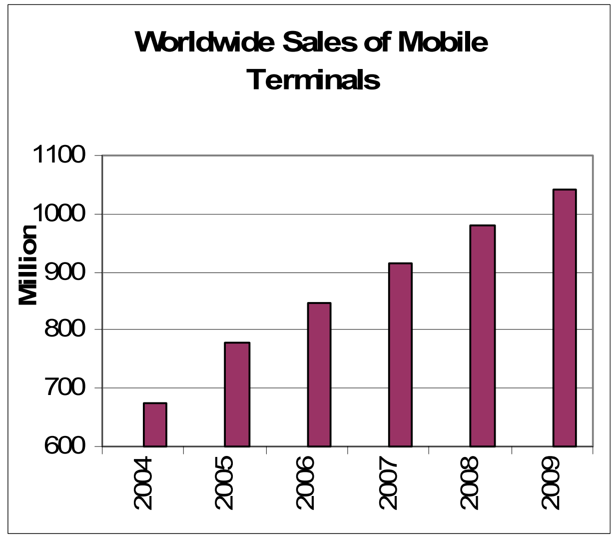
B. Razavi

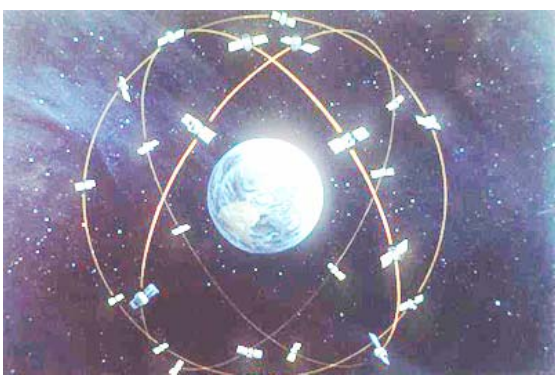
Introduction to RF and Wireless

• **Wireless is everywhere …**

Cellphones:

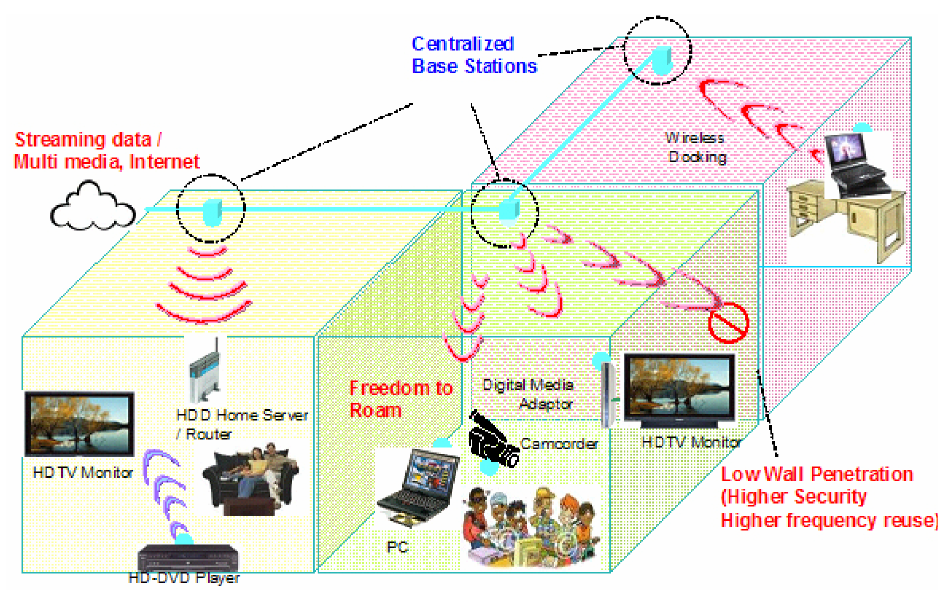


GPS:



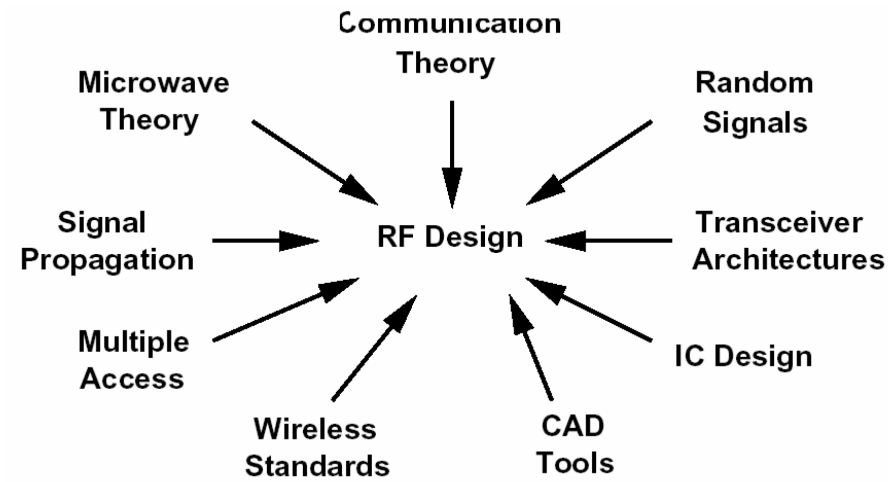
RFIDs:

Wireless Home:

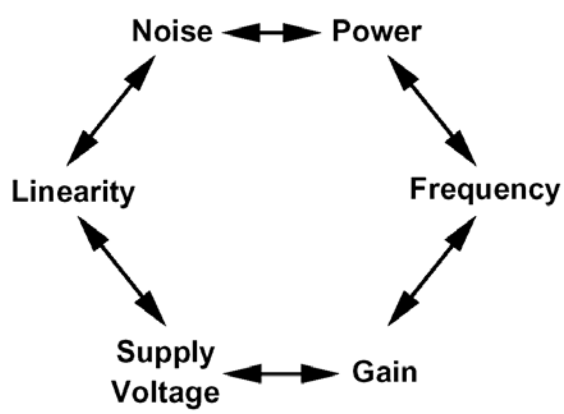


- **Wireless design is challenging …**

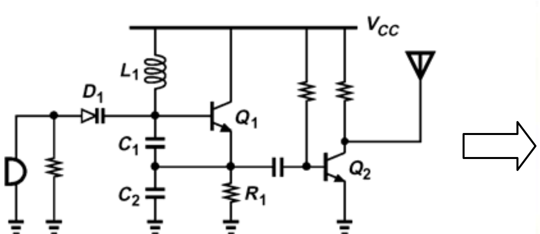
- Draws upon many fields:

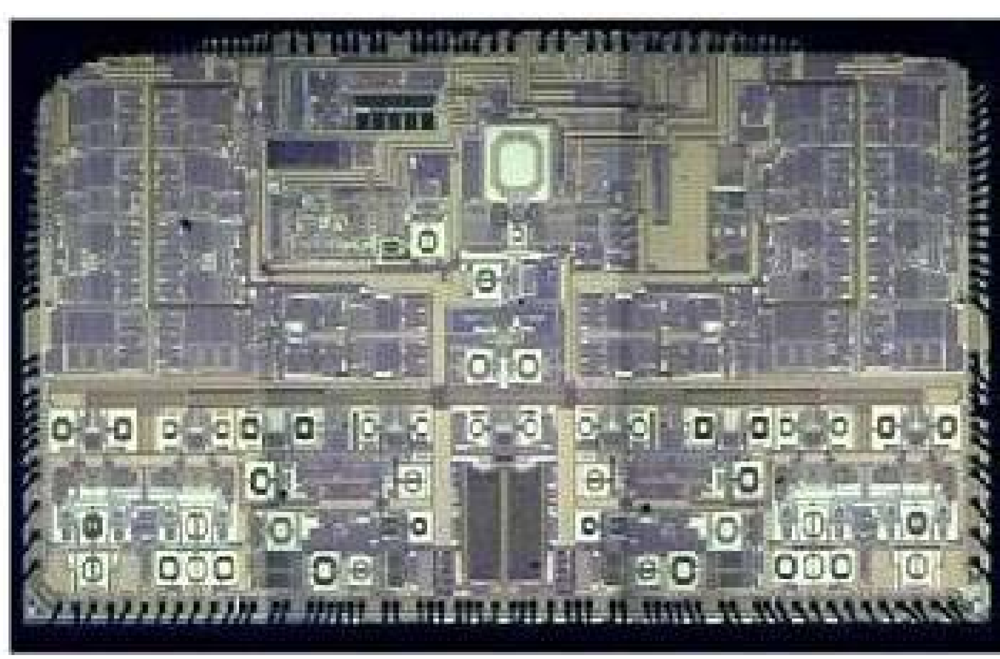


- Competitors must push for: cost, talk time, sensitivity, functions, size, weight, …



- **Wireless has come a long way …**





[A. Behzad et al, ISSCC07]

2x2 802.11a/b/g/n (18 mm^2)

Nonlinearity and Distortion

Linearity and Time Variance

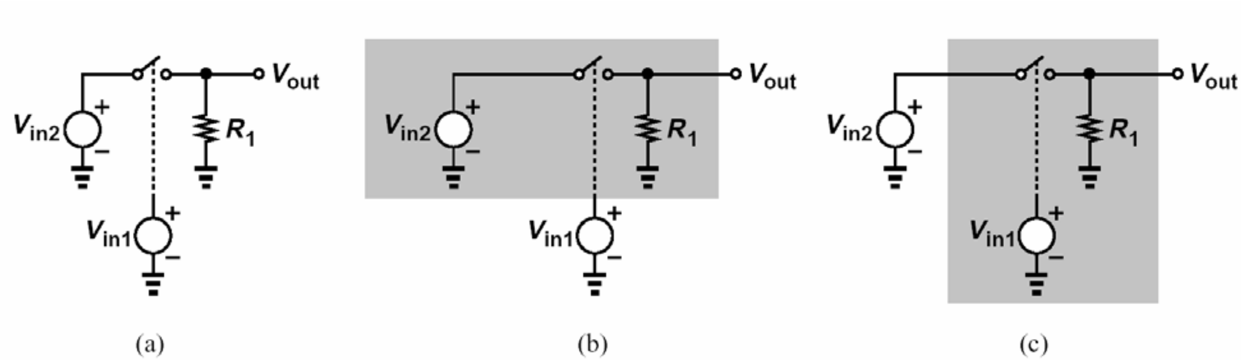
- A system is linear if it satisfies the superposition principle:

$$x_1(t) \rightarrow y_1(t), \quad x_2(t) \rightarrow y_2(t)$$
$$ax_1(t) + bx_2(t) \rightarrow ay_1(t) + by_2(t)$$

- A system is time-variant if $x(t) \rightarrow y(t)$, then $x(t-\tau) \rightarrow y(t-\tau)$

Example:

If the switch turns on at the zero crossings of Vin1, is the system



nonlinear or time-variant?

- A linear system <u>can</u> generate frequency components that do not exist in the input:

$$V_{out}(f) = V_{in2}(f) * \sum_{n=-\infty}^{+\infty} \frac{\sin(n\pi/2)}{n\pi}\delta(f - \frac{n}{T_1})$$

Graphically:

Classes of Systems

- Memoryless vs. Dynamic Systems: A memoryless ("static") system produces an instantaneous output, i.e., the output does not depend on past values:

$$y(t) = \alpha x(t)$$

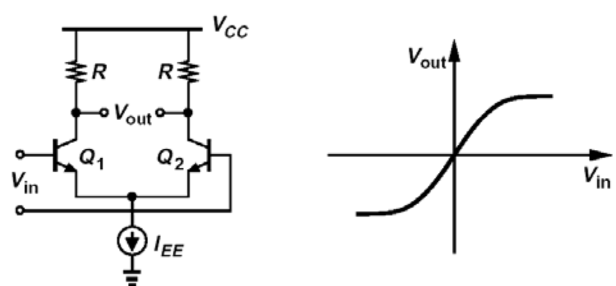
A dynamic system produces an output that may depend on past values. A linear dynamic system satisfies the convolution integral:

$$y(t) = h(t) * x(t)$$

- For a static, nonlinear system, we can approximate the input/output relationship by a polynomial:

$$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \cdots$$

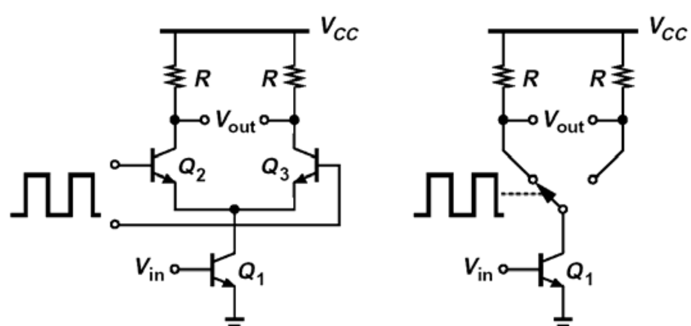
Most types of nonlinearity encountered in practice are "compressive:



(a) (b)

- For a dynamic, nonlinear system, one may need to resort to Volterra series or "harmonic balance" techniques.

Example:



(a) (b)

Effects of Nonlinearity

Nonlinearity introduces "harmonic distortion," "gain compression," "desensitization," "intermodulation," etc.

- **Harmonic Distortion**

If a sinusoid is applied to a nonlinear time-invariant system, the output contains components that are integer multiples of the input frequency: If

$$x(t) = A\cos\omega t$$

then

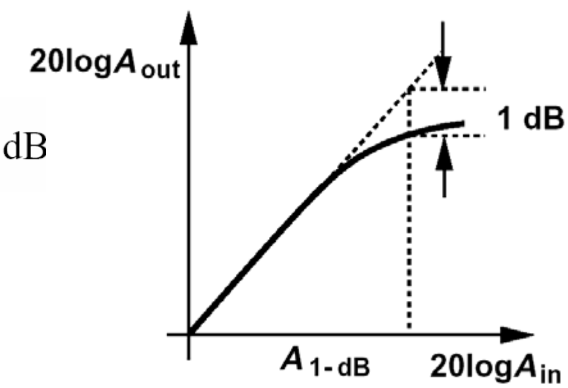
$$y(t) \;=\; \alpha_1 A\cos\omega t + \alpha_2 A^2\cos^2\omega t + \alpha_3 A^3\cos^3 t$$

- **With "odd symmetry," even harmonics are absent.**
- **Amplitude of nth-order harmonic is roughly proportional to A^n.**
-

- **Gain Compression**

In compressive systems, the small-signal gain (slope of the charac.) falls at high input levels. This is quantified by the "1-dB compression point:"

$$20\log\left|\alpha_1 + \frac{3}{4}\alpha_3 A_{1-\mathrm{dB}}^2\right| = 20\log|\alpha_1| - 1\ \mathrm{dB}$$



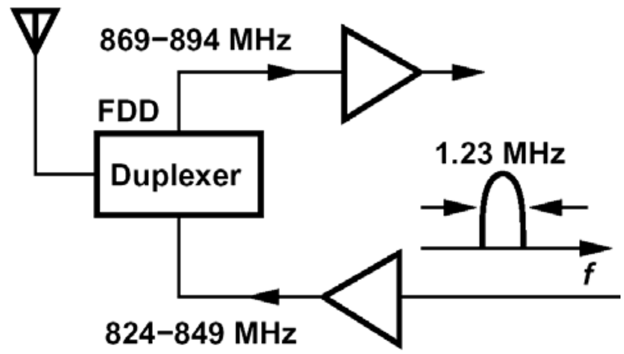
- **Desensitization and Blocking**

If a small signal is accompanied with a large interferer, then:

$$x(t) = A_1\cos\omega_1 t + A_2\cos\omega_2 t$$

$$y(t) = (\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2)\cos\omega_1 t + \cdots$$

The interferer is sometimes called a "blocker."
Example:



- **Intermodulation**

Suppose two interferers are applied to a nonlinear system ("two-tone test"):

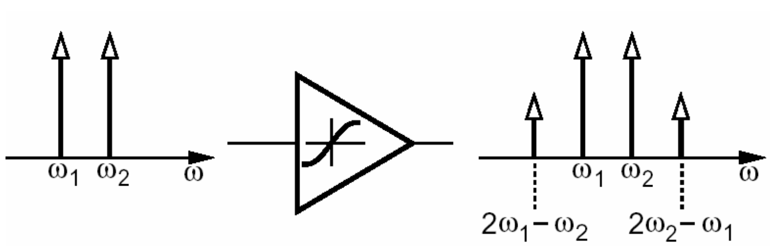
$$x(t) = A_1 \cos\omega_1 t + A_2 \cos\omega_2 t$$

$$\begin{aligned} y(t) &= \alpha_1(A_1 \cos\omega_1 t + A_2 \cos\omega_2 t) + \alpha_2(A_1 \cos\omega_1 t + A_2 \cos\omega_2 t)^2 \\ &+ \alpha_3(A_1 \cos\omega_1 t + A_2 \cos\omega_2 t)^3. \end{aligned}$$

We therefore have these "intermodulation" (IM) components:

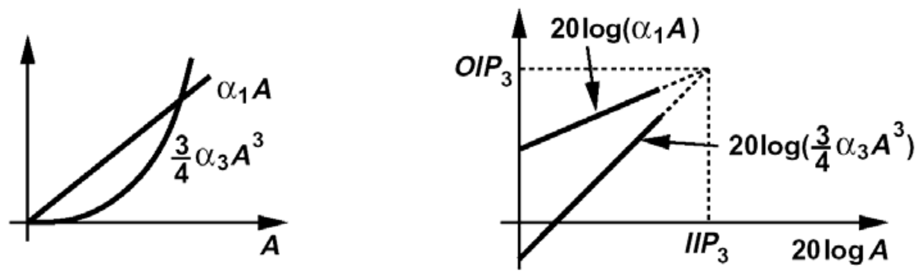
$$\begin{aligned} \omega &= \omega_1 \pm \omega_2 : \alpha_2 A_1 A_2 \cos(\omega_1 + \omega_2)t + \alpha_2 A_1 A_2 \cos(\omega_1 - \omega_2)t \\ \omega &= 2\omega_1 \pm \omega_2 : \frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1 + \omega_2)t + \frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1 - \omega_2)t \\ \omega &= 2\omega_2 \pm \omega_1 : \frac{3\alpha_3 A_2^2 A_1}{4}\cos(2\omega_2 + \omega_1)t + \frac{3\alpha_3 A_2^2 A_1}{4}\cos(2\omega_2 - \omega_1)t. \end{aligned}$$

Which ones are troublesome?



- "Third Intercept Point" (IP3): Two tones with equal amplitudes are applied:

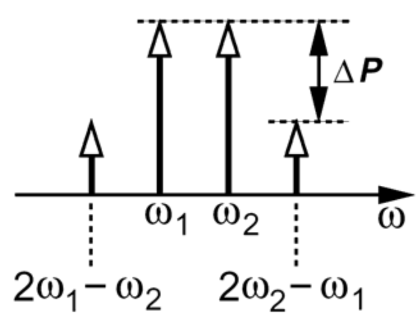
$$
\begin{aligned}
y(t) &= (\alpha_1 + \frac{9}{4}\alpha_3 A^2)A\cos\omega_1 t + (\alpha_1 + \frac{9}{4}\alpha_3 A^2)A\cos\omega_2 t \\
&+ \frac{3}{4}\alpha_3 A^3 \cos(2\omega_1 - \omega_2)t + \frac{3}{4}\alpha_3 A^3 \cos(2\omega_2 - \omega_1)t + \cdots
\end{aligned}
$$



(a) (b)

Thus, IIP3 is obtained as:

- Shortcut Method:



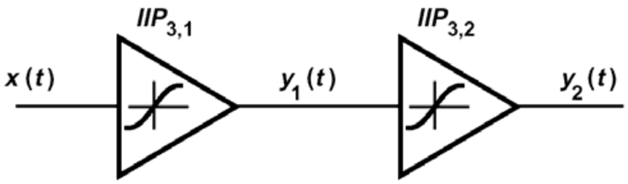
$$IIP_3|_{dBm} = \frac{\Delta P|_{dB}}{2} + P_{in}|_{dBm}$$

- Relationship between IP3 and P1dB:

$$
\begin{aligned}
\frac{A_{1-dB}}{A_{IP3}} &= \frac{\sqrt{0.145}}{\sqrt{4/3}} \\
&\approx -9.6\ \text{dB}
\end{aligned}
$$

Typical receiver IP3 is around -10 dBm.

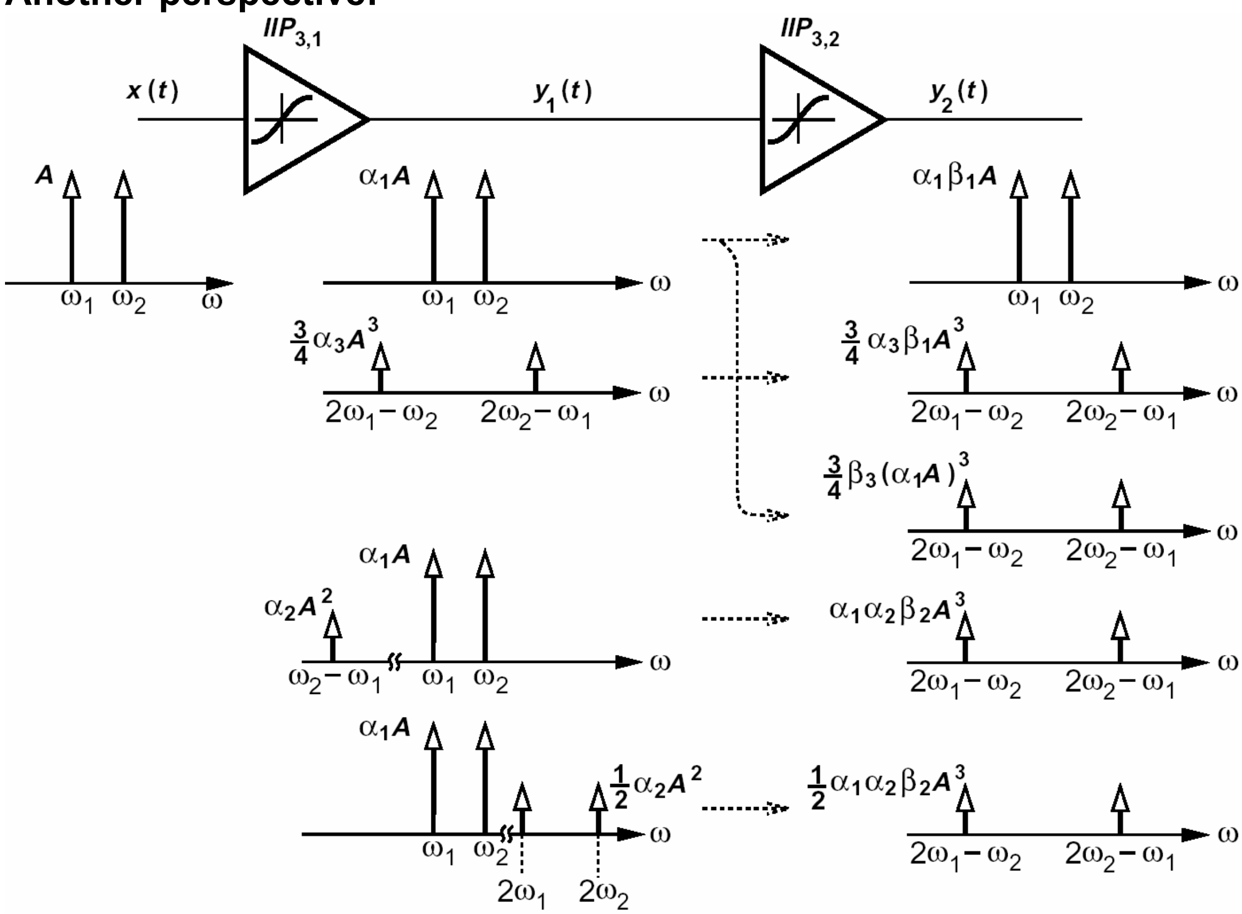
- **Cascaded Nonlinear Stages**



$$\begin{aligned} y_2(t) &= \beta_1[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)] + \\ &\quad \beta_2[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^2 + \\ &\quad \beta_3[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^3. \end{aligned}$$

$$\begin{aligned} \frac{1}{A_{IP3}^2} &= \frac{3}{4}\frac{|\alpha_3\beta_1| + |2\alpha_1\alpha_2\beta_2| + |\alpha_1^3\beta_3|}{|\alpha_1\beta_1|} \\ &= \frac{1}{A_{IP3,1}^2} + \frac{3\alpha_2\beta_2}{4\beta_1} + \frac{1}{A_{IP3,2}^2}, \end{aligned}$$

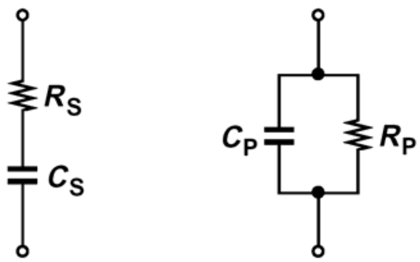
Another perspective:



B. Razavi

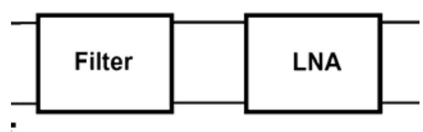
EE101 Concepts

- **Definitions of Q**

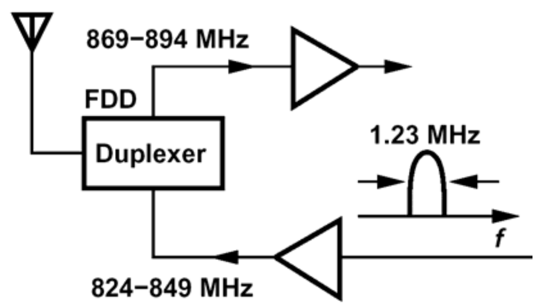


For a second-order tank:

- **Conjugate Matching**



Do we want to have conjugate matching here?



- **dB's and dBm's**

dB is used for <u>dimensionless</u> quantities to make them algebraically manageable:

- Voltage Gain: Vout/Vin → in dB 20log (Vout/Vin)

- Power Gain: Pout/Pin → in dB 10log (Pout/Pin)

Are the voltage gain and power gain equal if expressed in dB?

dBm is used for <u>power</u> quantities is a 50-ohm matched system:

Power P1 in dBm = 10 log (P1/ 1mW)

What do we do in on-50-ohm systems?

- A 50-ohm signal source delivers the specified power only if it is terminated into a 50-ohm load.

- **Other Basics**

  - **Fourier transform of sine and cosine**
  - **Sifting property of impulses**
  - **Trig. Identities: cos a +/- cos b, cos a cos b, cos (a+b), $\cos^2 a$, $\cos^3 a$**

B. Razavi

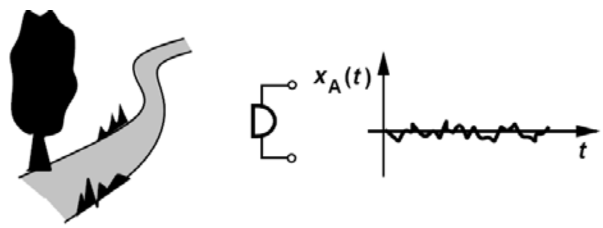
Noise in RF Design

What is Noise?

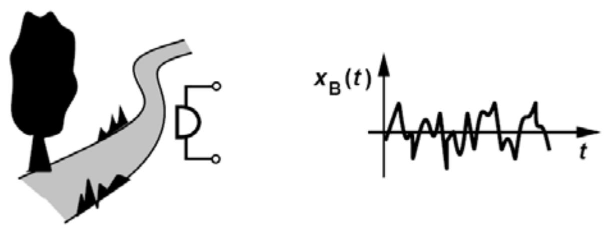
Noise is a random process. Since the instantaneous noise amplitude is not known, we resort to "statistical" models, i.e., some properties that can be predicted.

Average Power

Larger fluctuations mean that the noise is "stronger."

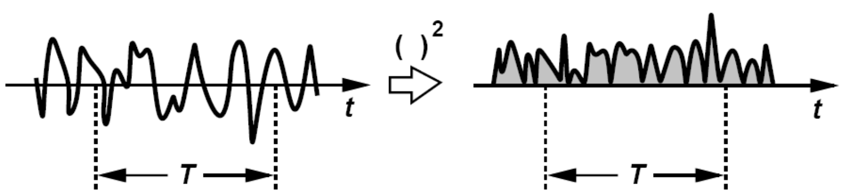


(a)



(b)

$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} \frac{x^2(t)}{R_L} dt,$$



Normalized average power:

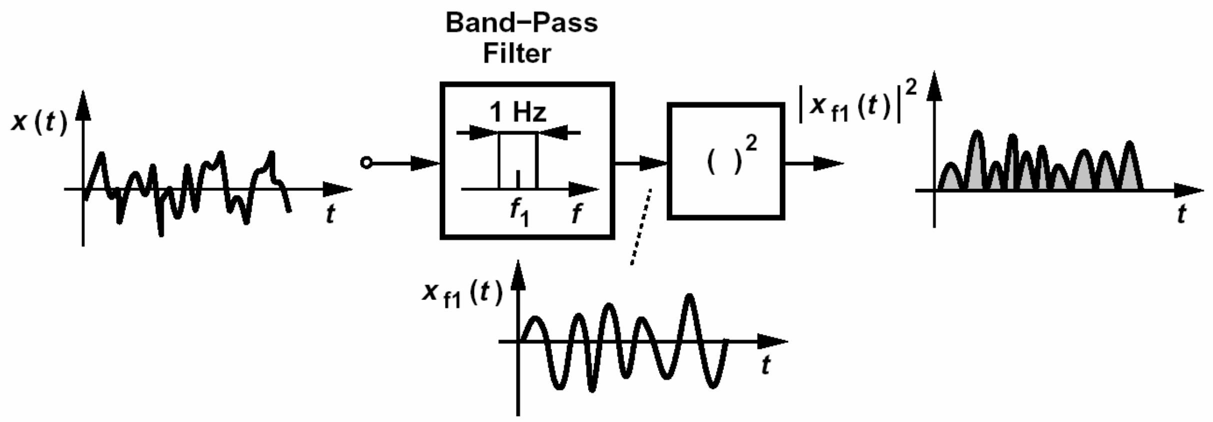
$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} x^2(t) dt,$$

Statistical Characterization

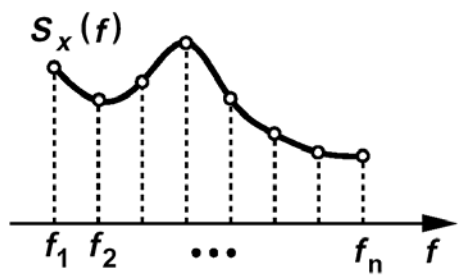
- **Frequency-Domain Behavior**

For random signals, the concept of Fourier transform cannot be directly applied. But we still know that men carry less high-frequency components in their voice than women do.

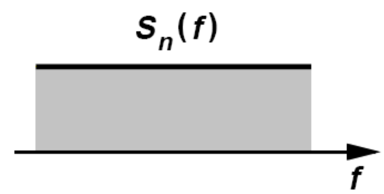
We define the "power spectral density" (PSD) (also called the "spectrum") as:



The PSD thus indicates how much power the signal carries in a small bandwidth around each frequency.



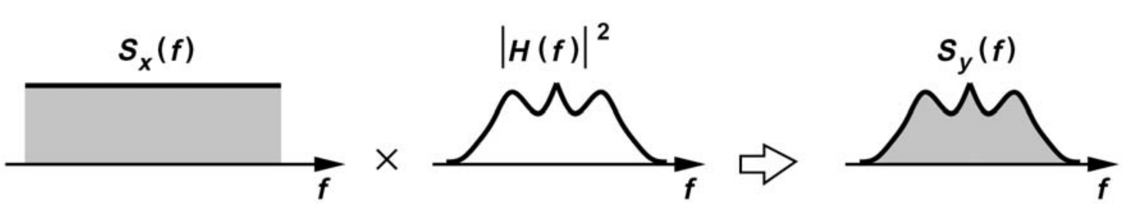
Example: Thermal Noise Voltage of a Resistor



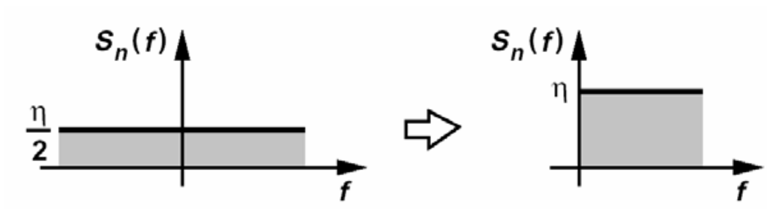
A flat spectrum is called "white."

- **Is the total noise power infinite?**
- **What is the total noise power in 1 Hz?**
- **What is the unit of S(f)?**

<u>Important Theorem</u>

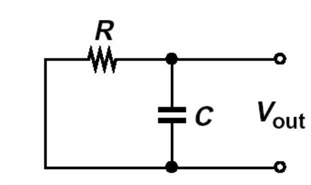


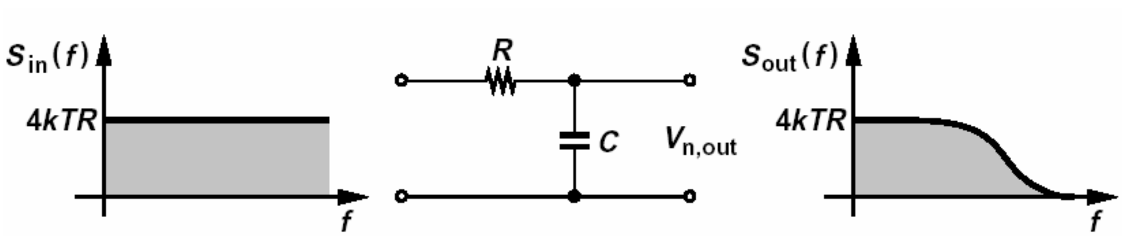
For mathematical convenience, we may "fold" the spectrum as shown here:



Example

Calculate the total rms noise at the output of this circuit.





Note:

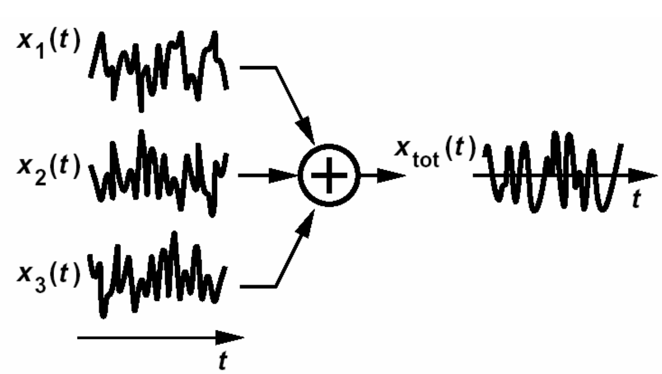
- **The PDF and PSD generally bear no relationship:**

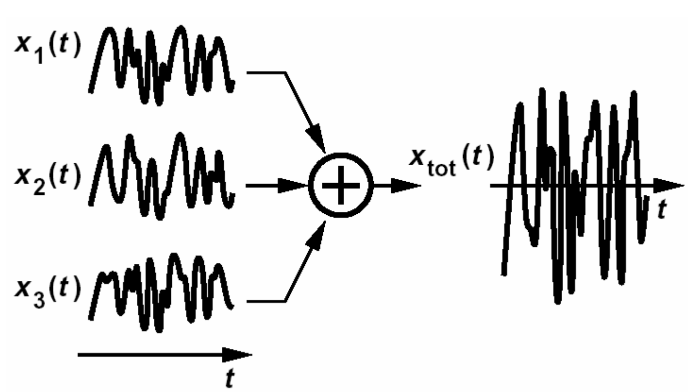
Thermal Noise: Gaussian, white
"Flicker" Noise: Gaussian, not white

Correlated and Uncorrelated Sources

Can we use superposition for noise components?

$$P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} [x_1(t) + x_2(t)]^2 dt$$

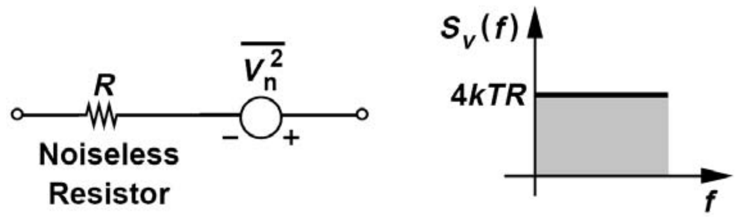




Types of Noise

1. Thermal Noise

Random movement of charge carriers in a resistor causes fluctuations in the current. The PDF is Gaussian because there are so many carriers. The PSD is given by:

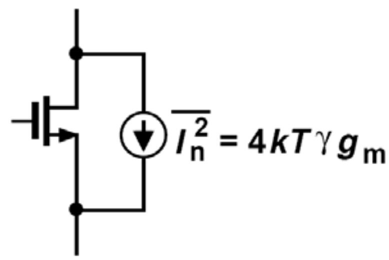


Note that the polarity of the voltage source is arbitrary.

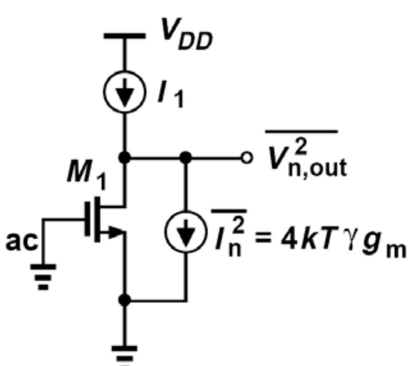
- **Example: A 50-Ώ resistor at room temperature exhibits an RMS noise voltage of .**

If this resistor is used in a system with 1-MHz bandwidth, then it contributes a total rms voltage of .

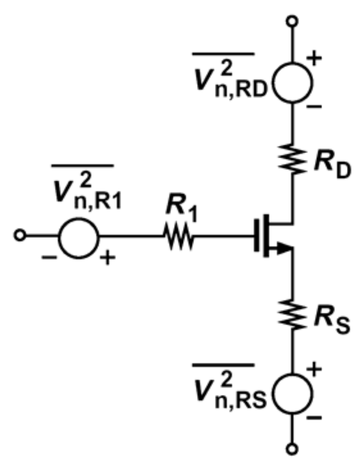
The ohmic resistances in transistors contribute thermal noise:

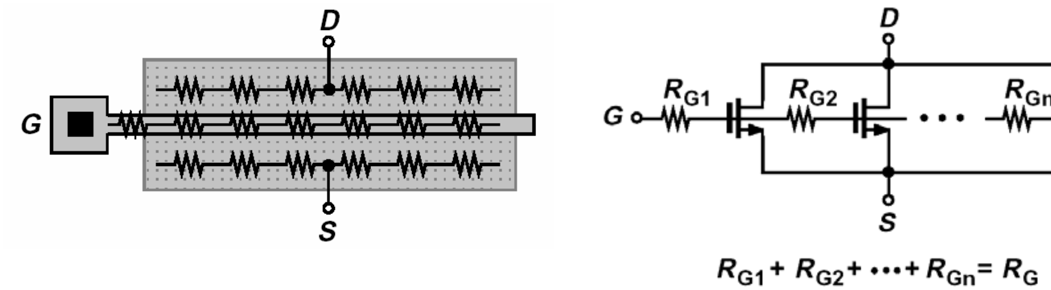


Example:

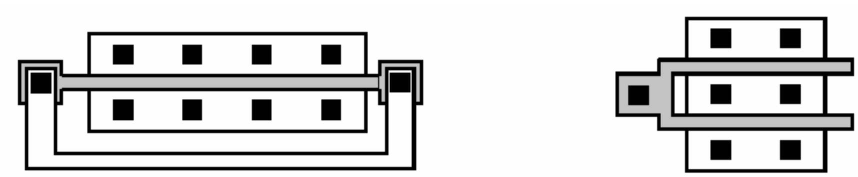


The ohmic sections also contribute thermal noise:





In a well-designed layout, only the channel thermal (and flicker) noise may be dominant:



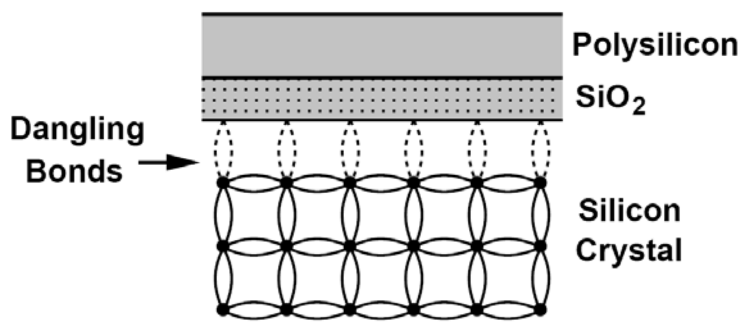
2. Shot Noise

If carriers cross a potential barrier, then the overall current actually consists of a large number of random current pulses. . The random component of the current is called "shot noise" and given by:

Note that shot noise does not depend on the temperature.

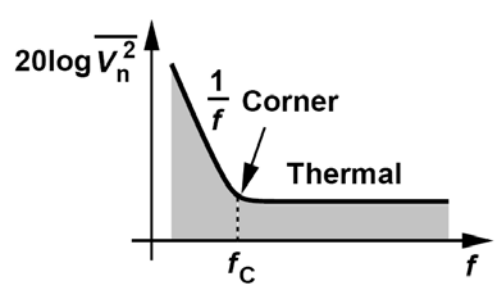
Shot noise occurs in pn-junction diodes, bipolar transistors, and MOSFETs operating in subthreshold region.

3. Flicker (1/f) Noise

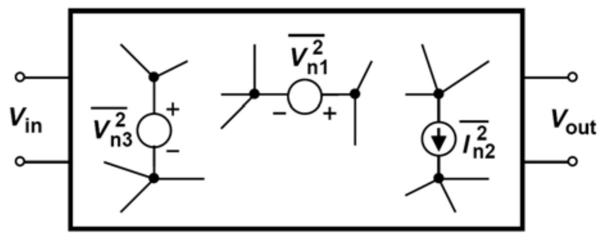


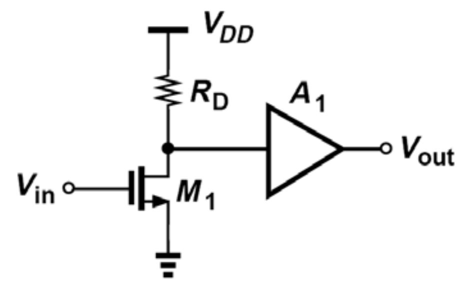
In MOSFETs, the extra energy states at the interface between silicon and oxide trap and release carriers randomly and at different rates. The noise in spectrum referred to the gate is given by:

Where k is a constant and its value heavily depends on how "clean" the process is. We often characterize the seriousness of 1/f noise by considering the 1/f "corner" frequency.



Representation of Noise in Circuits

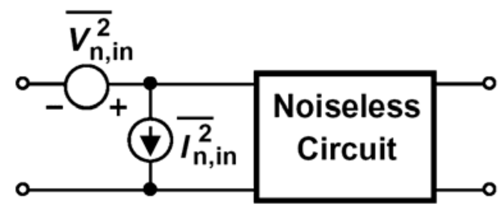




- **Input-Referred Noise**

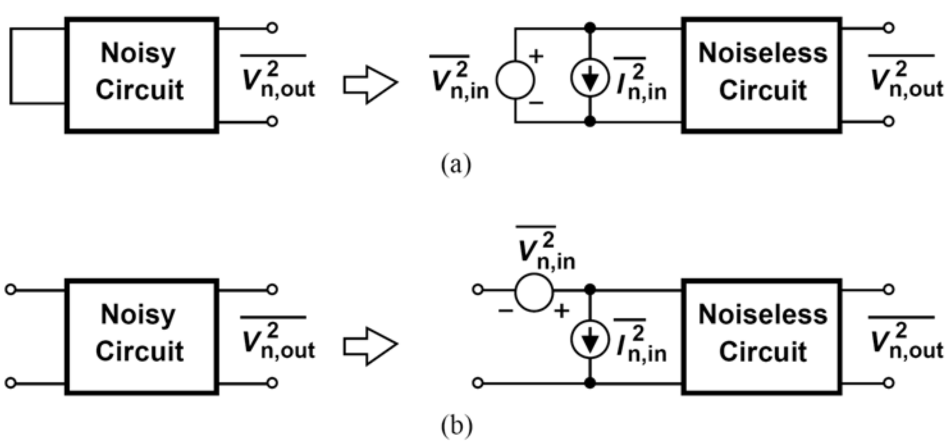
Input-referred noise is the noise voltage or current that, when applied to the input of the noiseless circuit, generates the same output noise as the actual circuit does.

In general, we need both a voltage source and a current source at the input to model the circuit noise:



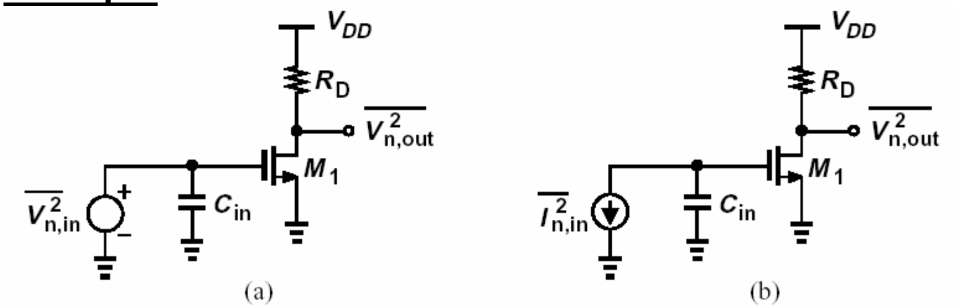
If the source impedance is high with respect to the input impedance of the circuit, then both must be considered.

- How do we calculate the input-referred noise?



Important Note: These two components may be correlated in many cases.

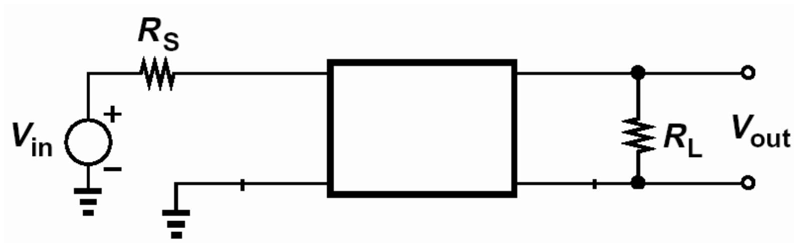
Example



- **Noise Figure**

At high frequencies, it becomes difficult to measure the input-referred noise voltage and current and their correlation. We therefore seek a single metric that represents the noise behavior:

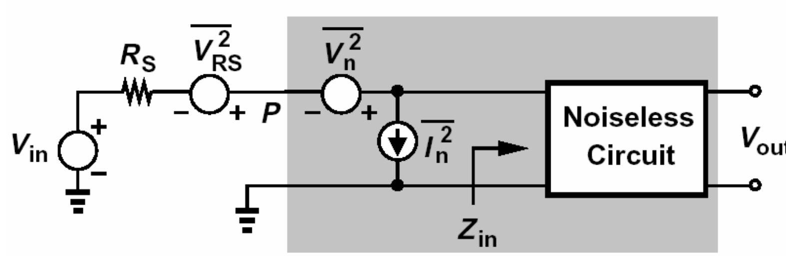
$$\text{Noise Figure} = \frac{SNR_{in}}{SNR_{out}}$$



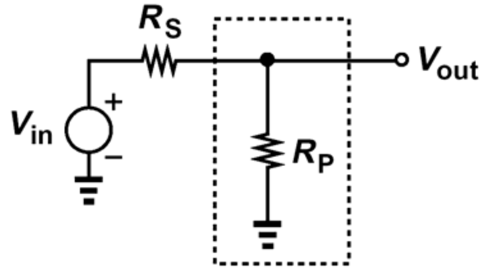
Notes:
- NF measures how much the SNR degrades as the signal passes thru the system.
- If the input has no noise, NF is meaningless.

Calculation of NF:

$$\textbf{NF} = \frac{V^2_{n,out}}{A^2}\frac{1}{4kTR_S}$$

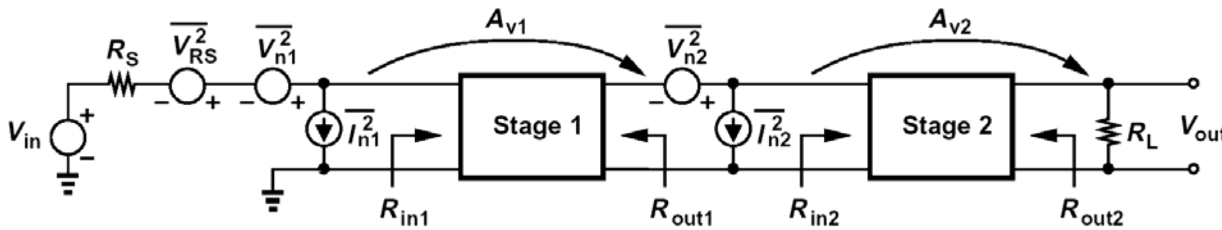


Example



Typical LNAs achieve a noise figure of about 2dB.

- **NF of Cascaded Stages**



$$V^2_{n,in1} = \overline{[I_{n1}(R_S||R_{in1}) + V_{n1}\frac{R_{in1}}{R_{in1}+R_S}]^2} + \overline{V^2_{RS}}\frac{R^2_{in1}}{(R_{in1}+R_S)^2}$$

$$V^2_{n,in2} = V^2_{n,in1}A^2_{v1}(\frac{R_{in2}}{R_{out1}+R_{in2}})^2 + \overline{[I_{n2}(R_{out1}||R_{in2}) + V_{n2}\frac{R_{in2}}{R_{in2}+R_{out1}}]^2}.$$

The total voltage gain is equal to:

Thus,

$$NF_{tot} = \frac{4kTR_S + \overline{(I_{n1}R_S+V_{n1})^2}}{4kTR_S} + \frac{\overline{(I_{n2}R_{out1}+V_{n2})^2}}{A^2_{v1}}\frac{1}{(\frac{R_{in1}}{R_S+R_{in1}})^2}\frac{1}{4kTR_S}$$

Not much intuition here. In traditional microwave design, all interfaces are matched to 50 ohms, and

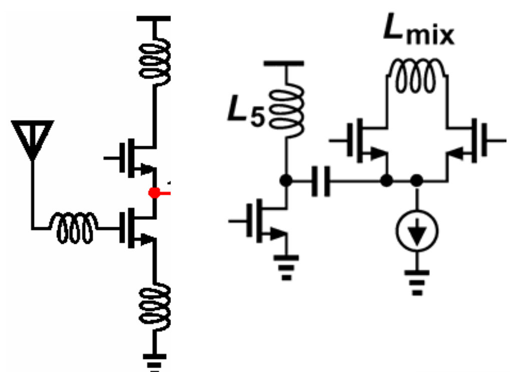
$$\begin{aligned} NF_{tot} &= NF_1 + \frac{\overline{(I_{n2}R_S+V_{n2})^2}}{A^2_{v1}}\frac{1}{4kTR_S} \\ &= NF_1 + \frac{NF_2-1}{A^2_{v1}}, \end{aligned}$$

More generally, the NF can be expressed in terms of the "available power gain," Ap, defined as the available power at the output divided by the available source power:

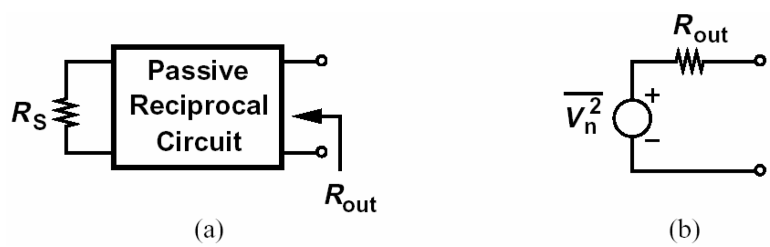
$$NF_{tot} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{p1}} + \cdots + \frac{NF_m - 1}{A_{p1} \cdots A_{p(m-1)}}$$

This is called Friis' Equation. Note that each NF must be calculated with respect to the output impedance of the <u>preceding</u> stage.

But how do we do this for this cascade:

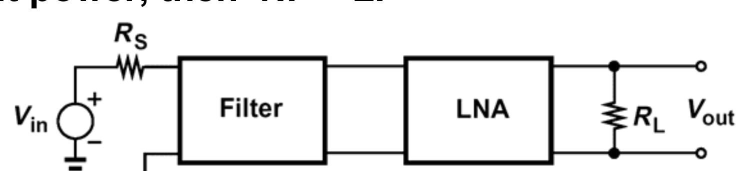


- **NF of Lossy Circuits**



If the available power loss L is defined as the available source power divided by the available output power, then NF = L.

For a cascade:



$$NF_{tot} = NF_{filt} + \frac{NF_{LNA} - 1}{L^{-1}}$$

Sensitivity and Dynamic Range

- Sensitivity is defined as the minimum signal level that can be detected with "acceptable" quality. With digital modulation schemes, the quality is measured by the "bit error rate" (BER).

$$NF = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{sig}/P_{RS}}{SNR_{out}} \Rightarrow \begin{aligned} P_{sig} &= P_{RS} \cdot NF \cdot SNR_{out} \\ P_{sig,tot} &= P_{RS} \cdot NF \cdot SNR_{out} \cdot B \end{aligned}$$

The available noise power for a resistor is given by:

Thus,

$$P_{in.min} = -174\,\text{dBm} + NF + 10\log B + SNR_{min}$$

Note that the sensitivity is a function of bandwidth and hence the bit rate. For example,

GSM:

11a:

- Spurious-free dynamic range (SFDR) in RF design is defined as the maximum level in a two-tone test that produces an IM3 product equal to the noise floor divided by the sensitivity.

Since

$$P_{IIP3} = P_{in} + \frac{P_{in} - P_{IM,in}}{2} = \frac{3P_{in} - P_{IM,in}}{2},$$

we have

For example, NF = 9 dB, IP3=-15 dBm, B= 200 kHz, SNR_{min}=12 dB

→ SFDR=53 dB.