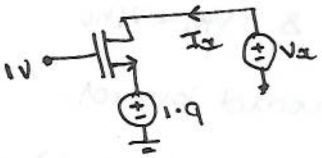
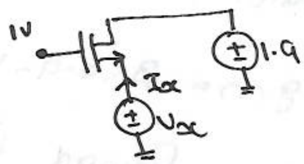
$\boxed{2.5c}$ Qn: I_x, g_m as a function of $V_x \in (0, V_{DD})$

$\boxed{\beta = \mu_n C_{ox} W/L \text{ in all question.}}$



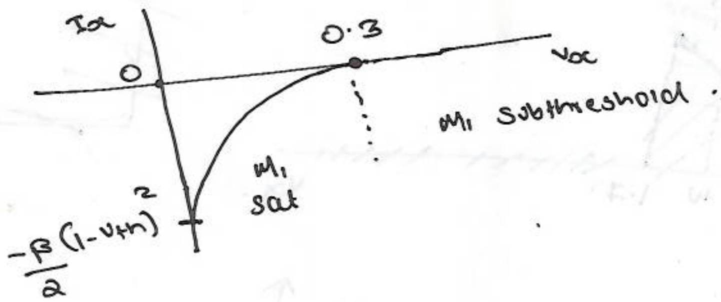
(i) Starting at $V_x = 0$ & showing source at lowest voltage

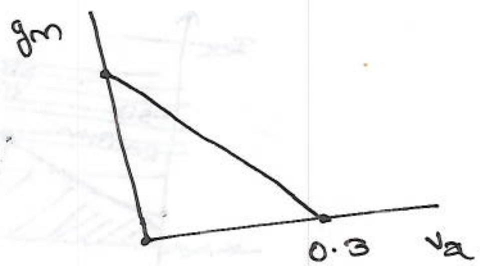


Devices starts in saturation as $V_D > V_g - V_{th}$

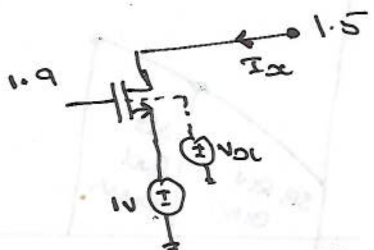
$$I_x = -\frac{\beta}{2}(1 - V_x - V_{th})^2 = -I_D \; ; \; g_m = \frac{\partial I_D}{\partial V_{gs}} = \beta(1 - V_x - V_{th})$$

Device goes off at $V_x = 1 - V_{th} \simeq 0.3V$ at which point $I_x = 0$





$\boxed{Q.5E}$



$$V_{th} = V_{th0} + \gamma\left[\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right]$$

$V_{SB} = (1 - V_x)$ — (1)

$$V_{th} = V_{th0} + \gamma\left[\sqrt{2\phi_F + 1 - V_x} - \sqrt{2\phi_F}\right]$$

(i) $0 \le V_x \le 1 \Rightarrow V_{th} > V_{th0}$ & device M_1 in saturation

$\Rightarrow$ SB diode is reverse biased

$\Rightarrow V_{th_{max}} (V_x = 0, \gamma = 0.45, 2\phi_F = 0.9) = 0.893V.$

$$I_x = \frac{\beta}{2}(0.9 - V_{th})^2 \qquad g_m = \beta(0.9 - V_{th})$$

(ii) $1 \le V_x \le 1.7 \Rightarrow V_{SB}$ is negative & $V_{th} < V_{th0}$

$\Rightarrow$ SB Diode is forward biased.

$\Rightarrow V_{th\,min} \{ V_{SB} = -.7, \gamma = 0.45, 2\phi_F = 0.9) = 0.47V.$

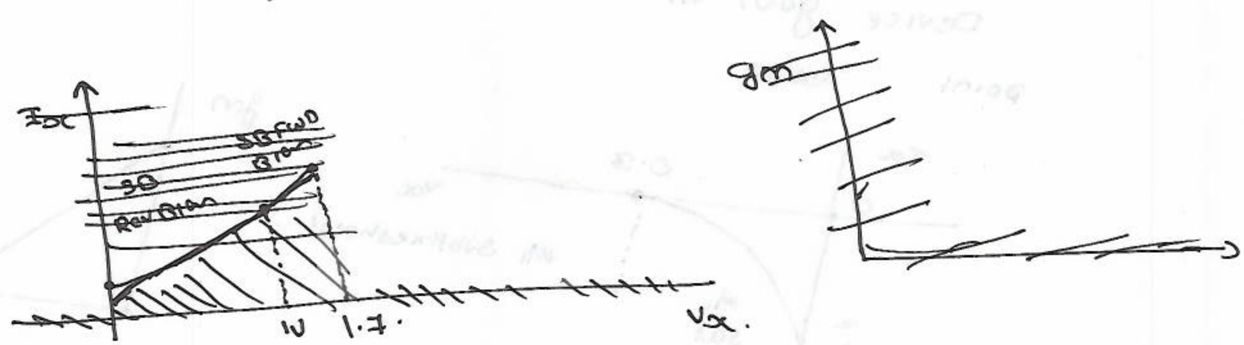
$\Rightarrow M_1$ is in saturation.

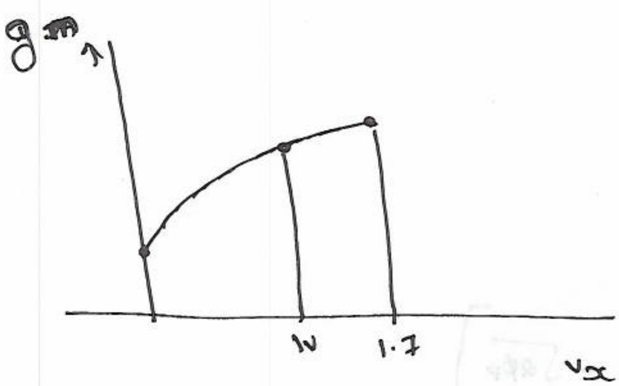
$$I_x = \frac{\beta}{2}(0.9 - V_{th})^2 \qquad g_m = \beta(0.9 - V_{th}).$$

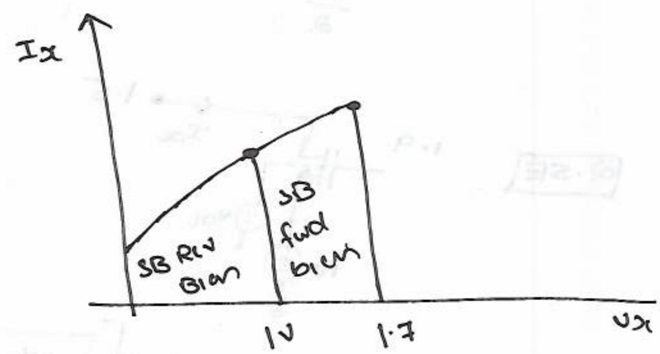
(iii) $V_x \ge 1.7 \Rightarrow$ SB diode heavily forward biased

$\Rightarrow$ Eqn (1) No longer valid

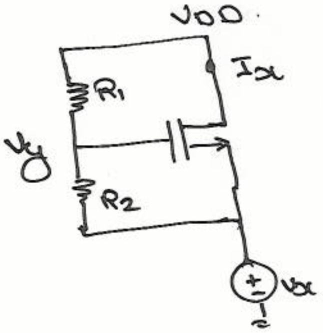
$\Rightarrow$ device will latch up







2.6b



$$V_y = \frac{V_{DD}R_2 + V_x R_1}{R_1+R_2}$$

$$V_{gs} = [V_{DD} - V_x]\frac{R_2}{R_1+R_2}$$

(i) starting $V_x = V_{DD}$ for convinienc where M_1 is off.

on $V_y = V_{DD}$. $V_{gs} = 0$.

Device will turn on when $V_{gs} = V_{th}$

$$\Rightarrow (V_{DD} - V_x)\frac{R_2}{R_1+R_2} = V_{th}$$

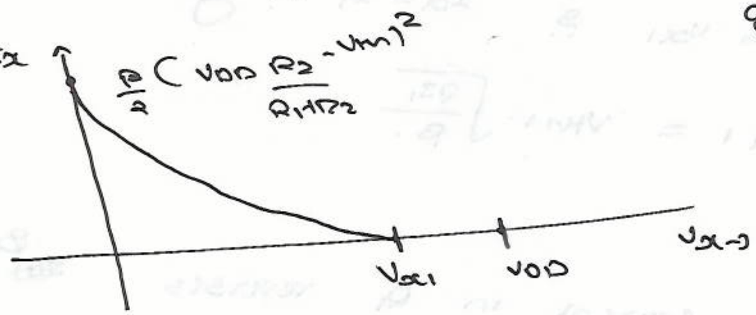
$$V_x = V_{DD} - \frac{V_{th}(R_1+R_2)}{R_2} = V_{x1}$$ till which $I_x = 0$.

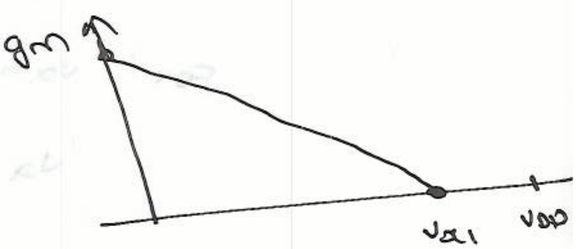
(ii) since $V_g < V_d$ device always in saturation. $V_x < V_{x1}$

so for $V_x < V_{DD} - \frac{V_{th}(R_1+R_2)}{R_2}$

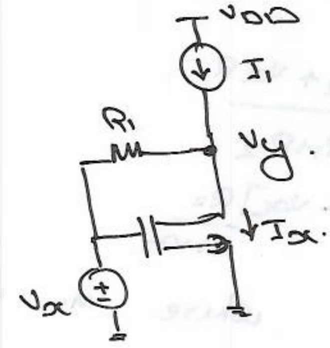
(iii)

$$I_x = \frac{\beta}{2}\left((V_{DD} - V_x)\frac{R_2}{R_1+R_2} - V_{th}\right)^2 \qquad g_m = \frac{\beta}{2}\left((V_{DD} - V_x)\frac{R_2}{R_1+R_2} - V_{th}\right)$$





(2.6 e)



(i) Starting at $V_x = 0$ for convinience. Device M_1 is off at this point & $V_y =$ [redacted] $0 + I_1R_1$

As V_x increase V_y also increases as $V_x + I_1R_1$

$I_x = 0$ & $V_y = V_x + I_1R_1$ until $V_x = V_{th}$.

(ii) $V_x > V_{th}$ & Device ~~tarm~~ on in saturation as $V_D > V_g - V_{th}$

$\{V_y > V_x$ at $V_x = V_{th}\}$.

$$I_x = \frac{\beta}{2}(V_x - V_{th})^2 \; ; \quad g_m = \beta(V_x - V_{th})$$

hence V_y [redacted] $= V_x + (I_1 - I_x)R_1$

For $V_x = V_{x1}$ ∴ $I_x = I_1$ $V_y = V_x$

$$V_{x1} = V_{th} + \sqrt{\frac{2I_1}{\beta}}$$

[redacted] & $V_y < V_x$.

(iii) $V_x > V_{x1}$ current in R_1 reverses [redacted]

Device still in saturation until [redacted] $V_x = V_{x2}$ ∴ $V_d = V_g - V_{th}$.

[redacted]

$\Rightarrow V_{x2} + (I_1 - I_x)R_1 = V_{x2} - V_{th}$

$\Rightarrow (I_x - I_1)R_1 = V_{th}$.

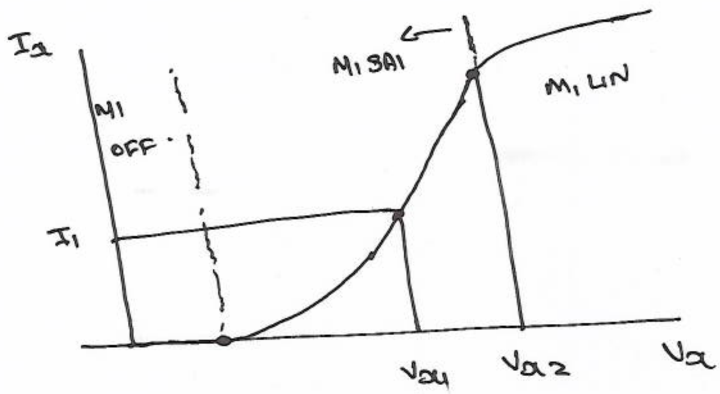
$\Rightarrow \left(\frac{\beta}{2}(V_{x2} - V_{th})^2 - I_1\right)R_1 = V_{th}$.

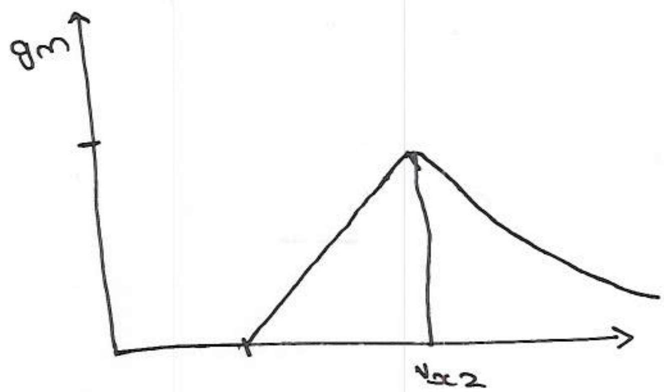
$I_x = \frac{\beta}{2}(V_x - V_{th})^2$ in this region ; $g_m = \beta(V_x - V_{th})$.

(iv) $V_x > V_{x2}$ device is in linear region.

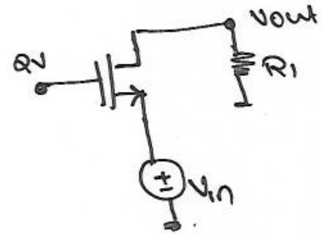
$$I_x = \beta\left[(V_x - V_{th})(V_y) - \frac{V_y^2}{2}\right] \quad \text{where} \quad V_y = V_x + (I_1 - I_x)R_1$$

$g_m =$ [redacted] βV_y.



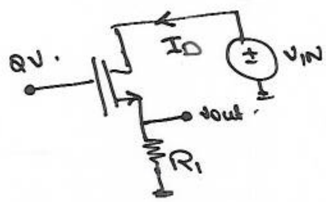


(2) (2.7b) Qn Vout as f(vin) as $v_{in} \in (0, V_{DD})$.



$V_{IN} > V_{out}$.

Redrawing



(i) starting from $V_{IN} = 0 \Rightarrow I_x = 0$ & $V_{out} = 0$.

Device starts in linear region

$$I_D = \beta\left((2 - V_{out} - V_{th})(V_{IN} - V_{out}) - \frac{(V_{in} - V_{out})^2}{2}\right).$$

$$V_{out} = I_D R_1$$

this is valid until $V_{IN} = V_{IN1} = 2 - V_{th}$ when

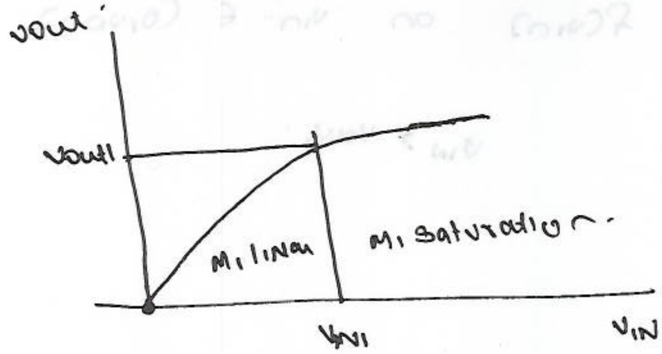
$$V_{th} = V_{th0} + \gamma\left[\sqrt{2\phi_F + V_{out}} - \sqrt{2\phi_F}\right].$$

(ii) when $V_{IN} > V_{IN1}$ Device is in saturation.

$$I_D = \frac{\beta}{2}(2 - V_{out} - V_{th})^2 \quad \& \quad V_{out} = I_D R_1 = V_{out1}$$

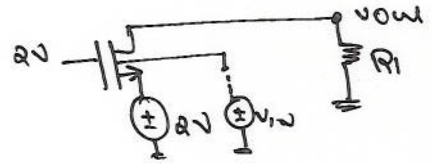
$$= \frac{\beta}{2}(2 - I_D R_1 - V_{th})^2$$

For high V_{IN} Vout increases slightly due to channel length modulation but otherwise stays constant.



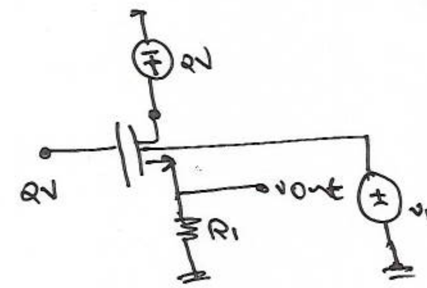
(2.7c) Same as 2.7b with Vg = 3V.

(2.8c)



$V_{out} < 2V$.

Redrawing



(i) Starting with $V_{IN} = 0$

Device in in saturation $\Rightarrow$ (2)

$$V_{th} = V_{th0} + \gamma\left[\sqrt{2\phi_F + (V_{out} - V_{IN})} - \sqrt{2\phi_F}\right].$$

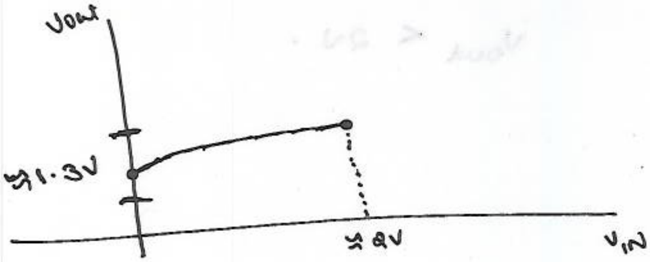
$$V_{out} = I_D R_1$$

$$I_D = \frac{\beta}{2}(2 - V_{out} - V_{th})^2$$ (ignoring channel modulation).

As V_{in} increases V_{th} reduces increasing I_D & hence V_{out} slowly. in V_{out} increase in much slow compared to V_{IN} due to negative feedback via V_{SB} & R1

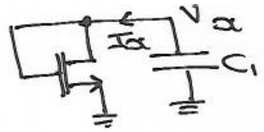
(ii) As V_{IN} increases it will catch up with V_{out} at some point. (V_{out} stays in the vicinity of $2 - 0.7 = 1.3V$) and beyond which SB diode will forward bias taking V_{th} below V_{th0}.

(iii) For large values of V_{IN} SB junction will be heavily forward biased & equation 2 won't be valid



Vout
≈1.3V
≈2V
VIN

Q.9b



$t = 0^-$ $V_x = 3V$ Device is in saturation

$t > 0$ $$I_x = \frac{\beta}{2}(V_x - V_{th})^2 = -C_1 \frac{dV_x}{dt}$$

$$\frac{dV_x}{(V_x - V_{th})^2} = \frac{-\beta\, dt}{2C_1}$$

integrating $\Rightarrow$

$$\frac{(V_x - V_{th})^{-1}}{-1} = -\frac{\beta t}{2C_1} + K.$$

$$\frac{1}{V_x - V_{th}} = \frac{\beta t}{2C_1} - K$$

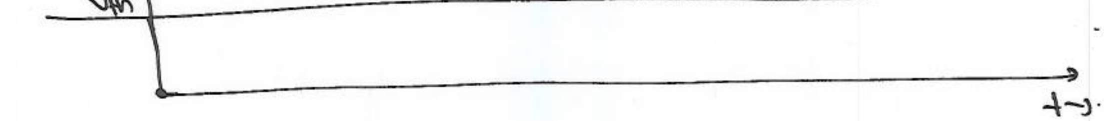
$t = 0$ $V_x = 3$ $\Rightarrow$

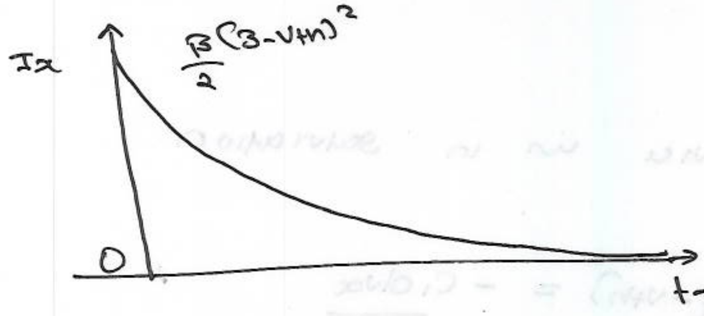
$$\frac{1}{3 - V_{th}} = -K.$$

$$\Rightarrow \frac{1}{V_x - V_{th}} = \frac{\beta t}{2C_1} + \frac{1}{3 - V_{th}}.$$

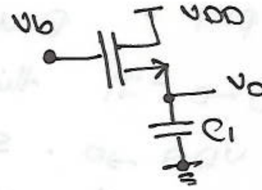
$$V_x = V_{th} + \frac{1}{\frac{\beta t}{2C_1} + \frac{1}{3 - V_{th}}}.$$

$$I_x = \frac{\beta}{2}\left(\frac{1}{\frac{\beta t}{C_1} + \frac{1}{3 - V_{th}}}\right)^2$$



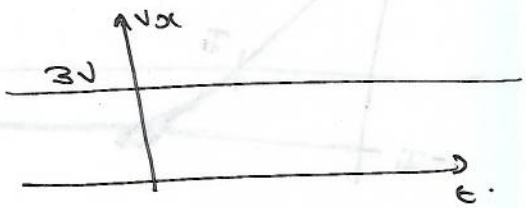

Ix
β/2 (3-Vth)²
O
t→

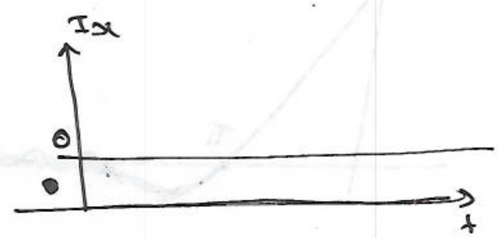
(2.9c)



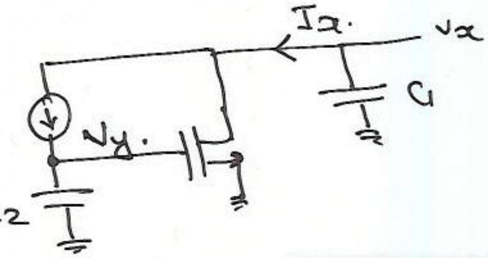
$V_b < V_{DD} = 3V$

$t = 0^-$ $V_x = 3V$ $\Rightarrow$ device M_1 is off so V_x stays at 3V. & $I_x = 0$





(2.10a)



$t = 0^-$ $V_y = 3$ $V_x = 1$ Device is in linear region

$t > 0$

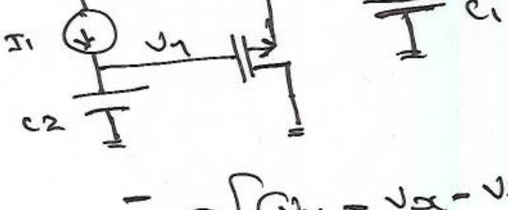
$$V_y = 3 + \frac{I_1 t}{C_1}$$

$$I_1 + \beta\left[(V_y - V_{th})V_x - \frac{V_x^2}{2}\right] + C_1\frac{dV_x}{dt} = 0$$

$\Rightarrow$ V_x discharges towards zero. & $V_x(t = T_1) = 0$.

$t > T_1$ $V_x < 0$ $\Rightarrow$ source & drain swaps. Device still in linear region

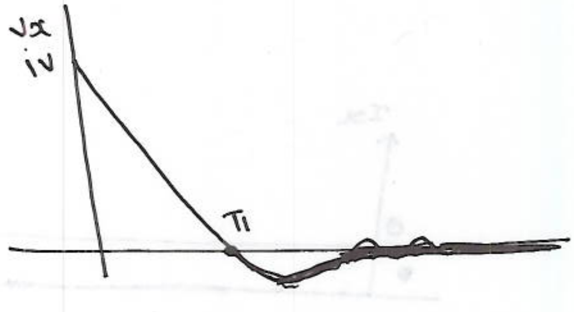
$V_y = 3 + I_1 t$.

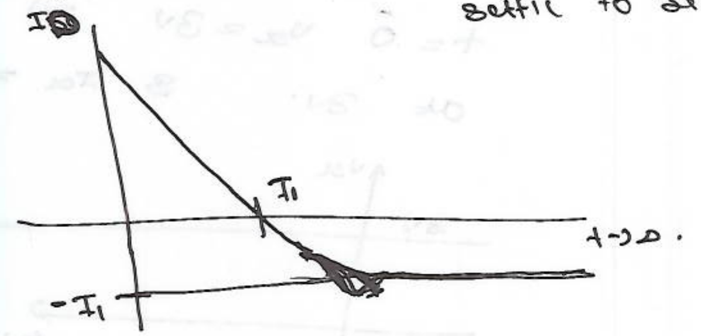


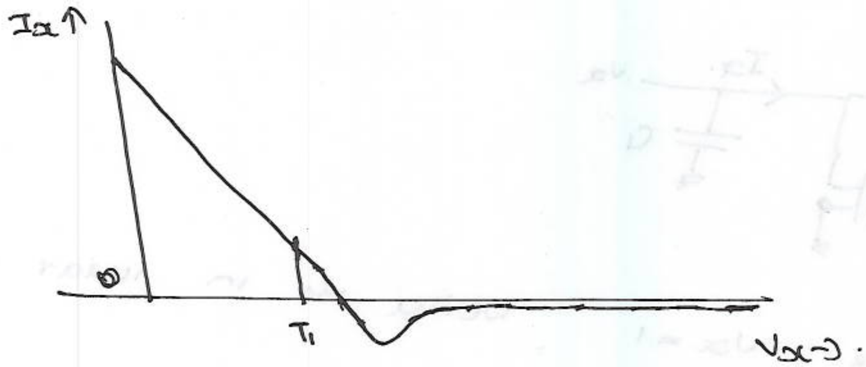
$$I_1 - \beta\left[(V_y - V_x - V_{th})(-V_x) - \frac{V_x^2}{2}\right] + C_1\frac{dV_x}{dt} = 0.$$

Clearly V_x will reduce & I_D will tend to I_1 at steady state.

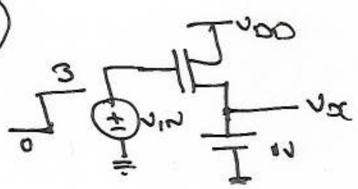
Solving this we can get a plot. But intuitively at $t = \infty$ $V_{gs} = \infty$ $I_D \to -I_1$ this will be achieved by a small $V_{DS} \to 0$. So V_x will overshoot to a negative value & then will slowly settle to zero.





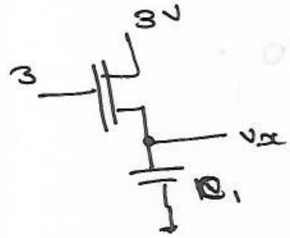


(2.11a)



$t<0^-$ M_1 is off as $v_{gs}<0$.

$t>0$



ignoring body effect & channel length modulation.

$$\frac{\beta}{2}(3-V_{th}-V_x)^2 = \frac{C_1 dV_x}{dt}.$$

$$\frac{dV_x}{(3-V_{th}-V_x)^2} = +\frac{\beta dt}{2C_1}.$$

$$\frac{1}{+(3-V_{th}-V_x)} = +\frac{\beta t}{2C_1} + K.$$

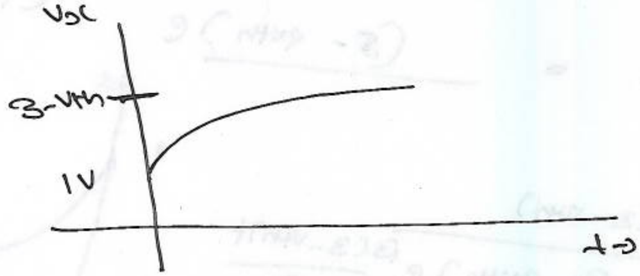
$t=0$ $V_x = 1$

$$K = \frac{1}{2-V_{th}}$$

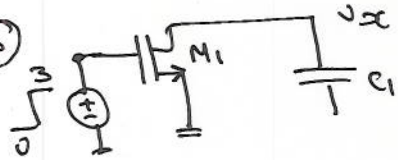
$$3-V_{th}-V_x = \frac{1}{\frac{\beta t}{2C_1}+K.}$$

$$V_x = 3-V_{th} - \frac{1}{\frac{\beta t}{2C_1}+K}$$

$$= 3-V_{th} - \frac{1}{\frac{\beta t}{2C_1}+\frac{1}{2-V_{th}}}.$$



2.11b



$t = 0^-$ $V_{gs} = 0$ M_1 in off.

$t > 0$ $V_{gs} = 3V$ M_1 in on in linear region

$$\beta\left[(3 - V_{th})V_x - \frac{V_x^2}{2}\right] + C_1\frac{dV_x}{dt} = 0.$$

$$\frac{dV_x}{V_x\left[2(3 - V_{th}) - V_x\right]} = -\frac{\beta\, dt}{2C_1}$$

$$\frac{dV_x}{2(3 - V_{th})}\left[\frac{1}{V_x} + \frac{1}{2(3 - V_{th}) - V_x}\right] = -\frac{\beta\, dt}{2C_1}$$

integerating $\ln\left[\frac{V_x}{2(3 - V_{th}) - V_x}\right] = -\frac{\beta(3 - V_{th})t}{C_1} + \text{unk}.$

$$\frac{V_x}{2(3 - V_{th}) - V_x} = K e^{-\frac{\beta(3 - V_{th})t}{C_1}}$$

$t = 0$ $V_x = 1$ $\Rightarrow$ $K = \frac{1}{5 - 2V_{th}}.$

$$\frac{V_x}{2(3 - V_{th}) - V_x} = \frac{1}{5 - 2V_{th}} e^{-\frac{\beta(3 - V_{th})t}{C_1}}.$$

$$\frac{2(3 - V_{th})}{V_x} - 1 = (5 - 2V_{th}) e^{\frac{\beta(3 - V_{th})t}{C_1}}$$

$$V_x = \frac{2(3 - V_{th})}{1 + (5 - 2V_{th}) e^{\frac{\beta(3 - V_{th})t}{C_1}}}.$$

(2.26) (a) $t=0^-$ $v_x = V_{DD}$, $I_D = I_1$, $v_y = v_{y0} = V_{DD} - V_{th} - \sqrt{\frac{2I_1}{\beta}}$.

$t=0^+$ voltage across capacitor can't change instantly so the step at V_{in} propagates to v_x & v_y

$v_y(0^+) = v_{y0} + V_0$ & $v_x(0^+) = V_{DD} + V_0$

$t>0$

Since $I_C = 0$ $\quad v_x - v_y = \text{constant} = V_{DD} - v_{y0}$.

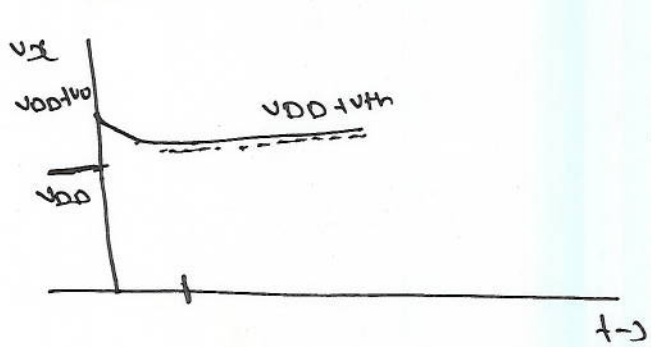
This means v_{gs} of device is constant and same as $t=0^-$. However since $V_0 > V_{th}$ device enters linear region & $I_D < I_1$ given by.

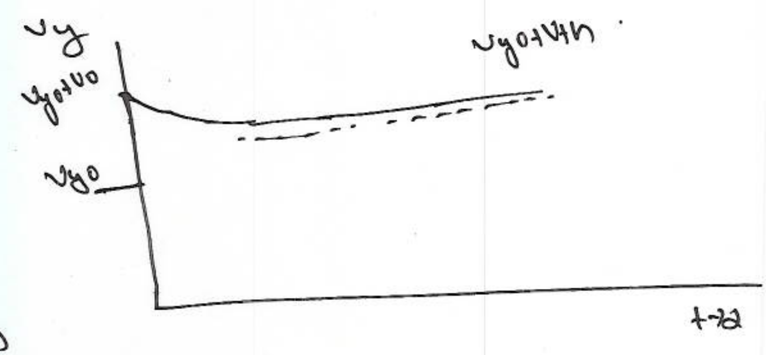
$$I_D = \beta\left[(V_{DD} - v_{y0} - V_{th})(V_{DD} - v_y) - \frac{(V_{DD} - v_y)^2}{2}\right].$$

Thus will lead to discharge of node v_y.

as $C_2 \frac{d}{dt}(V_0 - v_y) = I_1 - I_D$.

As v_y falls v_x tracks the change & this will continue until v_x reaches $V_{DD} + V_{th}$ at which point M_1 enters saturation making $I_D = I_1$. The voltage at v_y in steady state hence known to $v_{y0} + V_{th}$.





2.26b $t=0^-$ $V_x = V_{DD}$ $V_y = V_{y0} = V_{DD} - V_{th} - \sqrt{\frac{2I_0}{\beta}}$.

$t=0^+$ $V_x = V_{DD} - V_0$ $V_y = V_{y0} - V_0$.

$t>0$ Since $I_{C1} = 0$ $V_x - V_y = V_{DD} - V_{y0}$.

Since device is in saturation.

$I_D = I_1 \Rightarrow I_{C2} = 0$ so no change in voltages after initial step.