Phase Noise

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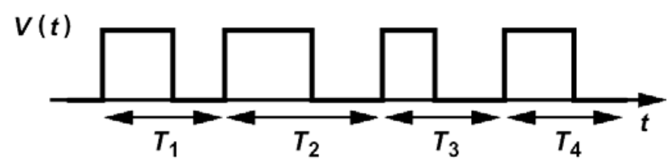
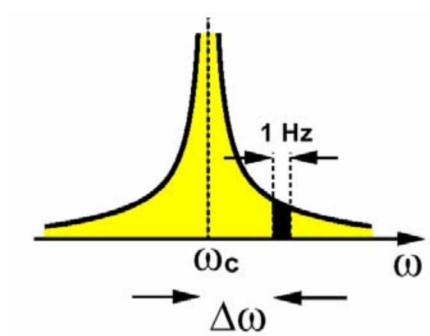
If you are phase-noise challenged ...

- **Phase noise is random variation of period of a nominally-periodic waveform:**

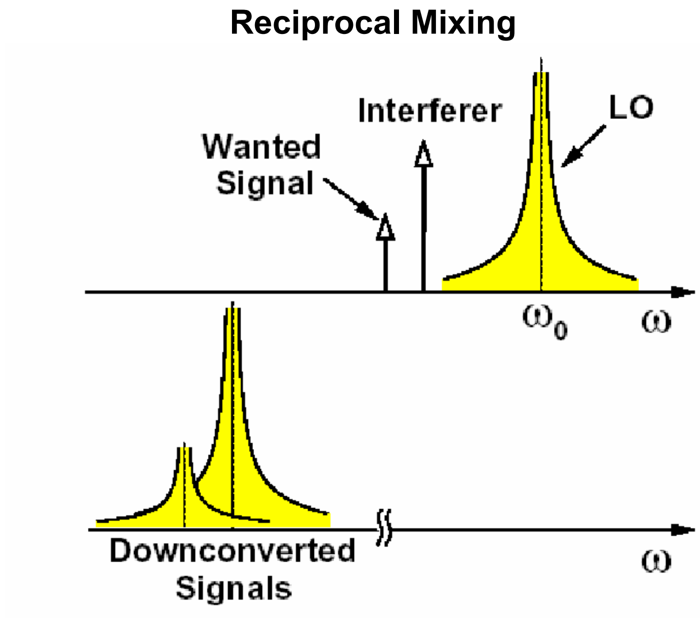


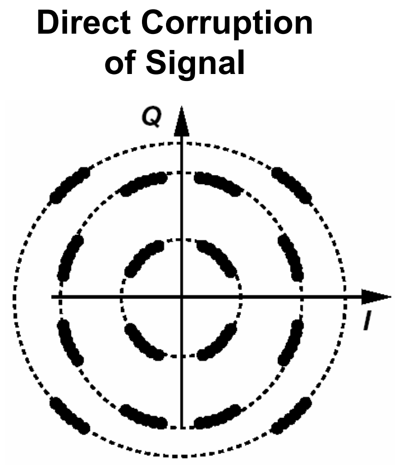
$$A\cos[\omega_c t + \phi_n(t)]$$
$$\approx A\cos\omega_c t - A\phi_n(t)\sin\omega_c t$$



- **Phase noise is measured in dBc/Hz at a certain frequency offset.**

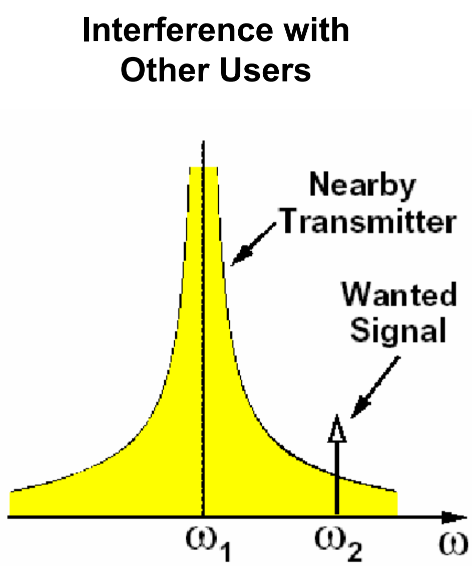
Effect of Phase Noise on RX

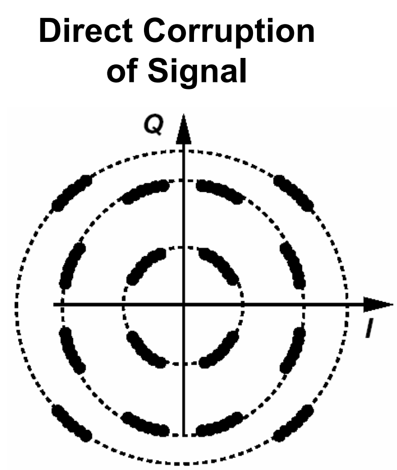




- In typical systems, if phase noise is low enough to make reciprocal mixing negligible, corruption of constellation is also negligible.

Effect of Phase Noise on TX

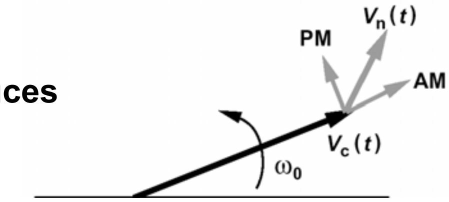


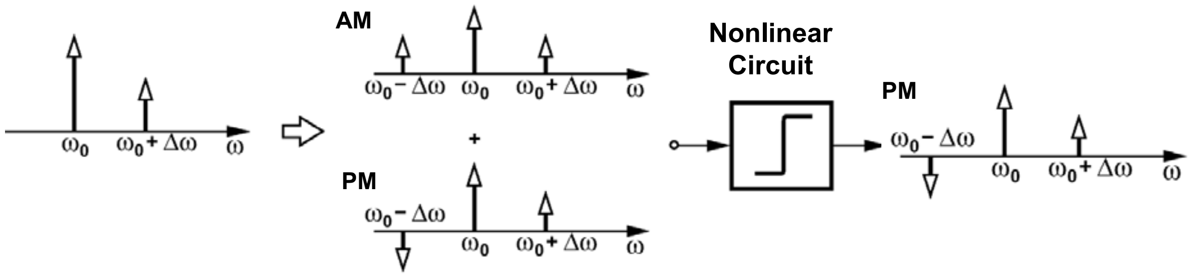


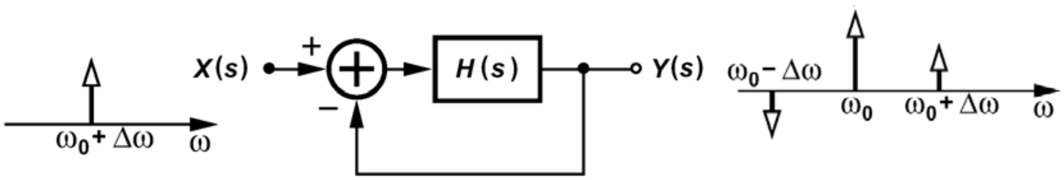
- Except for GSM, cellular standards place tougher phase noise specs on RX than on TX.
- In IEEE802.11a/g (with 64QAM), TX phase noise is as stringent as that of RX.

Injection of Noise into An Oscillator

- Noise injected into an oscillator introduces both phase and amplitude modulation:



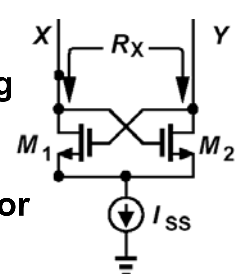




- The AM component is suppressed by the loop nonlinearity (and subsequent stages).

Analysis of Phase Noise

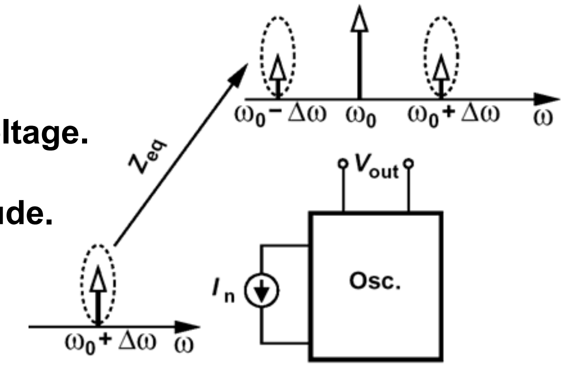
- Tens of papers have been published on phase noise in oscillators. Many mechanisms result in phase noise.
- No single approach has been sufficient to give insight into all mechanisms.
- We follow two approaches here:
  - Approach I: based on time averages →
    - (a) the average spectrum of noise of a device while the noise spectrum varies with time.
    - (b) the "average resistance," defined as the "dc" term in the Fourier series of a periodically-varying resistance.



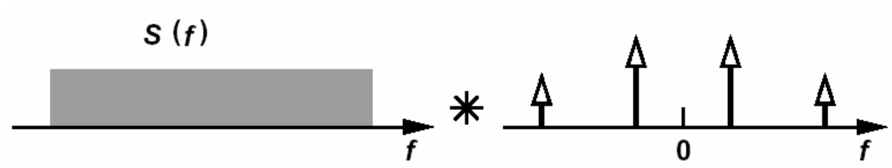
  - Approach II: based on phase response of an oscillator to an injected impulse in the time domain [Hajimiri & Lee, JSSC, Feb. 98].

Phase Noise Analysis: Approach I

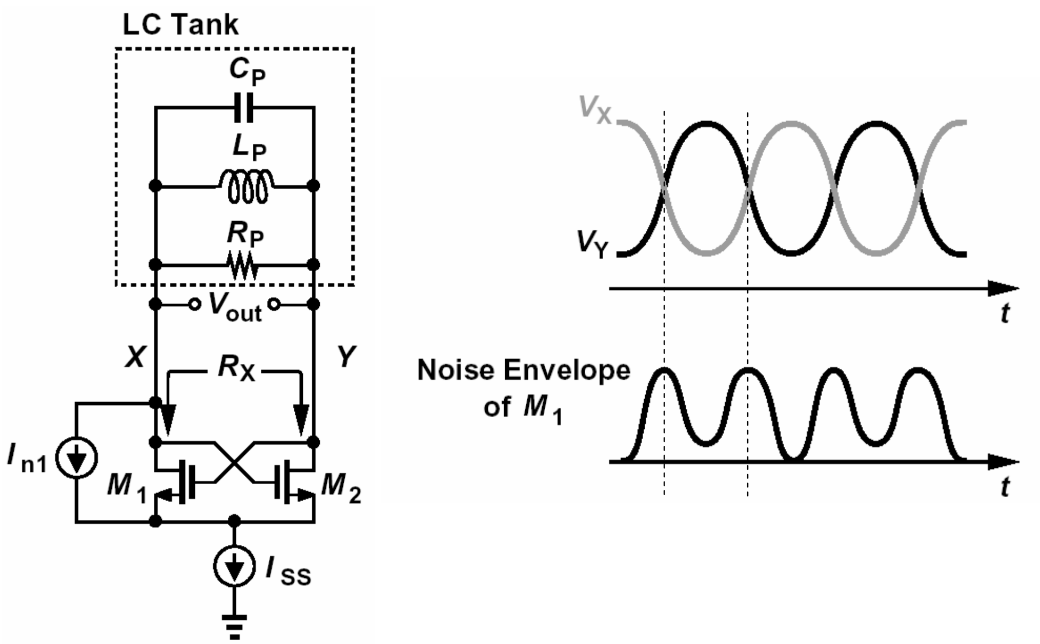
- Three steps:
1. Determine the transfer function from injected noise to output voltage.
2. Multiply by device noise.
3. Normalize to oscillation amplitude.



- Periodially-switched white noise is white:

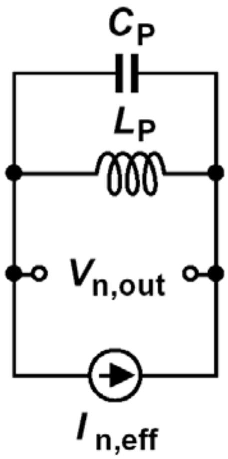


Example



- Average value of Rx is equal to –Rp.
- Noise of M1 and M2 is modulated periodically.
- Maximum noise is injected at zero crossings.

Example of Phase Noise Calculation



$$\frac{\overline{V_{n,out}^2}}{\overline{I_{n,eff}^2}} = \frac{L_P^2\,\omega^2}{(1 - L_P C_P \omega^2)^2}$$

$$= \frac{R_P^2}{Q^2 (2 L_P C_P \Delta\omega)^2}$$

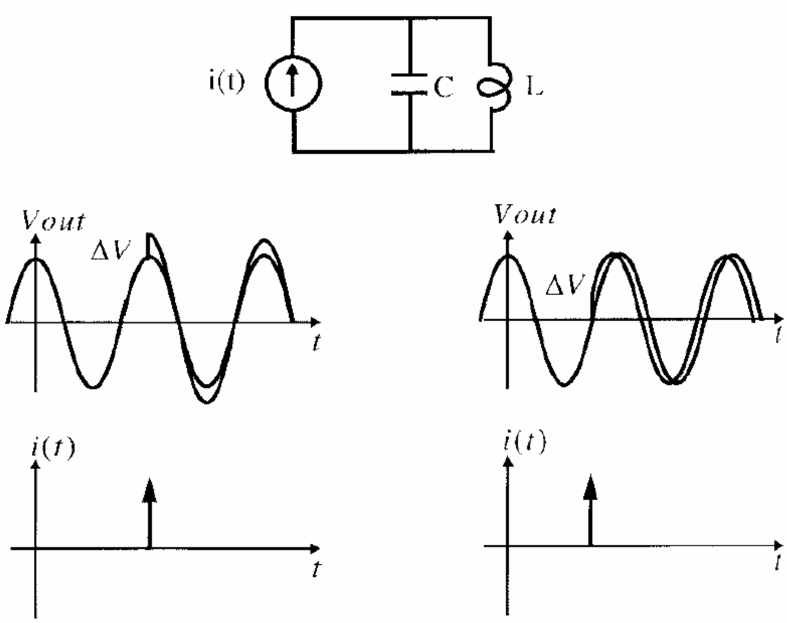
$$= \frac{R_P^2\,\omega_0^2}{4Q^2 (\Delta\omega)^2}$$

$$\omega = \omega_0 + \Delta\omega$$

- **Need to multiply by spectrum of $I_{n,eff}$ and normalize to carrier power, $(2/\pi)^2 I_{SS}^2 R_p^2$.**

Phase Noise Analysis: Approach II

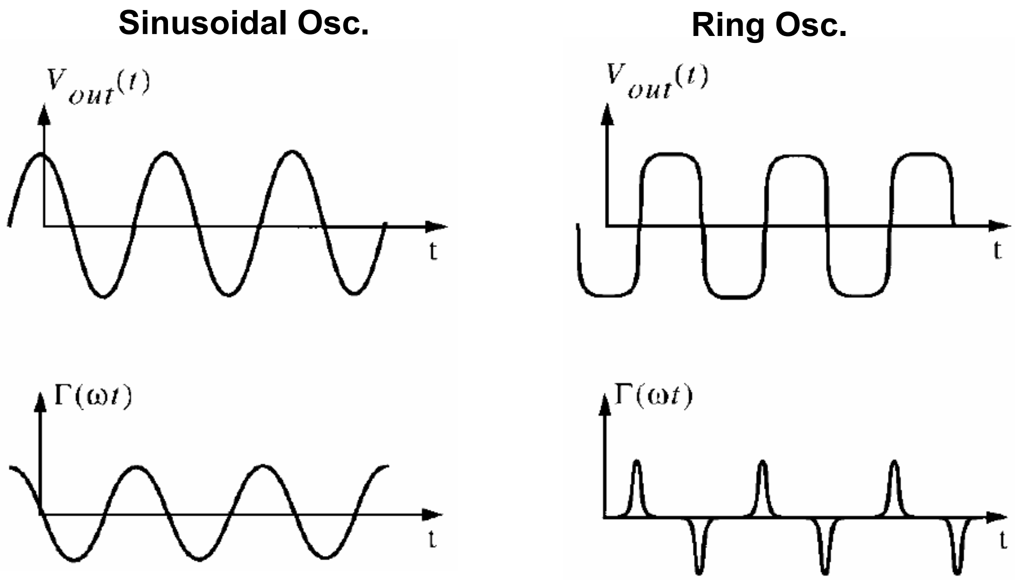
- **An impulse of current changes the phase and/or amplitude depending on when it is injected.**



[Hajimiri & Lee, JSSC, Feb. 98]

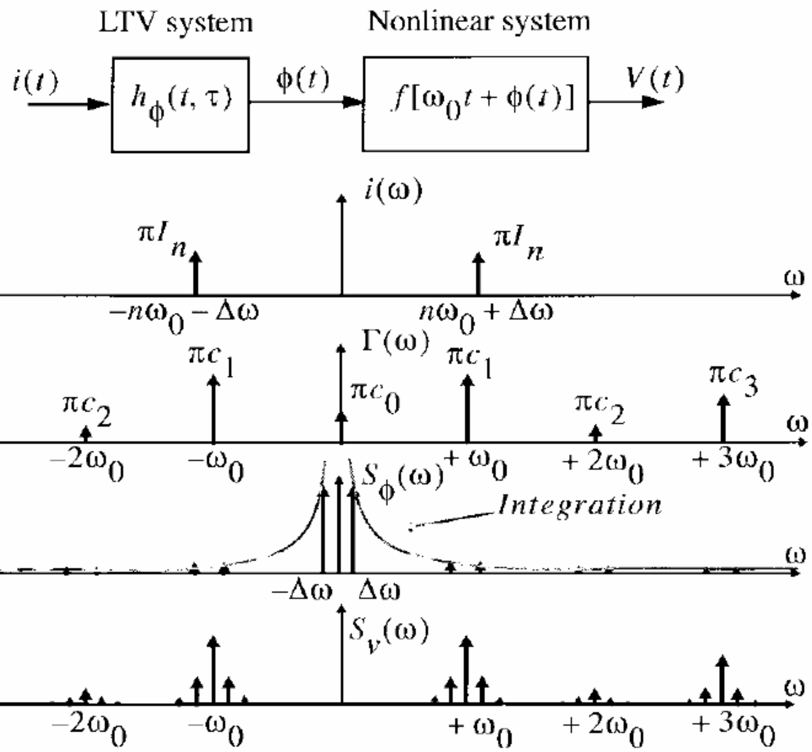
Impulse Sensitivity Function

- ISF is obtained by injecting an impulse whose arrival time slides along the period of oscillation.



[Hajimiri & Lee, JSSC, Feb. 98]

Conversion of Noise to Phase Noise



[Hajimiri & Lee, JSSC, Feb. 98]

Computation of Phase Noise

- Since ISF is periodic, it can be expanded as a Fourier series:

$$\Gamma(\omega_0\tau) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_0\tau + \theta_n)$$

- Phase modulation is obtained by convolving injected noise with ISF:

$$\phi_n(t) = \frac{1}{q_{\max}}\left[\frac{c_0}{2}\int_{-\infty}^{t} i(\tau)\,d\tau + \sum_{n=1}^{\infty} c_n \int_{-\infty}^{t} i(\tau)\cos(n\omega_0\tau)\,d\tau\right]$$

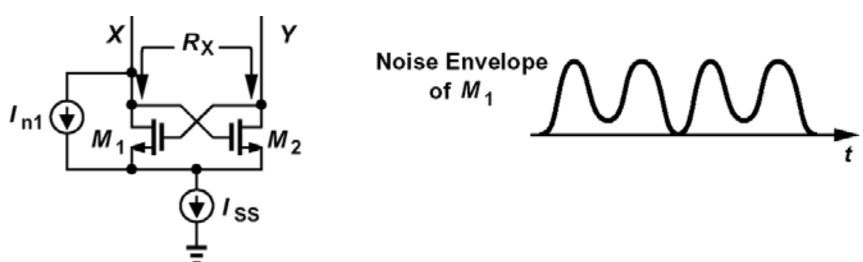
- Phase noise spectrum is obtained by noting that:

$$\begin{aligned} x(t) &= A\cos[\omega_c t + \phi_n(t)] \\ &\approx A\cos\omega_c t - A\phi_n(t)\sin\omega_c t \end{aligned}$$

[Hajimiri & Lee, JSSC, Feb. 98]

Cyclostationary Noise

- If noise envelope of a device varies periodically, the ISF can absorb the periodic behavior:

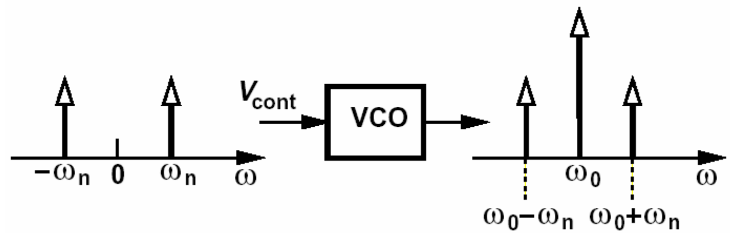


$$i_n(t) = i_{n0}(t)\cdot\alpha(\omega_0 t)$$

$$\begin{aligned} \phi(t) &= \int_{-\infty}^{t} i_n(\tau)\frac{\Gamma(\omega_0\tau)}{q_{\max}}\,d\tau \\ &= \int_{-\infty}^{t} i_{n0}(\tau)\frac{\alpha(\omega_0\tau)\Gamma(\omega_0\tau)}{q_{\max}}\,d\tau \end{aligned}$$

[Hajimiri & Lee, JSSC, Feb. 98]

Sidebands Due to Modulation

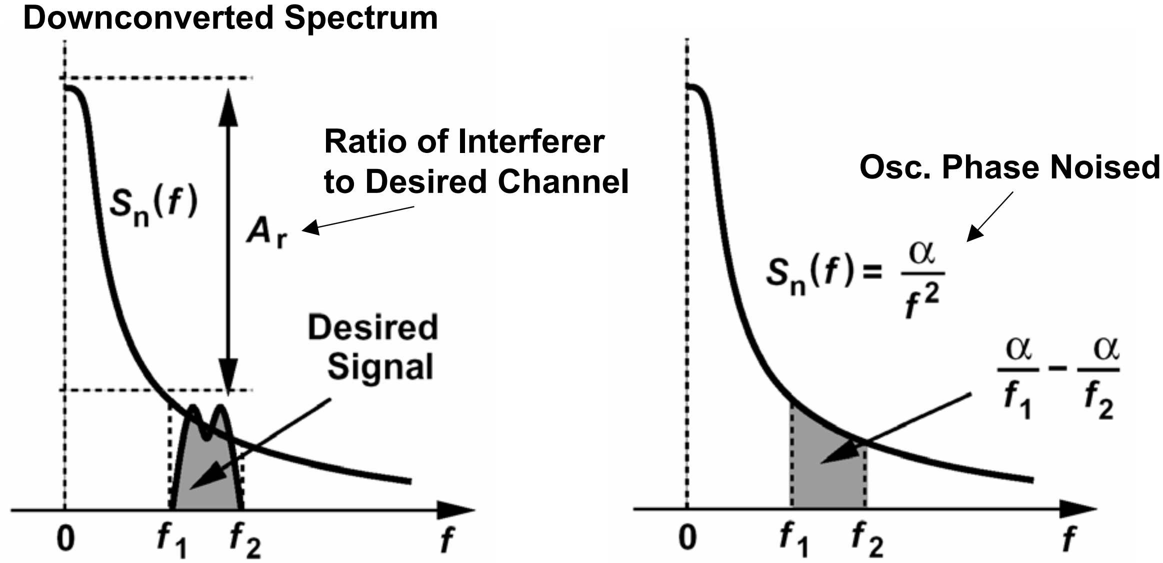


- **Period modulation of oscillator frequency results in spurs.**
- **Sources of spurs:**
  - **Charge pump activity**
  - **Reference coupling through the substrate and supply**
  - **Fractional periodicity in fractional-N synthesizers.**

Computation of Phase Noise Requirements

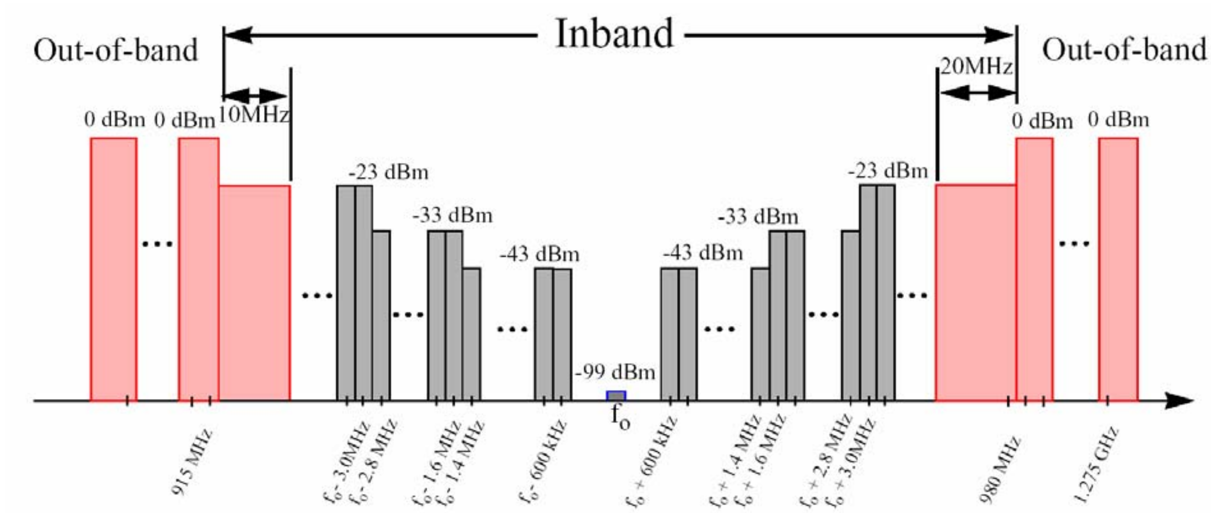
- **Assume a narrow-band interferer at zero frequency offset and a desired channel from f1 to f2:**

Downconverted Spectrum



$$\text{SNR} = \frac{P_{sig}}{P_{carr}\left(\frac{\alpha}{f_1} - \frac{\alpha}{f_2}\right)} \Rightarrow \alpha = \frac{1}{A_r\left(\frac{1}{f_1} - \frac{1}{f_2}\right)\text{SNR}}$$

GSM Example



GSM RX Phase Noise Computation

- Assume SNR=20 dB to leave margin for other imperfections.