

**EE215A**

**Midterm Exam**

**Fall 2014**

**Time Limit: 1 hour and 50 minutes**

**Open Book, Open Notes**

**Calculators are allowed.**

**Your Name:**

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**Name of Person to Your Left:**

**Name of Person to Your Right:**

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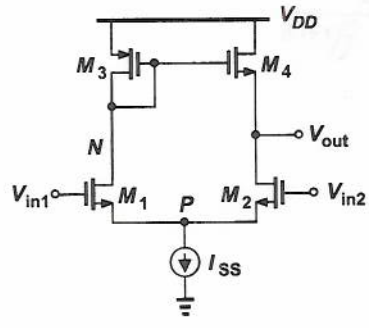
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- 1.
- 2.
- 3.
- 4.



1. In the circuit shown below, an NMOS device has been mistakenly used in the active load. Assume  $\lambda = \gamma = 0$ . Also, assume that  $M_1$  and  $M_2$  are identical and all transistors are in saturation.

- (a) Determine the dc value of the output voltage in terms of the transistor parameters and  $I_{SS}$ . 10  
 (b) Compute the small-signal voltage gain  $V_{out}/(V_{in1} - V_{in2})$ . 10



(a)  $\because \lambda = \gamma = 0$  & all devices are in saturation the tail current will split equally b/w  $M_1$  &  $M_2$

$$\Rightarrow I_3 = I_4 = \frac{I_{SS}}{2}$$

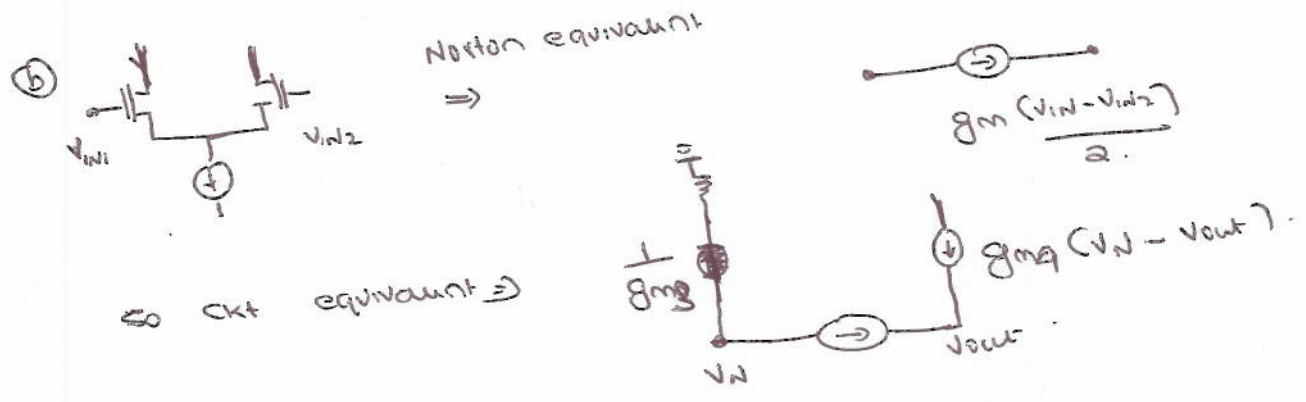
$$\Rightarrow V_N = V_{DD} - \left[ |V_{thp}| + \sqrt{\frac{2 \cdot I_{SS}/2}{\beta_p}} \right]$$

$$\Rightarrow V_{out} = V_N - \left[ V_{thn} + \sqrt{\frac{2 I_{SS}/2}{\beta_n}} \right]$$

$$= V_{DD} - |V_{thp}| - V_{thn} - \sqrt{I_{SS}/\beta_p} - \sqrt{I_{SS}/\beta_n}$$

$$\beta_p = \mu_n C_{ox} (w/L)_p$$

$$\beta_n = \mu_p C_{ox} (w/L)_n$$



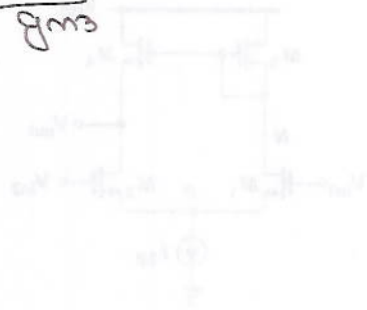
$$V_N = -\frac{g_{m1}(V_{in1} - V_{in2})}{2} \times \frac{1}{g_{mp}}$$

$$g_{m4}(V_N - V_{out}) + g_{m1} \frac{(V_{in1} - V_{in2})}{2} = 0$$

$$g_{m4} v_{out} = -\frac{g_{m4}}{g_{m3}} \cdot \frac{g_{m1}}{2} (v_{i1} - v_{i2}) + \frac{g_{m1}}{2} (v_{i1} - v_{i2})$$

$$= \left(1 - \frac{g_{m4}}{g_{m3}}\right) \cdot \frac{g_{m1}}{2} (v_{i1} - v_{i2})$$

$$\frac{v_{out}}{v_{i1} - v_{i2}} = \frac{g_{m1}}{2g_{m3}} \left(1 - \frac{g_{m4}}{g_{m3}}\right)$$



... to current mirror ...

... (1/2) ...

$$\left[ \frac{g_{m1}}{2} + g_{m3} \right] v_{out} = g_{m1} v_{id}$$

$$v_{out} = \frac{g_{m1}}{g_{m1} + 2g_{m3}} v_{id}$$

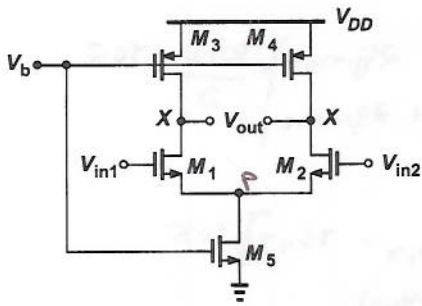


2. In the circuit shown below, the gates of the tail and PMOS current sources are mistakenly shorted.

(a) Is it possible to choose  $V_b$ , the input CM level, the output CM level, and the tail current such that all transistors are in saturation? Explain in detail why or why not.

(b) Assuming all transistors are in saturation, compute the small-signal gain from  $V_{DD}$  to the output CM level. You can neglect body effect.

10  
10



$$V_p = V_{inCM} - V_{thN} - \sqrt{\frac{I_S}{\beta_1}}$$

For a given  $V_b$

For  $M_5$  to be in sat-

$$V_p > V_b - V_{thN}$$

$$\Rightarrow V_{inCM} - \sqrt{\frac{I_S}{\beta_1}} > V_b$$

$$\Rightarrow V_{inCM} > V_b + \sqrt{\frac{I_S}{\beta_1}}$$

$$I_S = I_{M5}$$

$$\beta_1 = \mu_n C_{ox} (W/L)_{M1,2}$$

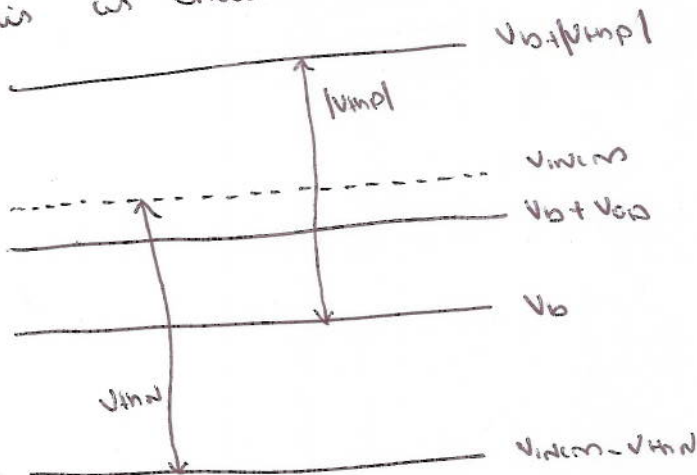
||y For  $M_{1,2}$  to be in saturation  $\curvearrowright$

$$V_{outCM} > V_{inCM} - V_{thN}$$

& For  $M_{3,4}$  to be in saturation  $\curvearrowright$

$$V_{outCM} < V_b + |V_{thP}|$$

This is shown below



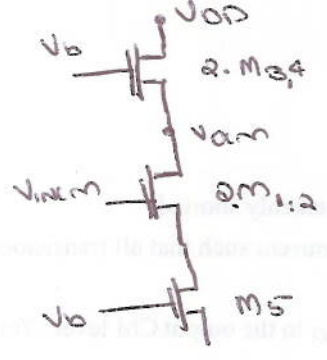
$$\Rightarrow \textcircled{a} V_b + \sqrt{\frac{I_S}{\beta_1}} \leq V_{inCM} \leq V_b + |V_{thP}| + V_{thN}$$

$$\textcircled{b} V_{inCM} - V_{thN} \leq V_{outCM} \leq V_b + |V_{thP}|$$

there exist such a solution.

$$\textcircled{c} V_b > V_{thN} \Rightarrow V_b < V_{DD} - |V_{thP}|$$

(b)



common mode equivalent

gain is like that of a cascode with load impedance

$$R_D = \frac{r_{o12}}{2} + r_{o5} + \left( \frac{2g_{m1,2}}{1 + 2g_{mb1,2}} \right) \cdot \frac{r_{o12}}{2} \cdot r_{o5}$$

$$= \frac{r_{o12}}{2} + r_{o5} + \left[ \frac{2(g_{m1,2} \cdot r_{o1,2})}{1 + 2g_{mb1,2}} \right] r_{o5}$$

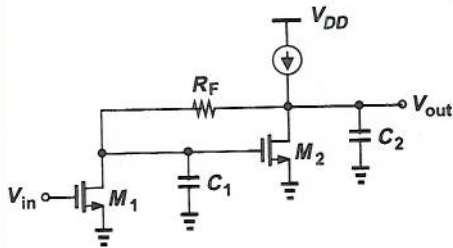
⇒ gain from VDD to output common mode

$$= \frac{1 + (g_{m3,4} + g_{mb3,4}) (r_{o3,4})}{\frac{r_{o3,4}}{2} + R_D} \cdot R_D$$

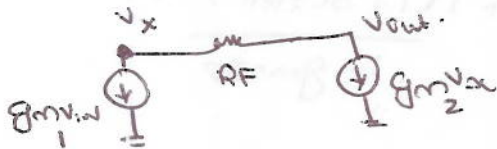

3. The circuit shown below is called the "Cherry-Hooper" amplifier. Neglecting channel-length modulation and all other capacitances,

(a) Determine the low-frequency voltage gain of the circuit.

(b) Without using Miller's theorem, compute the transfer function of the circuit. Verify that it reduces to the result obtained in (a) if  $s = 0$ .



(a) small signal equivalent at low frequency.



$$g_{m1} v_{in} + g_{m2} v_x = 0$$

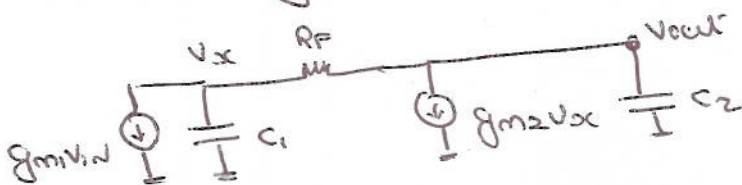
$$v_x = -\frac{g_{m1} v_{in}}{g_{m2}}$$

also  $\frac{(v_x - v_{out})}{R_F} = g_{m2} v_x$

$$v_{out} = v_x (1 - g_{m2} R_F)$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = -\frac{g_{m1} (1 - g_{m2} R_F)}{g_{m2}}$$

(b) small signal model



$$g_{m1}v_{in} + v_x s c_1 + \frac{(v_x - v_{out})}{R_F} = 0 \quad - (1)$$

$$g_{m2}v_x + v_{out} s c_2 + \frac{(v_{out} - v_x)}{R_F} = 0 \quad - (2)$$

$$(2) \Rightarrow v_{out} \frac{(1 + s c_2 R_F)}{R_F} = v_x \frac{(1 - g_{m2} R_F)}{R_F}$$

$$v_x = \frac{(1 - g_{m2} R_F) v_{out}}{1 + s c_2 R_F}$$

$$v_x = \frac{v_{out} (1 + s c_2 R_F)}{1 - g_{m2} R_F}$$

$$(1) \Rightarrow R_F g_{m1} v_{in} + \frac{(1 + s c_1 R_F)(1 + s c_2 R_F) v_{out}}{1 - g_{m2} R_F} - v_{out} = 0$$

$$v_{out} = g_{m2} R_F v_x$$

$$R_F g_{m1} v_{in} (1 - g_{m2} R_F) + v_{out} \left[ (1 + s c_1 R_F)(1 + s c_2 R_F) - (1 - g_{m2} R_F) \right] = 0$$

$$\frac{v_{out}}{v_{in}} = \frac{-g_{m1} R_F (1 - g_{m2} R_F)}{g_{m2} R_F + s(c_1 + c_2) R_F + s^2 c_1 c_2 R_F^2}$$

$$s = 0$$

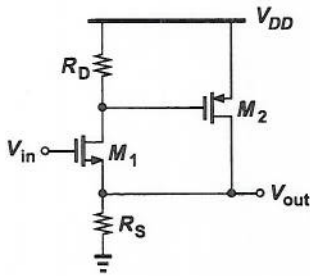
$$\Rightarrow \frac{v_{out}}{v_{in}} = \frac{-g_{m1} (1 - g_{m2} R_F)}{g_{m2}} = \text{Solution } \uparrow \text{ part a.}$$



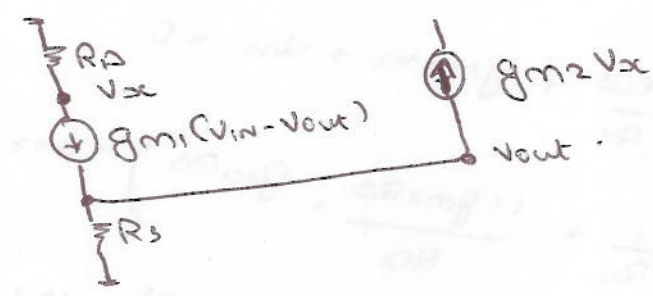
4. Consider the circuit shown below. Assume channel-length modulation and body effect are negligible. Also, neglect the noise contributed by the resistors.

- (a) Determine the voltage gain.
- (b) Compute the total output noise voltage.
- (c) Determine the input-referred noise voltage.

8  
5+5  
2  
 $\gamma = 0 \quad \chi = 0$



① small signal equivalent.



$$\frac{V_x}{R_D} + g_{m2} V_x + \frac{V_{out}}{R_S} = 0$$

$$\Rightarrow V_x \left( 1 + g_{m2} R_D \right) + \frac{V_{out}}{R_S} = 0$$

$$\Rightarrow V_x = -V_{out} \cdot \frac{R_D}{R_S} \cdot \frac{1}{1 + g_{m2} R_D}$$

Also  $g_{m1} (V_{in} - V_{out}) = -\frac{V_x}{R_D} = \frac{V_{out}}{R_S (1 + g_{m2} R_D)}$

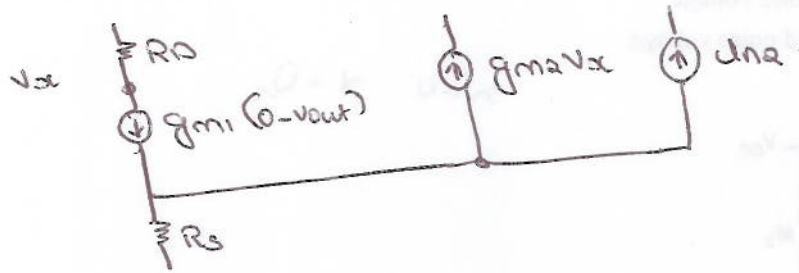
$$g_{m1} V_{in} = \left[ g_{m1} + \frac{1}{R_S + g_{m2} R_D R_S} \right] V_{out}$$

$$\frac{V_{out}}{V_{in}} = \frac{g_{m1} R_S + g_{m1} g_{m2} R_D R_S}{1 + g_{m1} R_S + g_{m1} g_{m2} R_D R_S}$$

⑥ Noise due to  $M_1$  at output

$$V_{nout1}^2 = \frac{4KT\gamma}{g_{m1}} \cdot \left[ \frac{g_{m1}R_S + g_{m1}g_{m2}R_S R_D}{1 + g_{m1}R_S + g_{m1}g_{m2}R_S R_D} \right]^2$$

Noise due to  $M_2$  at output



$$g_{m1}(0 - V_{out}) + \frac{V_x}{R_D} = 0$$

$$V_x = g_{m1}R_D V_{out}$$

Also

$$\frac{V_x}{R_D} + \frac{V_{out}}{R_S} + g_{m2}V_x + I_{n2} = 0$$

$$V_{out} \left( \frac{1}{R_S} + \frac{1 + g_{m2}R_D}{R_D} \cdot g_{m1}R_D \right) + I_{n2} = 0$$

$$V_{out} \left( \frac{1}{R_S} + g_{m1}(1 + g_{m2}R_D) \right) + I_{n2} = 0$$

$$V_{out} = \frac{I_{n2} R_S}{1 + g_{m1}R_S + g_{m1}g_{m2}R_S R_D}$$

$$V_{nout2}^2 = 4KT\gamma g_{m2} \frac{R_S^2}{(1 + g_{m1}R_S + g_{m1}g_{m2}R_S R_D)^2}$$

Total noise = ~~4KT\gamma~~  $V_{nout2}^2 + V_{nout1}^2$

$$\begin{aligned} \text{⑦ I/P referred noise} &= \frac{4KT\gamma}{g_{m1}} + \frac{4KT\gamma g_{m2} R_S^2}{(g_{m1}R_S + g_{m1}R_S g_{m2}R_D)^2} \\ &= \frac{4KT\gamma}{g_{m1}} + \frac{4KT\gamma g_{m2}}{g_{m1}^2} \cdot \frac{1}{(1 + g_{m2}R_D)^2} \end{aligned}$$