Non-Pilot-Aided Iterative Decoding for Joint Data Recovery and Channel Estimation in Fading

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Abstract

Decoding in wireless rapidly fading channels requires that the receiver has access to explicit or implicit channel-state information (CSI). This paper presents a joint channel estimation and decoding scheme for frequency-flat, time-selective Rayleigh fading with relatively high Doppler, that does not require external phase information or even pilot symbols. This system outperforms a pilot-aided iterative channel estimation algorithm that does not decode and estimate jointly. The joint decoding algorithm models the phase trajectory of the channel as a suitable Markov process, the dynamics of which depend on the Doppler rate. At each iteration it considers all possible phase trajectories to determine the maximum a posteriori probability (MAP) value for each bit. This decoding procedure is a natural extension of standard turbo decoding, but introduces a significant complexity increase. Performance is assessed via an upper bound to the capacity of a Markov-phase channel.

1 Introduction

Since the advent of turbo-codes [1], researchers have been exploring ways to utilize the enormous potential of the Forward-Backward algorithm [2] to perform joint channel estimation and decoding (e.g. [3] for the ISI channel), and thus combat distortion due to channels more severe than AWGN. Here, we propose an iterative algorithm for joint channel estimation and data decoding using PSK turbo-codes in a frequency-flat, time-selective Rayleigh fading channel, whereby significant time variation of the channel is present within a packet, without having access to explicit CSI or pilot symbols [4] at the receiver. This channel arises, for instance, in the reverse link of a CDMA system.

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In such a flat fading channel with high Doppler and low SNR, it is no longer reasonable to assume that perfect CSI is available at the receiver and design trellis codes accordingly, as in [5]. In particular, the low SNR where turbo-codes operate, combined with higher Doppler rates, makes it increasingly difficult for practical channel acquisition mechanisms to accurately track the channel variations and provide accurate CSI. Decision directed phase-locked loops (PLLs) are plagued by the fast channel phase variation and the noise. Note that in PSK transmission CSI refers predominantly to the channel phase, because amplitude fading merely scales the noise, and a good estimate of the fading amplitude is the received amplitude itself, in SNR sufficient for reliable decoding.

For an ideal finite-state Markov chain (FSMC), [6] and later [7] proposed trellis codes with decision-feedback-aided recursive channel estimation, but the recursions are vulnerable to error propagation and unreliable when the channel quality degrades. A different approach using turbo-codes was presented in [8], where the *a priori* statistics of a binary Markov channel are incorporated into the equations of the Forward-Backward decoding algorithm, each of the two channel states being a different level of BSC crossover probability. We extend this technique to a more realistic fading scenario, whereby the channel has more than 2 states, which correspond to different intervals of fading phase. By maintaining a notion of quantized fading phase at the receiver, phase estimation and turbo-decoding proceed jointly along a supertrellis via the Forward-Backward algorithm, achieving communication with no explicit pilots at relatively high Doppler rates. Performance is assessed with respect to the same turbo-code enjoying perfect CSI, and a state-of-the-art system based on pilot averaging [9], as well as an upper bound to the capacity of a Markov-phase channel, designed to approximate both the values and the dynamics of the Rayleigh fading phase.

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The idea of quantizing the channel phase was also explored in [10], using pilot symbols and joint decoding and channel estimation on separate trellises, while work on pilot-aided turbo-codes for flat fading was also reported in [11], using adaptive Wiener filters.

2 Channel Model

The flat fading channel is accurately modeled as:

$$y_t := a_t \cdot x_t + n_t, \tag{1}$$

where x_t is the transmitted M-PSK constellation point, $\{n_t\}$ is an i.i.d. (white) complex Gaussian noise process, with variance $N_o/2$ per dimension, and $\{a_t\}$ is the correlated channel fading process, modeled as a circular complex Gaussian random process. Assuming absence of line of sight and a continuum of scatterers in the vicinity of the omnidirectional mobile receiver antenna, we write $a_t = X_t + jY_t = r_t e^{j\phi_t^a}$, where $\{X_t\}$ and $\{Y_t\}$ are mutually uncorrelated Gaussian processes, each with correlation properties determined by the Doppler frequency f_D:

$$R_c(\tau) \doteq E[X_t X_{t+\tau}] = E[Y_t Y_{t+\tau}] = \gamma^2 \mathcal{J}_o(2\pi f_D \tau)$$

where $\mathcal{J}_o(\cdot)$ is the zero-order modified Bessel function of the first kind, and $\gamma^2 = 0.5$ for normalized power. This autocorrelation gives rise to the well-known U-shaped normalized power spectral density in Jakes [12]:

$$S_{xx}(f) = S_{yy}(f) = \frac{1}{\sqrt{1 - \left(\frac{f}{f_D T}\right)^2}}. \tag{2}$$

This model for the fading process has been found to quite accurately match field measurements of physical channels. However, the non-Markovian autocorrelation properties of the amplitude process r_t, and, mainly, the phase process ϕ_t^a are difficult to analyze and exploit in a practical receiver. Therefore, we derive a suitable finite-state Markov model for the channel phase, depending on the Doppler rate $f_D T$. A similar approach, constructing an FSMC to model the amplitude fading of a phase-coherent Rayleigh fading channel was demonstrated in [13], while in [14] a discrete Markov process has been shown to capture most of the dynamics of Jakes' fading process. The receiver uses a K-state Markov model for the quantized version Q_t of the phase fading process ϕ_t^a, where $\{Q_t\}, t = 0, 1, 2, \ldots$ is a time-homogeneous, discrete-time, stationary Markov chain, taking values in the finite state space $\mathcal{Q} = \{q_0, q_1, \ldots, q_{K-1}\}$, a set of "quantized phase distortion channel states" q_i:

$$q_i = \frac{2\pi i}{K}, i = 0, 1, 2, \ldots, K-1, \tag{3}$$

in the following fashion, introducing a quantization operator $\Pi(.)$:

$$Q_t = q_i \Leftrightarrow \Pi(\phi_t^a) = q_i \Leftrightarrow \phi_t^a \in \left[q_i - \frac{\pi}{K}, q_i + \frac{\pi}{K}\right).$$

The transition probabilities $P_{i,j}$, $i, j = 0, 1, \ldots, K-1$ of the Markov chain are independent of t by stationarity, and can be computed from the joint pdf of two successive sampled fading phases:

$$\begin{aligned} P_{i,j} &= \Pr(Q_{t+T} = q_j \mid Q_t = q_i) & (4)\\ &= \frac{\int_{q_i-\pi/K}^{q_i+\pi/K} \int_{q_j-\pi/K}^{q_j+\pi/K} p(\phi_t^a, \phi_{t+T}^a) d\phi_t^a d\phi_{t+T}^a}{\int_{q_i-\pi/K}^{q_i+\pi/K} p(\phi_t^a) d\phi_t^a} & (5) \end{aligned}$$

where the marginal pdf is uniform, and the joint:

$$p(\phi_t^a, \phi_{t+T}^a) = \frac{1-\rho^2}{4\pi^2} \left[\frac{\sqrt{1-B^2} + B(\pi - B\cos^{-1}(B))}{(1-B^2)^{3/2}}\right]$$

where $B = \rho \cdot \cos(\phi_{t+T}^a - \phi_t^a)$, and $\rho = \mathcal{J}_o(2\pi f_D T)$.

The model described above is an approximation in a dual sense: First, it maps all real fading angles $\phi^a \in [-\pi, \pi)$ to K "quantized fading phase states" q_i, $i = 0, 1, \ldots, K-1$. Moreover, the model approximates the dynamics of the continuous process $\{\phi_t^a\}_{t=0,1,\ldots,\infty}$ with a discrete Markov chain, with stationary probabilities $p_i = 1/K$ and transition probabilities $P_{i,j}$. In the above setting, observe that the channel of Eq. (1) is not quantized; the channel state Q_t is only meant to represent *intervals* of the continuous phase ϕ_t^a, since a finite-state Markov model is needed for the Forward-Backward estimation. If K phase states are to effectively represent the rotation of a constellation point caused by the continuous fading phase ϕ^a, then it must hold that:

$$K \geq 2 \cdot M, \tag{6}$$

where M is the size of the PSK constellation, and preferably (but not necessarily) that K is an integer multiple of M. Clearly, $K < 2M$ will degrade performance. Finer quantization of the channel phase (larger K) results in better estimation but also increases the model complexity for diminishing performance gain. According to our simulations, the value $K = 2M$, e.g. 8 quantized channel phases for 4-PSK, represents the most reasonable tradeoff between performance and complexity.

Following Eq. (1), the total received angle ϕ_t^r at time t is the sum of three distinct angles:

$$\phi_t^r = \phi_t^x + \phi_t^* + \phi_t^a \tag{7}$$

where ϕ_t^x is the transmitted constellation point angle, as the trellis code transitions from state c' to c, i.e. $x_t(c' \to c) = 1 \cdot e^{j\phi_t^x}$, and ϕ_t^* denotes the noise-induced additional angle, whose distribution $P(\phi^*)$ is given by the formula (parameterized with $\lambda = \frac{|a|}{\sigma\sqrt{2}}$):

$$P(\phi^* \,; \lambda) \;=\; \frac{e^{-\lambda^2}}{2\pi}\Big[1 + \sqrt{\pi}\lambda \cos\phi^* e^{(\lambda\cos\phi^*)^2} \cdot \\ \cdot \operatorname{erfc}(-\lambda\cos\phi^*)\Big] \qquad (8)$$

where $\operatorname{erfc}(\cdot)$ is the complementary error function and $|a|$ the fading amplitude.

3 Capacity of an FSMC

We compute an upper bound to the constrained capacity of the FSMC with constant amplitude and K discrete phases given by (3) under i.i.d. M-PSK inputs, and use it to assess the receiver performance in the next section. For any stationary, ergodic FSMC, the capacity defined in Gallager [15]:

$$C_{\text{FSMC}} = \lim_{N\to\infty} \frac{1}{N} \cdot I\left(X^N; Y^N\right), \qquad (9)$$

where X^N and Y^N denote sequences of N channel inputs and outputs respectively, is impossible to compute via the algorithm in [7] (a generalization of [6]) because of the computation needed for any number of states K in the Markov model beyond $K = 2$ or 4, which is insufficient for our purposes, as pointed out before. An obvious but very loose upper bound on C_{FSMC} is the constrained capacity given the *current* channel state:

$$C_{\text{FSMC}} \le \lim_{n\to\infty} I(X_n; Y_n \mid Q_n), \qquad (10)$$

which, for the Markov-phase channel, is the PSK constrained AWGN capacity. But it is possible to compute a tighter upper bound on C_{FSMC}, from the following proposition:

Proposition 1 *For any finite-state Markov channel (FSMC) with states Q, a sequence of progressively tighter upper bounds to the capacity C_{FSMC} is:*

$$\lim_{n\to\infty} I\left(X_n; Y_n \mid Q_{n-1}, Q_{n+1}\right) \qquad (11)$$

$$\lim_{n\to\infty} I\left(X_n; Y_n \mid Q_{n-2}, X_{n-1}, Y_{n-1}, Q_{n+1}\right) \qquad (12)$$

$$\vdots$$

Proof : By the chain rule for mutual information:

$$I\left(X^n; Y^n\right) = \sum_{i=1}^{n} I\left(X_i; Y^n \mid X^{i-1}\right) \qquad (13)$$

For each term in the summation of (13) consider two chain-rule expansions:

$$\begin{aligned} & I\left(X_i; Y^n, Q_{i-1}, Q_{i+1} \mid X^{i-1}\right) \\ &\quad = I\left(X_i; Y^n \mid X^{i-1}\right) + I\left(X_i; Q_{i-1}, Q_{i+1} \mid X^{i-1}, Y^n\right) \\ &\quad = I\left(X_i; Q_{i-1}, Q_{i+1} \mid X^{i-1}\right) \\ &\qquad + I\left(X_i; Y^n \mid X^{i-1}, Q_{i-1}, Q_{i+1}\right) \end{aligned}$$

Note that $I\left(X_i; Q_{i-1}, Q_{i+1} \mid X^{i-1}\right) = 0$, since current input X_i is independent of the previous and the next channel states Q_{i-1}, Q_{i+1}, even conditioned on previous inputs. Also, $I\left(X_i; Q_{i-1}, Q_{i+1} \mid X^{i-1}, Y^n\right) \ge 0$, hence:

$$\begin{aligned} I\left(X_i; Y^n \mid X^{i-1}\right) &\le I\left(X_i; Y^n \mid X^{i-1}, Q_{i-1}, Q_{i+1}\right) \\ &= \sum_{j=1}^{n} I\left(X_i; Y_j \mid X^{i-1}, Y^{j-1}, Q_{i-1}, Q_{i+1}\right) \end{aligned} \qquad (14)$$

All the terms in this sum with $j \neq i$ vanish. For $j = i$, $I\left(X_i; Y_i \mid X^{i-1}, Y^{i-1}, Q_{i-1}, Q_{i+1}\right) = I\left(X_i; Y_i \mid Q_{i-1}, Q_{i+1}\right)$, which, combined with (14) proves (11). The other bounds are shown similarly. □

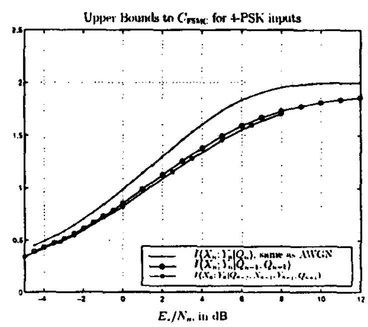

Figure 1: Bounds on C_{FSMC} for 4-PSK inputs

In Fig. 1 we plot the loose upper bound (10) and the tighter bounds (11), (12) against SNR for the FSMC with K Markov phases, derived from Jakes' channel with $f_D T = 0.05$, as in section 2. For rate 0.5 bit/sec/Hz, the overall rate of the turbo-code used n the next section, Eq. (12) gives the tightest bound to the FSMC capacity:

$$\text{SNR}_{0.5\ \text{bit/channel use}} \ge -3\ \text{dB} \triangleq C_1. \qquad (15)$$

4 Data and Channel Estimation

The system of Fig. 2 is a parallel concatenated (turbo) code [16]. Each constituent decoder runs the Forward-Backward algorithm on a supertrellis, whose state S_t at time t is an ordered pair consisting of the "fading phase state" Q_t and the code state C_t. Hence

$S_t = (Q_t, C_t) = (q, c) = m$, with $m = 0, 1, \ldots, 2^\nu K - 1$, for a code with ν memory elements. The need to preserve the channel correlation in order to estimate the fading phase precludes the use of channel interleaving with our iterative scheme. This leads to lack of diversity. However, implicit diversity exists due to the uniform interleaver connecting the constituent codes, two identical 8-state, rate-1/2 Gray-labeled 4-PSK codes, optimized for effective Hamming distance.

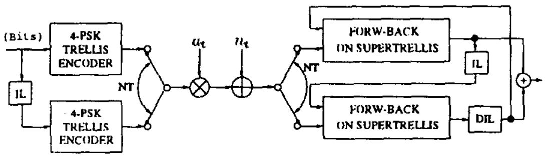

Figure 2: System block diagram

For iterative decoding [16], the crucial quantity to be computed is:

$$
\begin{aligned}
\gamma_t(m', m) &= \Pr\left(S_t = (q, c), \phi_t^r = \theta \,|\, S_{t-1} = (q', c')\right) \\
&= \Pr\left(S_t = (q, c) \,|\, S_{t-1} = (q', c')\right) \cdot \\
&\qquad f\left(\phi_t^r = \theta \,|\, S_t = (q, c), S_{t-1} = (q', c')\right). \quad (16)
\end{aligned}
$$

For the first RHS term of Eq. (16) we have:

$$
\begin{aligned}
&\Pr\left(S_t = (q, c) \mid S_{t-1} = (q', c')\right) \\
&= \Pr\left(\Pi(\phi_t^a) = q \mid \Pi(\phi_{t-1}^a) = q'\right) \cdot \\
&\quad \Pr\left(u_t \text{ such that } C_t = c \mid C_{t-1} = c'\right) \\
&= P_{i,j} \cdot P(u_t; I) \qquad (17)
\end{aligned}
$$

where $P_{i,j}$ was derived in (4)-(5), and $P(u_t; I)$ is the extrinsic information about u_t provided by the other soft decoder in the iterative structure of Fig. 2. For the second RHS term of (16):

$$
\begin{aligned}
&f(\theta \,|\, (q, c), (q', c')) \\
&\stackrel{\text{def}}{=} \Pr\left(\phi_t^r = \theta \,|\, S_t = (q, c), S_{t-1} = (q', c')\right) \\
&\stackrel{(7)}{=} \Pr\left(\phi_t^a + \phi_t^* = \theta - \angle x_t(c' \to c) \mid \right. \\
&\qquad \left. \phi_t^a \sim \mathcal{U}\left[q - \frac{\pi}{K}, q + \frac{\pi}{K}\right]\right) \\
&= \frac{K}{2\pi} \cdot \int_{\theta - \angle x(c' \to c) - q - \frac{\pi}{K}}^{\theta - \angle x(c' \to c) - q + \frac{\pi}{K}} p(\phi^*) d\phi^* \qquad (18)
\end{aligned}
$$

where $P(\phi^*)$ was given in Eq. (8), where for λ we use the approximation $|a_t| \approx |y_t|$ (true for PSK and high SNR), since the true $|a|$ is unknown.

This system relies on the joint estimation of the channel and the data, and does not require explicit CSI or pilots. However, if periodic pilots are available, the adaptation of the algorithm to take them into account is straightforward. Fig. 3 shows the simulated BER performance of the system of Fig. 2 in Jakes' fading channel with $f_D T = 0.05$. As a benchmark, we also plot the performance of the same turbo-code enjoying the benefits of ideal channel interleaving (infinite diversity) and perfect CSI, along with that of a sophisticated pilot-averaging system in [9], employing 3 pilots every 5 data symbols, just to demonstrate the difficulty of obtaining CSI in a practical system operating in moderate to high Doppler rates such as $f_D T = 0.05$.

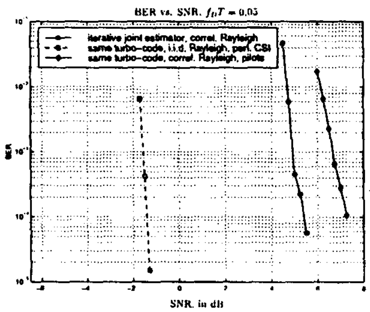

Figure 3: Iterative estimator in Jakes' fading, compared with that of the same turbo-code with perfect CSI and interleaving (left) and with 3 pilot symbols for every 5 coded symbols (right).

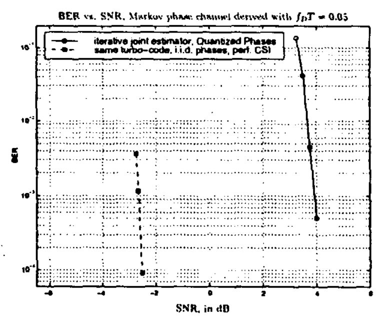

Figure 4: Performance in Markovian quantized phases, matching the FSMC.

In this fast fading channel, even with the significant sacrifice in rate by 3/8 = 37.5%, the pilot-aided system cannot perform as well as our joint data and channel estimation with no pilots at all. The intuitive explanation is that with joint iterative estimation every data symbol essentially becomes a pilot symbol, as its reliability increases with successive iterations. Fig. 4 shows the performance of the iterative decoder in the FSMC with constant amplitude and Markovian phases (3), whose capacity is bounded by $C_1 = -3$ dB. Albeit performing better overall than in Jakes' fading by about 1.5 dB (as expected in a more favorable channel), the joint estimator still performs about 6.5 dB worse than with perfect CSI, and 7 dB from $C_1 = -3$ dB.

Regarding the constituent code selection, the rate-

1/2 encoders chosen are suboptimal from coding perspective, because of the systematic bit repetition. However, this choice of low-rate encoders is unavoidable to maintain reasonable complexity. Their rate has to be low enough for a given constellation size, so that the algorithm can distinguish whether a change in the received phase is to be attributed to the code or to a change in the channel. For 4-PSK and our current encoders, only two possible outputs are possible at any given time, and one is always more likely given the previous channel phase state and the transition probabilities. If, on the other hand, each constituent code has two input bits (i.e. four possible outputs), then at least 8-PSK must be used and $K \geq 16$, which yields a supertrellis with at least 128 states. Therefore, although our scheme does not lose rate directly because of pilots that bear no information, the rate reduction that makes channel estimation possible is inherent in the requirements of the constituent encoder design. On a higher level, this can be viewed as incorporating the training in the code design, instead of having higher rate codes and then explicitly injecting pilot symbols in the coded data stream.

5 Conclusion

This paper proposed a PSK turbo-coded system that operates in the flat correlated Rayleigh fading channel and requires no CSI or pilot symbols. The data estimation is performed in an iterative (turbo) fashion, jointly with channel phase estimation along a supertrellis, formed by combining a Markov model, designed to approximate the values and dynamics of the channel phase, with the code trellis. The performance for moderate to high Doppler rates is assessed via a capacity bound and demonstrated to be superior to a system using a large number of pilots for channel estimation.

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