EE215A

Midterm Exam
Fall 2008

Name: Solutions

Time Limit: 1 hour and 50 minutes

Open Book, Open Notes

1. 10

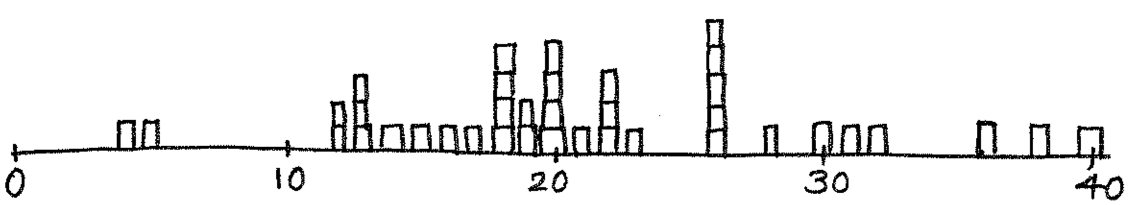
2. 10

3. 10

4. 10

40

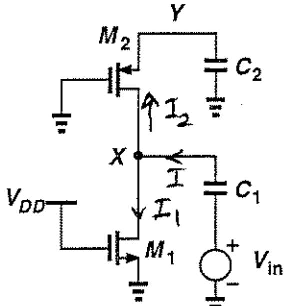
Distribution



Avg. = 20/40

Mean = 20/40

1. Consider the circuit shown below, where the initial voltage across C_1 and C_2 is zero and $\lambda = \gamma = 0$. At $t = 0$, V_{in} jumps from 0 to 3 V. If $C_1 = C_2$ and M_1 is much weaker than M_2, sketch V_X and V_Y as a function of time. Explain your plots in detail. Assume $V_{THN} = |V_{THP}| = 0.4$ V.



At $t = 0^+$, both transistors are in sat.

$I = I_1 + I_2$ Since M1 is much weaker than M2

$\Rightarrow I_1 \ll I_2$

$\Rightarrow I \approx I_2 \Rightarrow -C_1 \frac{dV_X}{dt} = C_2 \frac{dV_Y}{dt}$

$\xRightarrow{C_1 = C_2} -\Delta V_X = \Delta V_Y$

at $t = t_1$ $\begin{cases} V_X = 2.6 \\ V_Y = 0.4 \end{cases}$ $\longrightarrow$ both M1 and M2 enter triode

$-\Delta V_X = \Delta V_Y = 0.4$

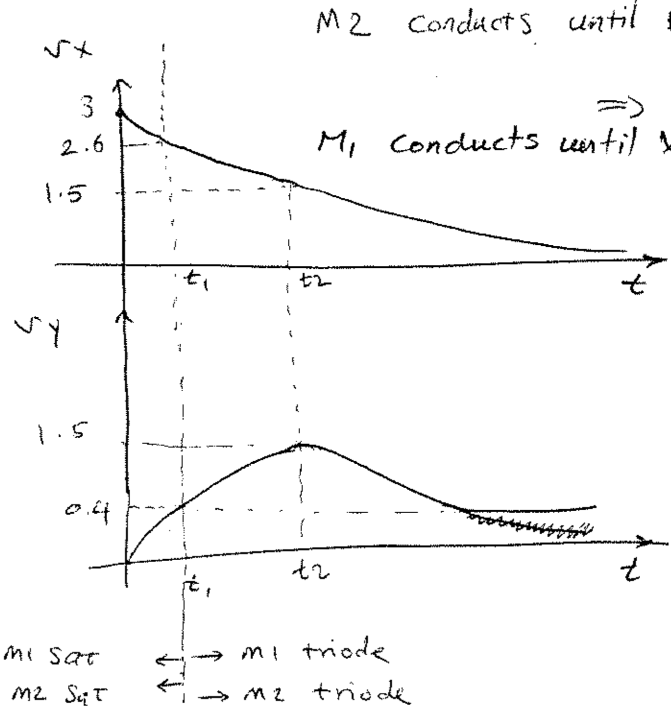
at $t = t_2$: $-\Delta V_X = \Delta V_Y = 1.5$ V $\Rightarrow$ M2 turns off

$\Rightarrow I = I_1 \Rightarrow C_1$ discharges through M1 and reduces voltage of $V_X \Rightarrow$ M2 conducts again and reduces V_Y

M2 conducts until V_Y reaches the threshold voltage

$\Rightarrow V_Y(\infty) = 0.4$

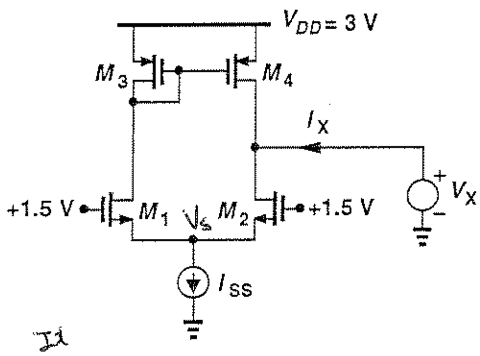
M_1 conducts until V_X reaches zero. $\Rightarrow V_X(\infty) = 0$.



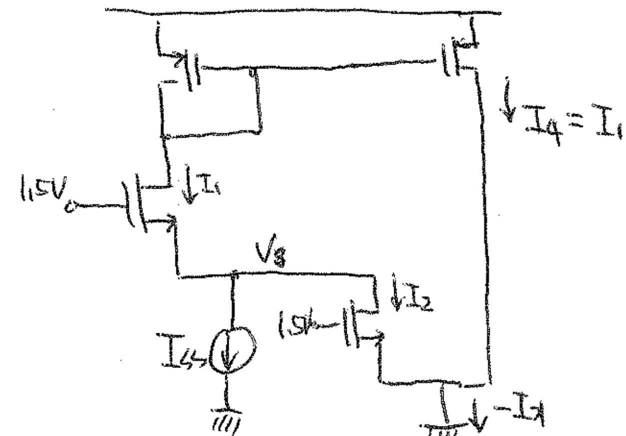
2. Assuming $\lambda = \gamma = 0$, $V_{THN} = |V_{THP}| = 0.4$ V, M_1 and M_2 are identical, and M_3 and M_4 are identical,

(a) Sketch I_X as a function of V_X as V_X goes from 0 to V_{DD}. Explain the details of your plot.

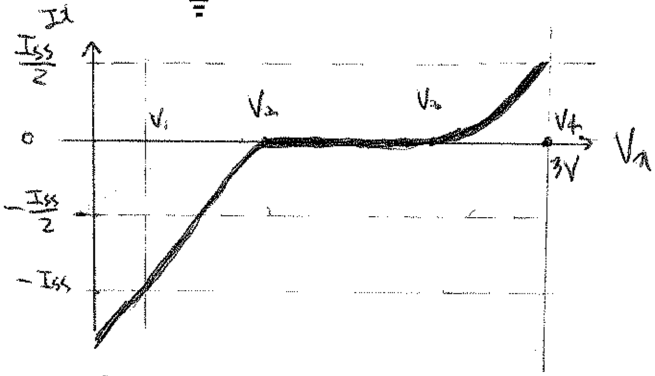
(b) For $V_X = 0$, construct *one* equation that expresses the value of I_X in terms of device and circuit parameters. You need not solve the equation.



(b) Eq CKT $V_x = 0$



(a)



MOS in SAT if not stated otherwise,

i) $0 \le V_x < V_2$ M2 triode, $V_2 = 1.1$, S ⇔ D @ V_1

ii) $V_x = V_1$ $I_{D2} = 0$, $V_1 = 1.1 - \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox}(\frac{W}{L})_{1,2}}}$

iii) $V_2 \le V_x \le V_3$ All sat ∴ $I_x = 0$ (∵) $I_i = \frac{I_{SS}}{2}$ $i = 1,2,3,4$ 6pnts

iv) $V_3 < V_x < V_4$ M4 triode $V_3 = 3 - 0.4 - \sqrt{\frac{I_{SS}}{\mu_p C_{ox}(\frac{W}{L})_{3,4}}}$

v) $V_x = V_{DD}$ $I_4 = 0$ ∴ $I_x = I_2 = \frac{I_{SS}}{2}$

(b) Let $\alpha_n \equiv \mu_n C_{ox}\left(\frac{W}{L}\right)_{1,2}$,

Using KCL 4pnts

$$\begin{cases} -I_x = I_1 + I_2 & \text{—①} \\ I_{SS} = I_1 - I_2 & \text{—②} \end{cases} \Leftrightarrow I_x = -(I_{SS} - 2I_2) = -(2I_1 - I_{SS}) \quad \text{—③}$$

$$I_1 = \frac{1}{2}\alpha_n (V_{GS} - V_{TH})^2 = \frac{1}{2}(V_G - V_S - V_{TH})^2 = \frac{1}{2}\alpha_n (1.1 - V_S)^2 \quad \text{—④}$$

$$I_2 = \frac{1}{2}\alpha_n \left(2(V_{GS} - V_{TH})V_{DS} - V_{DS}^2\right) = \frac{1}{2}\alpha_n (2.2\,V_S - V_S^2) \quad \text{—⑤}$$

plugging ④ & ⑤ into ②,

$$I_{ss} = \frac{1}{2}\alpha_n (1.1 - V_s)^2 - \frac{1}{2}\alpha_n (2.2 V_s - V_s^2)$$

$$\frac{2I_{ss}}{\alpha_n} + 1.1^2 = 2(V_s - 1.1)^2$$

$$\therefore V_s = 1.1 - \sqrt{\frac{1}{2}\left(\frac{2I_{ss}}{\alpha_n} + 1.1^2\right)} \quad \text{—⑥}$$

plug ⑥ into ④

$$I_1 = \frac{1}{2}\alpha_n \left(1.1 - 1.1 + \sqrt{\frac{1}{2}\left(\frac{2I_{ss}}{\alpha_n} + 1.1^2\right)}\right)^2$$

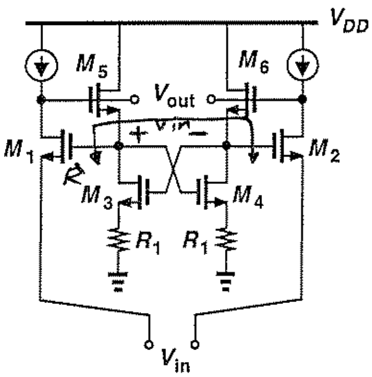
$$= \frac{1}{4}\alpha_n \left(\frac{2I_{ss}}{\alpha_n} + 1.21\right) \quad \text{—— ④'}$$

now using ④' & ③

$$I_x = -(2I_1 - I_{ss})$$

$$\therefore I_x = -\frac{1.21}{2}\alpha_n \qquad \text{(note)} \quad I_x\big|_{V_x=0} < -I_{ss} \;//$$

3. Assuming perfect symmetry and $\lambda = \gamma = 0$, determine the differential voltage gain and input impedance of the circuit shown below.



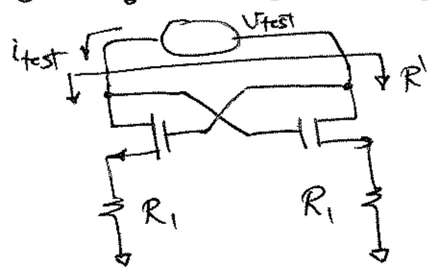
$$V_{out} = V_{gs5} + V_{in} - V_{gs6}$$

$$V_{gs5} = \frac{i_5}{g_{m5,6}}$$

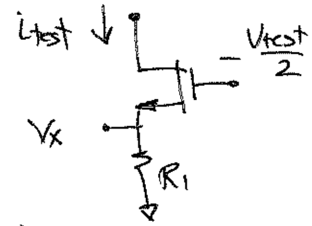
$$V_{gs6} = \frac{i_6}{g_{m5,6}}$$

but $i_6 = -i_5 \rightarrow V_{gs5} - V_{gs6} = \frac{2 \times i_5}{g_{m5,6}}$

But $\quad i_5 = \frac{V_{in}}{R'} \longrightarrow$



half circuit:



$$R' = \frac{V_{test}}{i_{test}}$$

$$V_x = i_{test} \cdot R_1$$

$$i_{test} = g_{m3,4}\left(-\frac{V_{test}}{2} - V_x\right)$$

$$\therefore i_{test} = \frac{-g_{m3,4}}{2} V_{test} - g_{m3,4} R_1 \, i_{test}$$

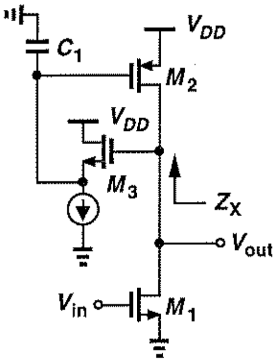
$$\rightarrow \frac{V_{test}}{i_{test}} = -\frac{2(1 + g_{m3,4}R_1)}{g_{m3,4}} = R'$$

$$\Rightarrow V_{out} = V_{in} + 2 \times \frac{V_{in}}{R'} \times \frac{2}{g_{m5/6}} =$$

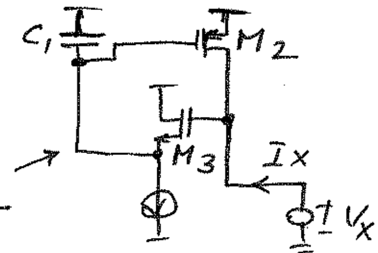
$$\boxed{V_{out} = V_{in}\left(1 + \frac{-g_{m3,4}/g_{m5,6}}{1 + g_{m3,4} R_1}\right)}$$

$\boxed{Z_{in} = \infty}$ (input current is always constant)

4. Neglecting all other capacitances in the circuit shown below and assuming $\lambda = \gamma = 0$,
(a) Determine the transfer function $V_{out}/V_{in}(s)$ and plot the magnitude as a function of frequency.
(b) Find an equivalent ciruit consisting of only resistors, capacitors, and inductors for the load impedance Z_X.



(a)

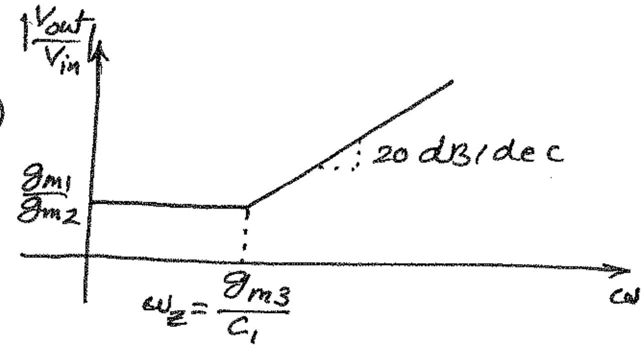


$$V_X \frac{\frac{1}{C_1 s}}{\frac{1}{C_1 s} + \frac{1}{g_{m3}}}$$

$$\Rightarrow I_X = V_X \frac{\frac{1}{C_1 s}}{\frac{1}{C_1 s} + \frac{1}{g_{m3}}} g_{m2}$$

$$\Rightarrow Z_X = \frac{1}{g_{m2}}\left(1 + \frac{C_1 s}{g_{m3}}\right)$$

$$\Rightarrow \frac{V_{out}}{V_{in}}(s) = \frac{g_{m1}}{g_{m2}}\left(1 + \frac{C_1 s}{g_{m3}}\right)$$



(b) $Z_X = \frac{1}{g_{m2}} + \frac{C_1}{g_{m2} g_{m3}} s$

$Z_X \equiv$