EE215C

Final Exam

Winter 2009

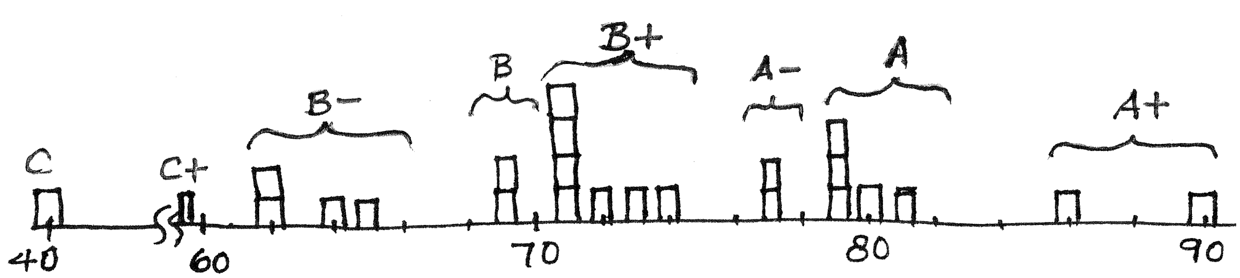
Name: Solutions

Time Limit: 3 Hours

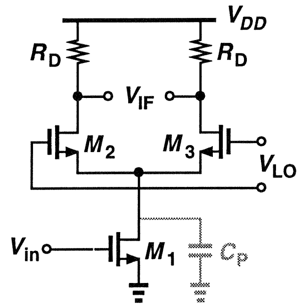
Open Book, Open Notes

1. 20
2. 10
3. 10
4. 10
5. 15

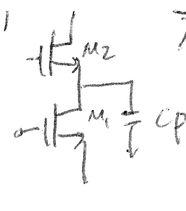
Total: 65



1. Consider the active mixer shown below, where all other capacitances are neglected and no transistor enters the triode region. Also, channel-length modulation and body effect are negligible.
(a) Suppose M_2 is on and M_3 is off. Derive an expression for the small-signal drain current of M_2 due to the thermal noise of M_2. Plot the spectrum of the drain current. Neglect all other sources of noise.
(b) Now suppose the impedance of C_P is much larger than $1/g_{m2}$ at the frequencies of interest, and the LO is applied with 50% duty cycle. If M_2 and M_3 switch abruptly, and only the first and third harmonics of the LO are considered, explain in detail how the thermal noise of M_2 appears at IF. Sketch the IF noise spectrum due to the first and third harmonics of the LO. Neglect all other sources of noise.



(a)

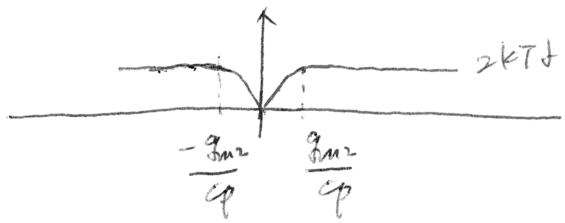


$$\overline{i_{dn}^2} = 4kT\gamma g_{m2} \left| \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m2}} + \frac{1}{sC_p}} \right|^2$$

$$= 4kT\gamma g_{m2} \frac{\omega^2 C_p^2}{\omega^2 C_p^2 + g_{m2}^2} \quad (6)$$

#

spectrum:



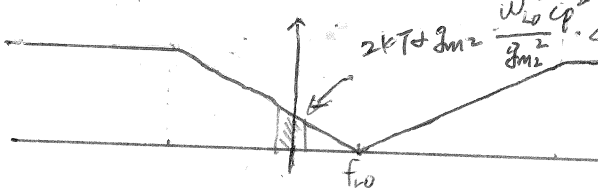
(4)

#

b) time domain: square wave × noise

⇒ frequency domain:

for $+f_{LO}$

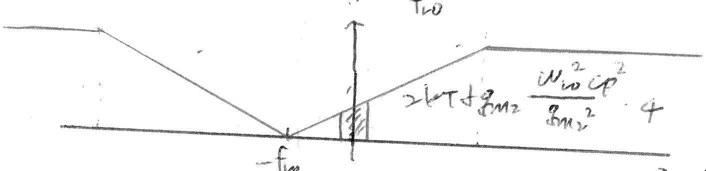


$\because \frac{1}{\omega C_p} \gg \frac{1}{g_{m2}}$

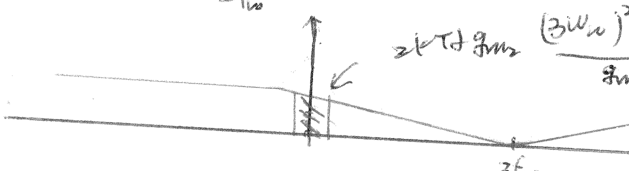
$\Rightarrow \omega C_p \ll g_{m2}$

$\therefore \overline{i_{d2}^2} = 4kT\gamma g_m \frac{(\omega C_p)^2}{g_{m2}^2}$

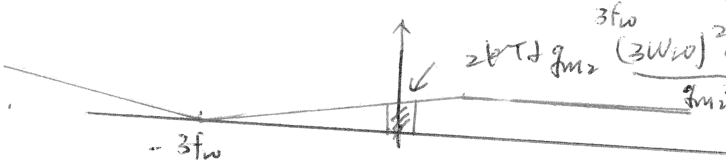
for $-f_{LO}$



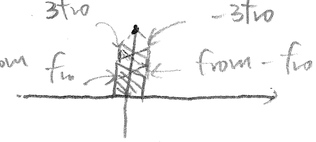
for $3f_{LO}$



for $-3f_{LO}$.



$\therefore$ the total noise at IF $\doteqdot 2kT\gamma g_{m2} \frac{\omega_{LO}^2 C_p^2}{g_{m2}^2} \cdot 16$ #

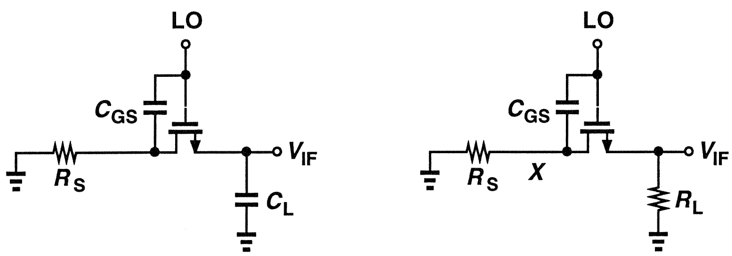


the 3rd harmonic contributes the same noise as the 1st one

2. A student considers the arrangement shown on the left, where C_{GS} introduces LO leakge at the input and hence a dc component at the output. (Note that the capacitor does not carry a dc current.) The student then decides that the arrangement on the right is *free* from dc offsets, reasoning that a dc voltage, V_{ds}, at the output would require a dc current, V_{dc}/R_L, through R_L and hence an equal current through R_S. But this is impossible because a dc current through R_S gives rise to a negative voltage at node X.

5 (a) Explain in detail whether the student's conjecture is correct and why.

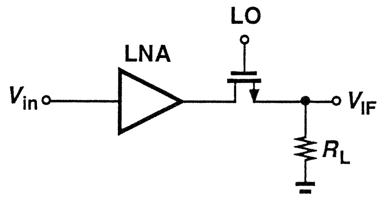
5 (b) Neglecting C_{GS}, calculate the voltage conversion gain of the circuit on the right. Assume a square-wave LO with 50% duty cycle. Also, assume the switch has an on-resistance of R_{on}.



(a) The flaw in the student's reasoning is that a dc current at the output requires a dc current at the input. In other words, KCL only holds for all of the frequency components summed together – not for each individual component. Thus, both circuits suffer from dc offsets.

(b) The mixer operates as an ideal return-to-zero topology, except that R_{on} and R_L attenuate the signal when the switch is on. That is, the input signal is multiplied by a square wave toggling between 0 and $\dfrac{R_L}{R_L + R_{on} + R_S}$. The conversion gain is therefore equal to $\dfrac{1}{\pi}\dfrac{R_L}{R_L + R_{on} + R_S}$.

3. Shown below is part of a direct-conversion receiver. Suppose the LNA input-output characteristic can be expressed as $y(t) = \alpha_1 x(t) + \alpha_2 x^2(t)$. If the mixer switches abruptly with a 50% duty cycle and if R_L is much greater than the on-resistance of the switch and the output resistance of the LNA, compute the IP_2 of the receiver.



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The LO signal can be represented as $\frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{2}\right) \cos(n\omega t)$

for a two tone test we have $V_{in} = A(\cos\omega_1 t + \cos\omega_2 t)$

$\therefore$ we have $V_{IF} = \left[\alpha_1 A(\cos\omega_1 t + \cos\omega_2 t) + \alpha_2 A^2(\cos\omega_1 t + \cos\omega_2 t)^2\right]\left[\frac{1}{2} + \frac{2}{\pi}\cos(\omega_{LO} t) + \ldots\right]$

for ω_2 slightly higher than ω_1, $(\omega_2 - \omega_1)$ component at the output of the LNA appears at the output of the mixer multiplied by its DC gain and located close to DC.

$\omega_2 - \omega_1$: @LO output $\alpha_2 A^2 \cos(\omega_2 - \omega_1)$

2

@ Mixer output $\frac{1}{2}\alpha_2 A^2 \cos(\omega_2 - \omega_1)$

This resides in the same band as the converted RF signal $\frac{\alpha_1 A}{\pi}$ 2

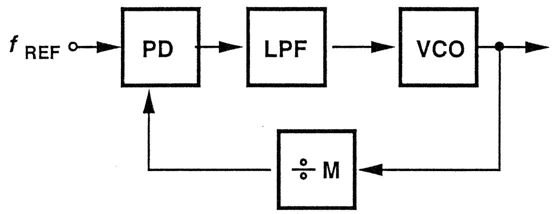
At the IP2 point $\frac{\alpha_1 A_{IP_2}}{\pi} = \frac{\alpha_2 A_{IP_2}^2}{2}$ 2

$\therefore A_{IP_2} = \frac{\alpha_1}{\alpha_2}\frac{2}{\pi}$ 2

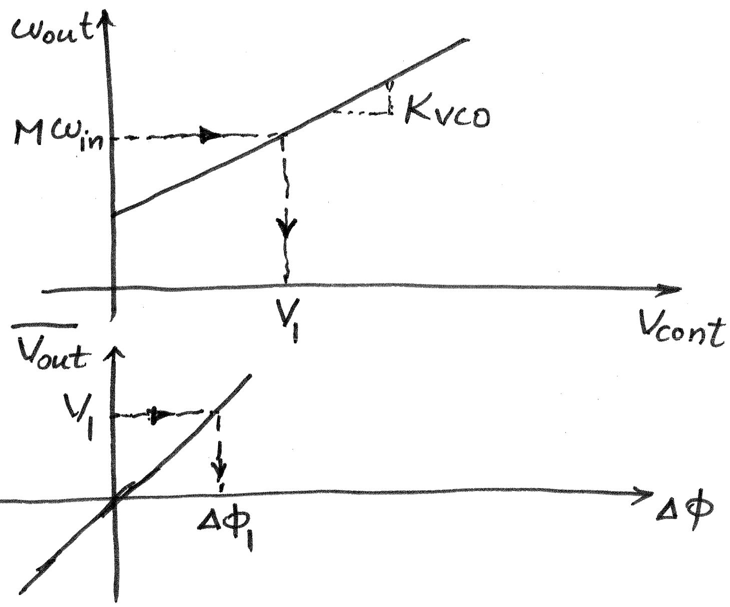
4. Consider the simple type-I PLL shown below. Assume the loop is locked.

(a) If the input frequency is known, describe in detail how you compute the control voltage and the phase error (between the two inputs of the PD).

(b) If the input frequency changes by $\Delta\omega$, compute the change in the phase error.



(a)



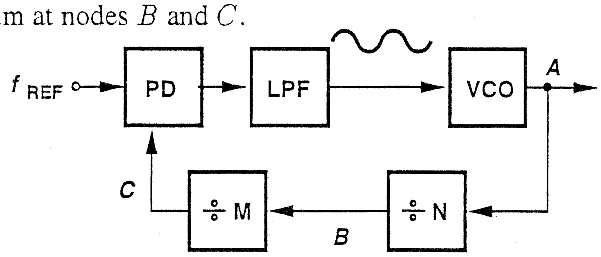
(b) If the input frequency changes by $\Delta\omega$, ω_{out} changes by $M\Delta\omega$ and the control voltage by $\frac{M\Delta\omega}{K_{VCO}}$. Thus, the input phase error changes by $\frac{M\Delta\omega}{K_{VCO}K_{PD}}$.

5. A type-I PLL is shown below, where the control voltage experiences a sinusoidal ripple with a frequency of f_{REF} and a peak amplitude of V_r. Assume the loop is locked.

(a) Explain the difference between a harmonic and a sideband.

(b) Using the narrow-band FM approximation, determine the spectrum at point A.

(c) Now determine the spectrum at nodes B and C.



(a) HARMONICS – A frequency component in a spectrum that is an integer multiple of the fundamental frequency. Appears when a sinusoid is applied to a non-linear system.

SIDEBANDS – Deterministic frequency component that appears around the carrier frequency. A generic example are tones produced by a modulation process of the original signal.

(b) (NOTE) Narrowband FM Approx.

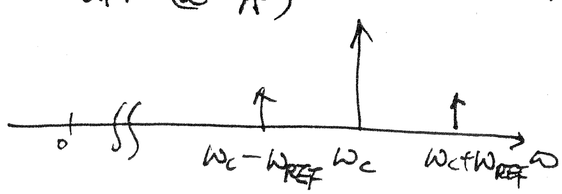
$$x_{FM}(t) = A_c \cos\left[\omega_c t + m\int_{-\infty}^{t} x_{BB}(t)\,dt\right] \simeq A_c \cos\omega_c t - A_c \sin\omega_c t \int x_{BB}(t)\,dt.$$

given $x_{BB}(t) = V_r \cos(\omega_{REF} t)$, $\quad \omega_{REF} = 2\pi f_{REF}$

$$x_{FM}(t) \cong A_c \cos(\omega_c t) - \frac{A_c V_r m}{\omega_{REF}} \sin(\omega_c t)\sin(\omega_{REF} t)$$

$$= A_c \cos(\omega_c t) - \frac{A_c V_r m}{2\omega_{REF}} \cos(\omega_c - \omega_{REF})t + \frac{A_c V_r m}{2\omega_{REF}} \cos(\omega_c + \omega_{REF})t_{//}$$

(SPECTRUM @ A)



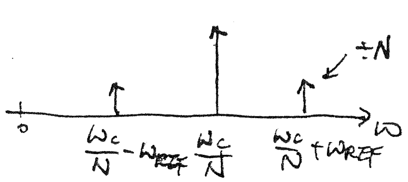
(c) When FM modulated signal is applied to a divider, (÷N)

fundamental : freq ÷ N

side band : Amp ÷ N

$$x_{FM} = A_c \cos\Big(\underbrace{\omega_c t + m\int x_{BB}(t)\,dt}_{\div N}\Big).$$

∴ @ B



@ C