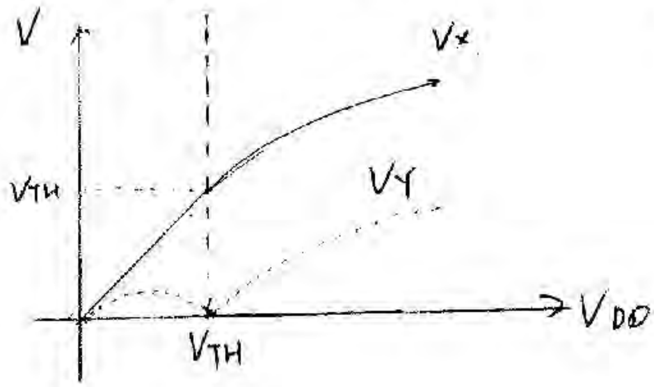
When $V_{DD} = V_{TH1,2}$, M_2 just turns on and hence M_1. Assume $V_{TH3,4} = V_{TH,12}$ then M_3 also turns on and hence M_4. M_1, M_4 are in deep triode regions. In the mean while, $V_X = V_{DD} = V_{TH1,2}$, $V_Y = V_{DD} - V_{TH3} = 0$

When $V_{DD} > V_{TH1,2}$, $V_X = V_{gs2}$, $V_Y = V_{DD} - V_{gs3}$

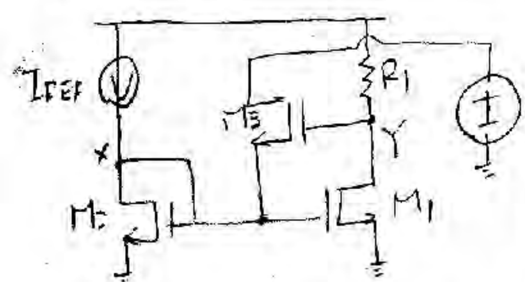
$$V_X - V_Y = V_{GS2} - V_{DD} + V_{gs3} \quad -(1)$$

(1) $> V_{TH}$ so both $|V_{GD}|$ of M_1, M_4 is larger than V_{TH}, $\Rightarrow M_1/M_2$ is kept in triode region.

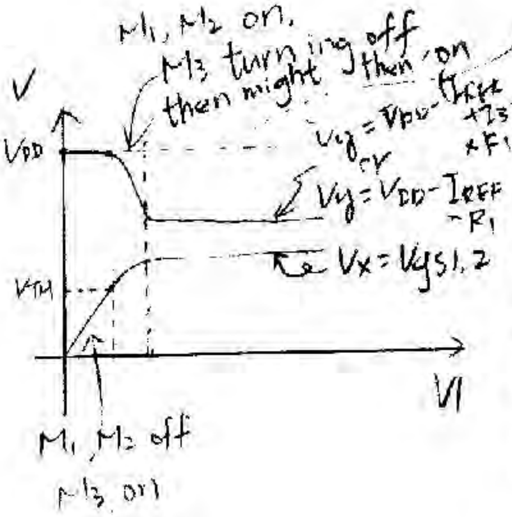
When V_{DD} increases further, M_4 never enters saturation, because if it is the case, I_4 will inject M_2 and mirrors more current back to itself, raising V_X and lowering V_Y until M_4 conduct less current than M_2. Therefore, $V_X - V_Y > V_{TH}$, and so is M_4 is always in triode region. The V_X V_Y VS V_{DD} plot is as follows.



5.12 (b)

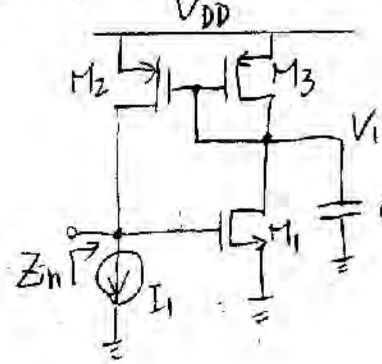


When $V_1 = 0$, S/D of M_3 swap, some current might flow out through M_3 to V_1, therefore, $I_{M1} = I_{M2} < I_{REF}$.
However, it it possible that I_{M3} sinks all the I_{REF} and make $I_{M2} = I_{M1} = 0$. Assume it is the case, then $V_Y = V_{DD}$, $V_X = V_1 + V_{DS3}$.
When $V_1 \uparrow$, V_X also increases, but M_2, M_1 remain of until V_X reaches V_{TH}. In the meanwhile V_{GS3} decreases, so V_{DS3} increase to conduct the same current. Therefore V_X increases faster than V_1.
When V_X reaches V_{TH}, M_1, M_2 starts to turn on. $V_Y = V_{DD} - I_{M1}R_1$, so V_Y starts to drop.
When V_1 keeps increasing, $V_{GS} = V_Y - V_1$ shrinks sharply and may finally get M_3 turned-off.
After M_3 turns off, increasing V_1 will not affect I_{M1}, I_{M2}, so V_X V_Y keep constant.



~~However, it is also possible that $V_1 > V_X$ before making M_3 off, then enters a stable state with conducting some current.~~
~~Then $I_{M1} = I_{M2} = I_{REF} + I_3$,~~
~~$V_y = V_{DD} - R_1(I_{REF} + I_3)$~~

5.19



The small-signal model is drawn below assuming $\lambda = 0$

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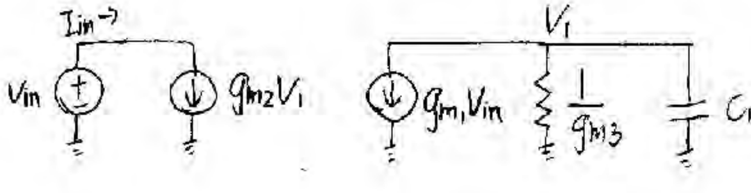
$I_{in} = g_{m2} V_1$, $V_1 = -g_{m1} V_{in} \times \frac{1}{g_{m3}} // \frac{1}{sC_1}$

$= -g_{m1} V_{in} \cdot \frac{1}{g_{m3} + sC_1}$

$\Rightarrow I_{in} = g_{m2} \times (-g_{m1} V_{in}) \frac{1}{g_{m3} + sC_1}$



$\Rightarrow Z_{in} = \frac{V_{in}}{I_{in}} = \frac{-(g_{m3} + sC_1)}{g_{m1} g_{m2}}$

$= -\frac{g_{m3}}{g_{m1} g_{m2}} - \frac{sC_1}{g_{m1} g_{m2}}$

negative $R = -\frac{g_{m3}}{g_{m1} g_{m2}}$, negative $= -\frac{sC_1}{g_{m1} g_{m2}}$

5.20(b)

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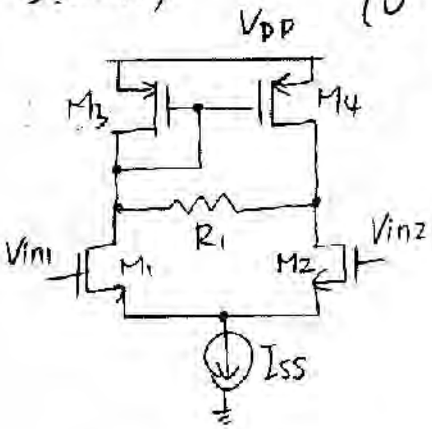


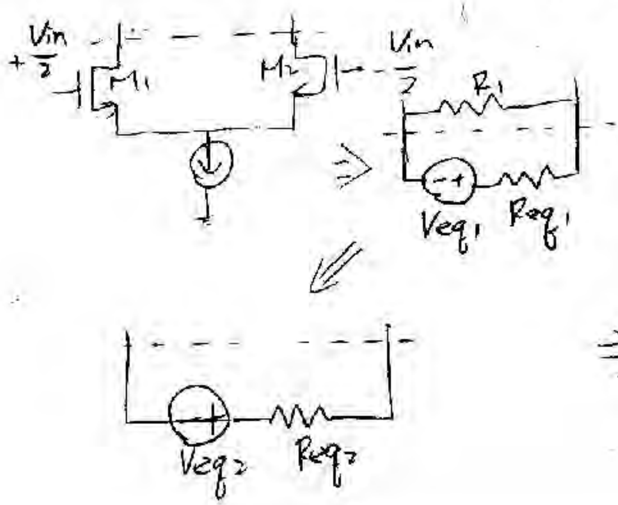
First we can treat M_1, M_2, I_{SS} as a one-port network and replace it with Thevenin equivalent circuit.

$V_{eq1} = g_{m1,2} r_{o1,2} V_{in}$, $R_{eq1} = 2 r_{o1,2}$

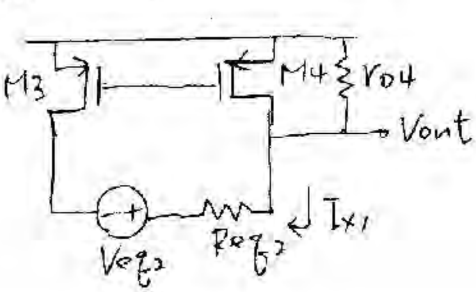
Combine R_1 with V_{eq1} and R_{eq1}, a second Thenevin equivalent ckt can be derived, too.

$\therefore V_{eq2} = \frac{R_1}{R_{eq1} + R_1} \times V_{eq1} = \frac{R_1}{R_{eq1} + R_1} \cdot g_{m1,2} r_{o1,2} V_{in}$

$R_{eq2} = R_{eq1} // R_1 = 2 r_{o1,2} // R_1$



The overall circuit is shown below



$\therefore I_x = \dfrac{V_o - V_{eq2}}{R_{eq2} + \frac{1}{g_{m3}} // r_{o3}}$ Since I_x is mirrored into M_4, we have

$$2 \frac{V_{out} - V_{eq2}}{R_{eq2} + \frac{1}{g_{m3}} // r_{o3}} \times \frac{r_{o3}}{r_{o3} + 1/g_{m3}} = \frac{-V_{out}}{r_{o4}} \quad \text{--- (1)}$$

Assume $r_{o3} \gg \frac{1}{g_{m3}} \Rightarrow \frac{1}{g_{m3}} // r_{o3} \approx \frac{1}{g_{m3}}$

Assume $R_{eq2} \gg \frac{1}{g_{m3}}$

$$(1) \Rightarrow 2 \frac{V_{out} - V_{eq2}}{R_{eq2}} = \frac{-V_{out}}{r_{o4}}$$

$$\Rightarrow V_{out}(2r_{o4} + R_{eq2}) = 2r_{o4} V_{eq2}$$

$$\Rightarrow V_{out} = \frac{2r_{o4} V_{eq2}}{2r_{o4} + R_{eq2}}$$

$$= \frac{2r_{o4}}{2r_{o4} + 2r_{o1,2} // R_1} \times \frac{R_1}{2r_{o1,2} + R_1} \times g_{m1,2} r_{o1,2} V_{in}$$

$$\Rightarrow A_v = \frac{V_{out}}{V_{in}} = \frac{2r_{o4} R_1 g_{m1,2} r_{o1,2}}{(2r_{o4} + 2r_{o1,2} // R_1)(2r_{o1,2} + R_1)}$$