

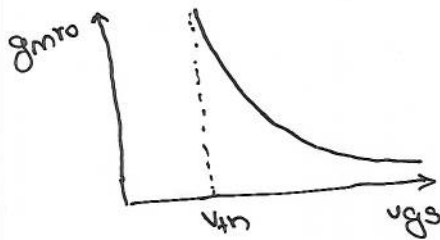
$$\beta = \mu n C_{ox} \frac{W}{L}$$

\Rightarrow Total = 33

3.6 a) $g_m = \frac{\partial I_D}{\partial V_{GS}}$ $r_o = \frac{1}{\partial I_D}$ ~~\Rightarrow~~

$$g_{mro} = \frac{\partial}{\partial [V_{GS} - V_{th}]}$$

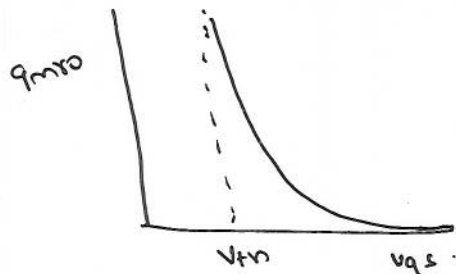
Eqn - 1 mark



plot - 1 mark
total (2)

b) $g_m = \mu n C_{ox} \frac{W}{L} (V_{GS} - V_{th})$

$$g_{mro} = \frac{\partial}{\partial (V_{GS} - V_{th})}$$



$$r_o = \frac{1}{\partial I_D} = \frac{1}{\frac{\partial}{\partial} \mu n C_{ox} \frac{W}{L} (V_{GS} - V_{th})^2}$$

Eqn - 1 mark

plot - 1 mark

total (2)

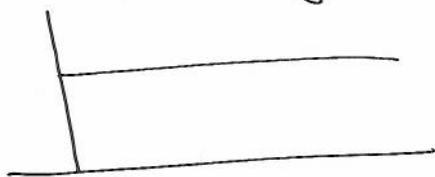
(3.7)

~~$g_{m0} = \frac{2 I_{D0}}{V_{GS} - V_{TH}}$~~

a) $g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$

$r_o = \frac{1}{\lambda I_D} = \frac{1}{\frac{\lambda}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2}$

$g_{mro} = \frac{2}{\lambda (V_{GS} - V_{TH})}$



Assuming λ is constant.

Ear - 1

Plot - 1

(2)

b)

$g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I}$

$r_o = \frac{1}{\lambda I}$

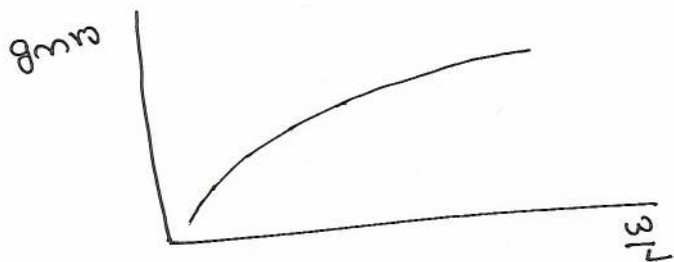
$g_{mro} = \sqrt{2 \mu_n C_{ox} \frac{W}{L}} \cdot \frac{1}{I} = \frac{1}{\lambda}$

Assuming λ is constant.

Ear - 1

Plot - 1

(2)



3.9

$$g_m = \beta (v_{gs} - v_{th})$$

$$= \beta (v_{gs} - [v_{th0} + \gamma \sqrt{2\phi_F + v_{SB}} - \sqrt{2\phi_F}]) \quad \text{--- ①}$$

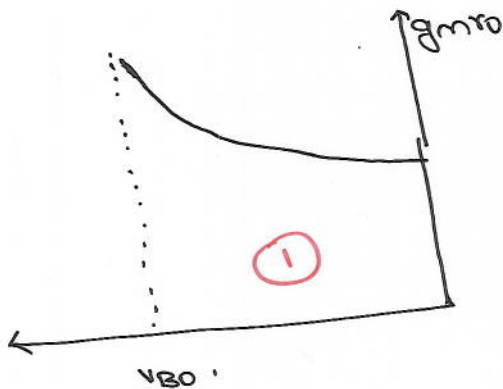
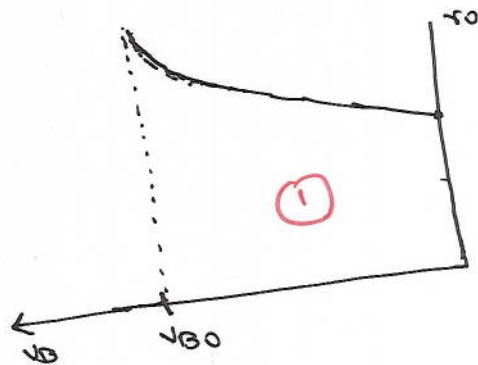
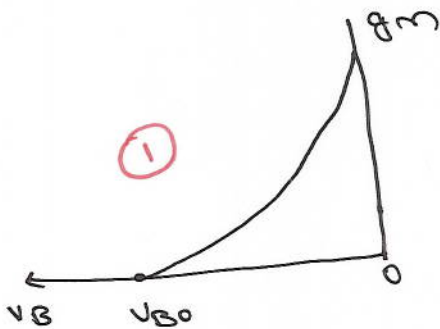
$$r_o = \frac{1}{\lambda I_D} = \frac{1}{\lambda \frac{\beta}{2} (v_{gs} - v_{th})^2}$$

$$= \frac{1}{\lambda \frac{\beta}{2} (v_{gs} - [v_{th0} + \gamma \sqrt{2\phi_F + v_{SB}} - \sqrt{2\phi_F}])^2} \quad \text{--- ①}$$

$$g_{mro} = \frac{2}{\lambda (v_{gs} - v_{th})} = \frac{2}{\lambda (v_{gs} - [v_{th0} + \gamma \sqrt{2\phi_F + v_{SB}} - \sqrt{2\phi_F}])} \quad \text{--- ①}$$

① -

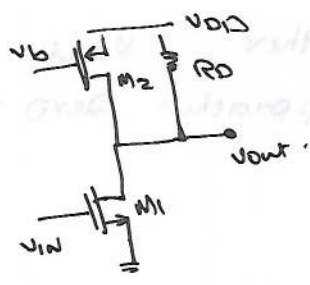
v_{th} increases as bulk potential goes negative and at a voltage v_{B0} $v_{th} = v_{gs}$ and device goes off.



$$v_{B0} \Rightarrow \left[\frac{v_{gs} - v_{th0} + \sqrt{2\phi_F}}{\gamma} \right]^2 + 2\phi_F + v_{SB}$$

total 6.

3-16a



(i) $V_{IN} = 0$ M_1 is off $v_{out} = V_{DD}$ & M_2 is in linear region
 this stays like this till $V_{IN} = V_{th}$ when M_1 turns on

(ii) $V_{IN} > V_{th}$ current through M_1 increases
 M_2 stays in linear region & v_{out} drops.

$$\frac{\beta_n}{2} (V_{IN} - V_{th})^2 = \beta_p \left((V_{DD} - V_B - |V_{thp}|)(V_{DD} - V_{out}) - \frac{(V_{DD} - V_{out})^2}{2} \right) + \frac{V_{DD} - V_{out}}{R_D}$$

(iii) when $v_{out} = V_B + |V_{thp}|$ M_2 enters saturation at this point. the
 assuming $V_{IN} = V_{IN1}$ in saturation given by:
 corresponding $\frac{\beta_n}{2} (V_{IN} - V_{th})^2 = \frac{\beta_p}{2} (V_{DD} - V_B - |V_{thp}|)^2 + \frac{V_{DD} - V_B - |V_{thp}|}{R_D}$

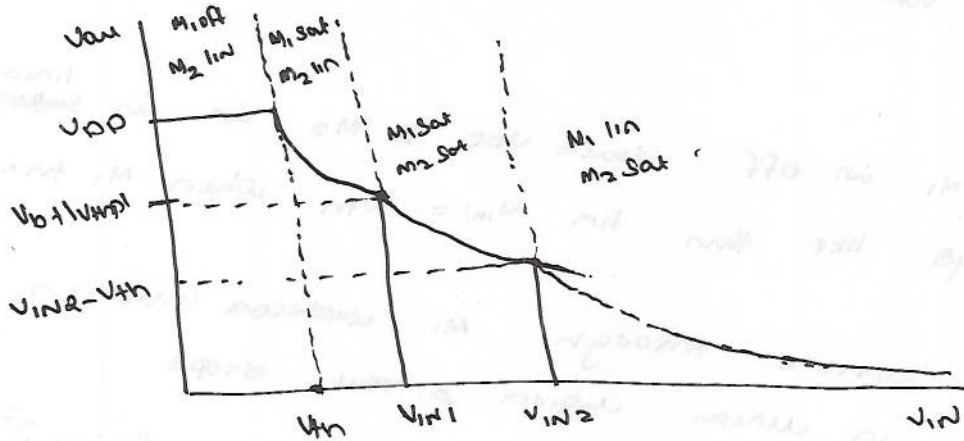
(iv) ~~for~~ For $V_{IN} > V_{IN1}$ M_2 stays in saturation
 conducting constant current. & v_{out} drops
 as per equation

$$\frac{\beta_n}{2} (V_{IN} - V_{th})^2 = \frac{\beta_p}{2} (V_{DD} - V_B - |V_{thp}|)^2 + \frac{V_{DD} - V_{out}}{R_D}$$

(v) when v_{out} drops to $V_{IN} = V_{th}$ M_1 will
 enter into linear region. & corresponding
 $V_{IN} = V_{IN2}$ in given by

$$\frac{\beta_n}{2} (V_{IN2} - V_{th})^2 = \frac{\beta_p}{2} (V_{DD} - V_B - |V_{thp}|)^2 + \frac{V_{DD} - (V_{IN2} - V_{th})}{R_D}$$

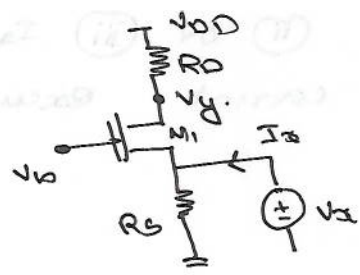
(vi) as V_{in} increases still further V_{out} continues to drop and asymptotically approaches zero.



Each region 1 mark total (4)

Each region $\frac{1}{2}$ for boundary condition relevant equation & trend.

[Faint handwritten notes and equations are visible in the background, including terms like VDD, Vb1, Vb2, and Vin.]



Starting from $v_{ds} = v_{DD}$ for convenience

(i) $v_{ds} = v_{DD}$ M_1 is in off $v_{gs} = v_{DD}$
 This continues until $v_{ds} = v_b - v_{th}$ at which point M_1 will turn on in saturation. Till then

$$I_d = \frac{v_{ds}}{R_s}$$

(ii) $v_{ds} < v_b - v_{th}$ M_1 is in saturation & v_{gs} starts dropping. M_1 will enter saturation when v_{gs} reaches $v_b - v_{th}$ at $v_{ds} = v_{ds1}$

$$\Rightarrow \frac{v_{DD} - v_{ds1}}{2} \cdot \frac{\beta_n}{2} (v_b - v_{ds1} - v_{th})^2 R_D = v_b - v_{th}$$

$$\Rightarrow v_{ds1} = v_{DD} - (v_b - v_{th}) \cdot \frac{2}{\beta_n R_D}$$

$$(v_b - v_{ds1} - v_{th})^2 = (v_{DD} - v_b + v_{th}) \frac{2}{\beta_n R_D}$$

$$v_b - v_{ds1} - v_{th} = \sqrt{(v_{DD} - v_b + v_{th}) \frac{2}{\beta_n R_D}}$$

$$v_{ds1} = v_b - v_{th} - \sqrt{(v_{DD} - v_b + v_{th}) \frac{2}{\beta_n R_D}}$$

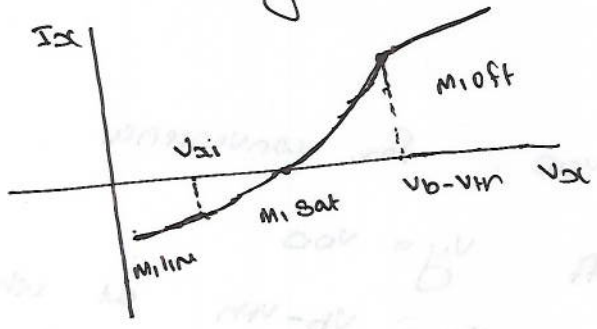
After then current = $\frac{v_{ds}}{R_s} = \frac{\beta_n}{2} (v_b - v_{ds} - v_{th})^2$

(iii) $v_{ds} < v_{ds1}$ M_1 is in linear region.

$$I_d = \frac{v_{ds}}{R_s} = \frac{\beta_n}{2} \left[(v_b - v_{ds} - v_{th})(v_{gs} - v_{ds}) - \frac{(v_{gs} - v_{ds})^2}{2} \right]$$

$$I_d = \frac{v_{ds}}{R_s} = \frac{\beta_n}{2} \left[(v_b - v_{ds} - v_{th})(v_{gs} - v_{ds}) - \frac{(v_{gs} - v_{ds})^2}{2} \right]$$

(iv) At some point in region (i) or (ii) I_d will change polarity when drain current exceeds $\frac{V_d}{R_S}$



Each region - 1
 total 3

(i) $V_{d1} < V_{th}$ $I_d < 0$ $V_{gs} = V_{th}$

(ii) $V_{d1} > V_{th}$ $I_d > 0$ $V_{gs} = V_{th} + \frac{V_d}{R_S}$

(iii) $V_{d1} > V_{th}$ $I_d < 0$ $V_{gs} = V_{th} + \frac{V_d}{R_S}$

$$I_d = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{gs} - V_{th})^2$$

$$I_d = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(V_{th} + \frac{V_d}{R_S} - V_{th} \right)^2$$

$$I_d = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(\frac{V_d}{R_S} \right)^2$$

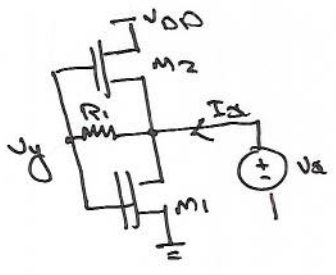
$$I_d = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \frac{V_d^2}{R_S^2}$$

$$I_d R_S^2 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} V_d^2$$

$$V_d^2 = \frac{2 I_d R_S^2 L}{\mu_n C_{ox} W}$$

$$V_{d1} = \sqrt{\frac{2 I_{d1} R_S^2 L}{\mu_n C_{ox} W}}$$

3.10d



Since there is no current flow through R_1 , $V_y = V_x$.

(i) At $V_x = 0$, M_1 is off & M_2 is on in saturation

This is the state till M_1 turns on at $V_x = V_{thn}$
 So for $0 \leq V_x \leq V_{thn}$

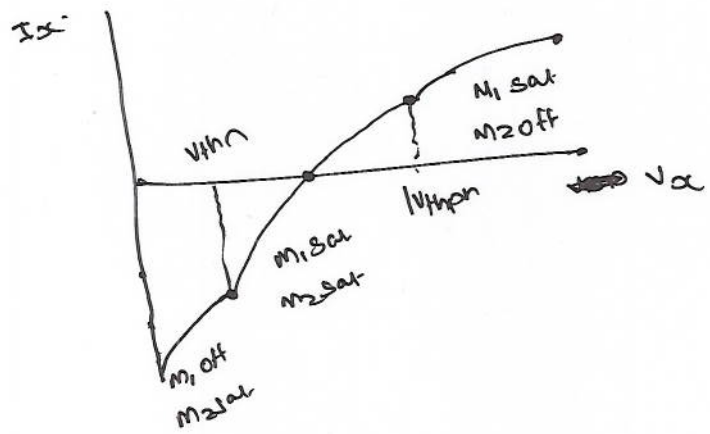
$$I_x = -\frac{\beta_p}{2} (V_{DD} - V_x - |V_{thp}|)^2$$

(ii) For $V_{thn} \leq V_x \leq |V_{thp}|$ both M_1 and M_2 are in saturation

$$I_x + \frac{\beta_p}{2} (V_{DD} - V_x - |V_{thp}|)^2 = \frac{\beta_n}{2} (V_x - V_{thn})^2$$

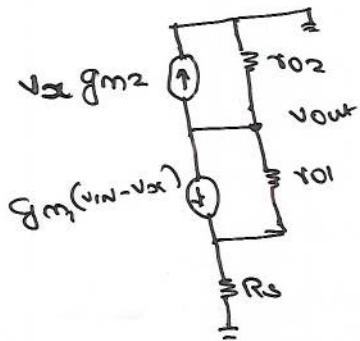
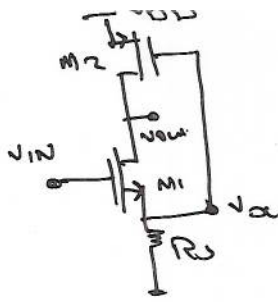
(iii) $V_x \geq |V_{thp}|$, M_2 is off.

$$I_x = \frac{\beta_n}{2} (V_x - V_{thn})^2$$



Each region 1
 total 3.

(3.20 e)



Small signal model - 1
 Analysis - 2 } 1 mark if
 approach in correct
 & final answer
 in wrong.

total (3)

$$g_{m2} V_x + \frac{V_{out}}{r_{O2}} + \frac{V_x}{R_S} = 0$$

$$\Rightarrow \frac{1 + g_{m2} R_S}{R_S} V_x = -\frac{V_{out}}{r_{O2}}$$

$$V_x = -V_{out} \cdot \frac{R_S}{r_{O2}} \cdot \frac{1}{1 + g_{m2} R_S}$$

Also

$$g_{m1} (V_{in} - V_x) + \frac{V_{out} - V_x}{r_{O1}} = \frac{V_x}{R_S}$$

$$g_{m1} V_{in} = g_{m1} V_x + \frac{V_x}{r_{O1}} + \frac{V_x}{R_S} - \frac{V_{out}}{r_{O1}}$$

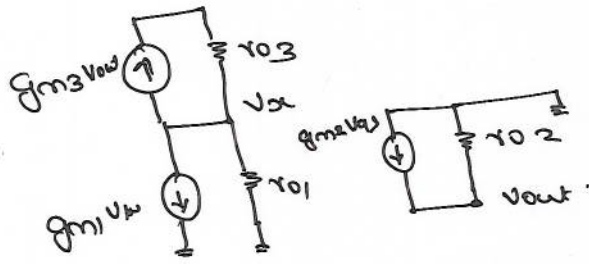
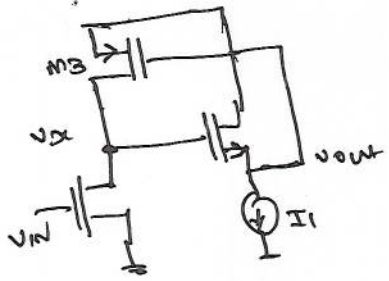
~~$$= \left[\frac{1}{r_{O1}} + \frac{r_{O1} + R_S + g_{m1} r_{O1} R_S}{r_{O1} R_S} \right] V_x - V_{out}$$~~

$$= \left[\frac{1}{r_{O1}} + \frac{r_{O1} + R_S + g_{m1} r_{O1} R_S}{r_{O1} R_S} \right] \frac{R_S}{r_{O2}} \cdot \frac{1}{(g_{m2} R_S + 1)} - V_{out}$$

$$= \left[\frac{(1 + g_{m2} R_S) r_{O2} + r_{O1} + R_S + g_{m1} r_{O1} R_S}{r_{O1} r_{O2} (1 + g_{m2} R_S)} \right] - V_{out}$$

$$\frac{V_{out}}{V_{in}} = \frac{-g_{m1} r_{O1} r_{O2} (1 + g_{m2} R_S)}{r_{O1} + (1 + g_{m1} R_S) r_{O1} + (1 + g_{m2} R_S) r_{O2}}$$

3 Q1h



Small signal model - 1
 Analysis 2 } 1 mark for correct approach

total 3

$$g_{m2}(v_x - v_{out}) = \frac{v_{out}}{r_{o2}}$$

$$g_{m2}v_x = \left[\frac{1 + g_{m2}r_{o2}}{r_{o2}} \right] v_{out}$$

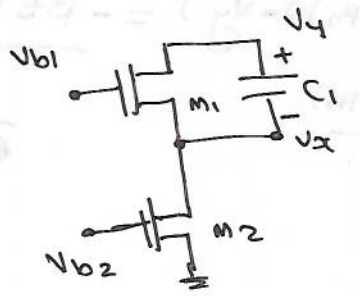
$$v_x = \frac{1 + g_{m2}r_{o2}}{g_{m2}r_{o2}} v_{out}$$

$$g_{m1}v_{in} + \frac{v_x}{r_{o1}} + \frac{v_x}{r_{o3}} + g_{m3}v_{out} = 0$$

$$g_{m1}v_{in} + \left[\left(\frac{1}{r_{o1}} + \frac{1}{r_{o3}} \right) \frac{1 + g_{m2}r_{o2}}{g_{m2}r_{o2}} + g_{m3} \right] v_{out} = 0$$

$$\frac{v_{out}}{v_{in}} = - \frac{g_{m1}}{g_{m3} + \left[\frac{1 + g_{m2}r_{o2}}{g_{m2}r_{o2}} \right] \left[\frac{1}{r_{o1}} + \frac{1}{r_{o3}} \right]}$$

3.22b



current through $M_2 = 0 \Rightarrow V_x = 0$ saturation at $t = 0$

(i) $V_y(0^-) = V_{DD} \Rightarrow M_1$ starts in

$$\frac{\beta}{2} (V_{b1} - V_{th})^2 + C \frac{dV_y}{dt} = 0$$

$$dV_y = - \frac{\beta}{2C} (V_{b1} - V_{th})^2 dt$$

$$V_y = V_{DD} - \frac{\beta}{2C} (V_{b1} - V_{th})^2 t$$

(ii) when $V_y(t=t_1) = V_{b1} - V_{th}$ M_1 enters linear region

$$\Rightarrow V_{DD} - \frac{\beta}{2C} (V_{b1} - V_{th})^2 t_1 = (V_{b1} - V_{th})$$

$$\Rightarrow t_1 = \frac{(V_{DD} - V_{b1} + V_{th})}{\frac{\beta}{2C} (V_{b1} - V_{th})^2}$$

(iii) $t > t_1$ M_1 is in linear region

$$\frac{\beta}{2} [(V_{b1} - V_{th}) V_y - \frac{V_y^2}{2}] + C \frac{dV_y}{dt} = 0$$

$$\frac{dV_y}{V_y [(V_{b1} - V_{th}) - V_y]} = - \frac{\beta}{2C} dt$$

$$\frac{1}{2(V_{b1} - V_{th})} \cdot dV_y \left[\frac{1}{V_y} + \frac{1}{2(V_{b1} - V_{th}) - V_y} \right] = - \frac{\beta}{2C} dt$$

$$u_n(v_y) - u_n(\cancel{v_y}) (v_{b1} - v_{th})^2 = -v_y = \frac{-\beta t (v_{b1} - v_{th})}{c}$$

$$\frac{K v_y}{2(v_{b1} - v_{th}) - v_y} = e^{-\frac{\beta t (v_{b1} - v_{th})}{c}} \quad \text{unk}$$

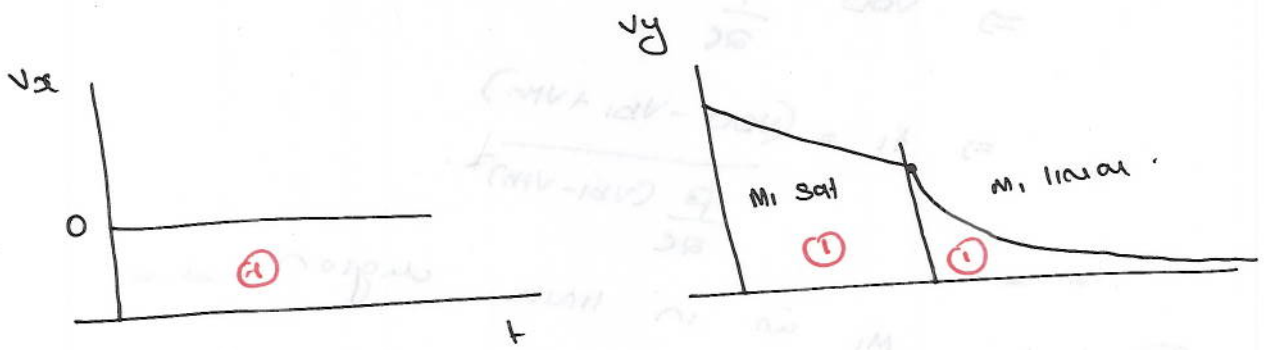
$$t = t_1 \quad v_y = v_b - v_{th}$$

$$\frac{K (v_{b1} - v_{th})}{v_{b1} - v_{th}} = e^{-\frac{\beta t_1 (v_{b1} - v_{th})}{c}}$$

$$\frac{v_y}{2(v_{b1} - v_{th}) - v_y} = e^{-\frac{\beta (t - t_1) (v_{b1} - v_{th})}{c}}$$

$$\frac{2(v_{b1} - v_{th}) - 1}{v_y} = e^{\frac{\beta (t - t_1) (v_{b1} - v_{th})}{c}}$$

$$v_y = \frac{2(v_{b1} - v_{th})}{1 + e^{\frac{\beta (t - t_1) (v_{b1} - v_{th})}{c}}}$$



- 1 mark for v_x .
- 1 mark for each region

total (3)

$$\left[\frac{1}{\beta v - (v_{th} - v_{b1})} + \frac{1}{\beta v} \right] \beta v = \frac{1}{(v_{th} - v_{b1})}$$

② Mos Params

NMOS

$$V_{th} = 476.7m$$

$$\mu_n C_{ox} = 248 \mu A/V^2$$

$$g_{ds} @ 400 \mu A = 167 \mu S$$

PMOS

$$|V_{th}| = 488m$$

$$\mu_p C_{ox} = 102 \mu A/V^2$$

$$g_{ds} @ 400 \mu A = 153 \mu S$$

③ To keep M_A in saturation

$$V_{b2} = V_{gs1} - V_{th4} + V_{gs3}$$

ignoring body effect.

$$= \sqrt{\frac{2I}{\beta_n}} + V_{thn} + \sqrt{\frac{2I}{\beta_p}}$$

$$= 0.754V$$

② $\frac{1}{2}$ approximately close & if expression in correct

④ To keep M_1 in saturation

$$V_{b1} = V_{gs1} - V_{th1} + V_{gs2}$$

$$V_{b1} = V_{DD} - (V_{gs1} - |V_{th1}|) + V_{gs2}$$

$$= 3 - |V_{th1}| + 2 \sqrt{\frac{2I}{\beta_p}}$$

$$= 2.0781V$$

$\frac{1}{2}$ approximately close & if expression in correct.

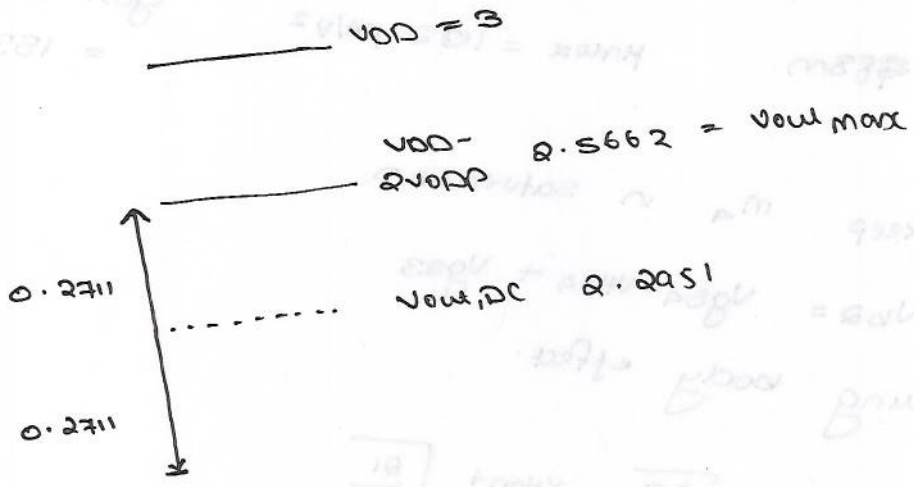
$$\begin{aligned} \text{⑤ } V_{out \text{ (max)}} &= V_{DD} - 2 \times V_{DDP} \\ &= 2.5662 \end{aligned}$$

$$\begin{aligned} V_{out \text{ min}} &= \cancel{V_{DD}} - 2 \times V_{DDN} \\ &= 0.28 \end{aligned}$$

$$\begin{aligned} V_{out \text{ DC}} &= V_{DD} - V_{gs1} \\ &= 2.2951 \end{aligned}$$

So even though the bias points $\{V_{b1}, V_{b2}\}$ are such that all allows $V_{DD} - 4V_{DS}$ swing, the output DC point prevents the swing to

$\pm 0.271V$



①

$V_{DS} = 0.2951$

② $A \approx g_{m1} * (g_{m2} r_{o2} r_{o1} || g_{m3} r_{o3} r_{o4})$

$g_{m1} = \sqrt{2\beta P I} = 3.7mS = g_{m2}$

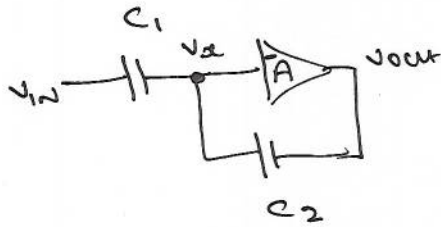
$g_{m3} = \sqrt{\beta_3 2I} = 5.8mS = g_{m4}$

$A \approx 3.7m * \left[\left(3.7m * \frac{1}{(153k)^2} \right) || \left(5.8m * \left(\frac{1}{167k} \right)^2 \right) \right]$

$= 3.7m * (158k || 208k)$

$= 330$

Adjustment may be required to keep m_1 & m_4 in saturation due to body effects of m_2 & m_3



$$(v_{IN} - v_x) C_1 = (v_x - v_{OUT}) C_2$$

$$\begin{aligned} C_1 v_{IN} &= v_x (C_1 + C_2) - v_{OUT} C_2 \\ &= \frac{-v_{OUT} (C_1 + C_2) - v_{OUT} C_2}{A} \end{aligned}$$

$$\frac{v_{OUT}}{v_{IN}} = \frac{C_1}{C_2 + \frac{C_1 + C_2}{A}}$$

$$\Rightarrow \frac{C_1}{C_2} = \underline{\underline{2}}$$

①

- ④ $A \Rightarrow$ ① $1 \Rightarrow$ Netlist / present sch
- ⑥ $Q \Rightarrow$ Bias currents & DC gain
 A reasonable. $\{I \pm 50\% \}$
 $\{A \geq 50\}$.
- ⑦ $1 \Rightarrow$ mid Band gain = $2 \pm 10\%$.

