An $O(n\log n)$ Algorithm for Obstacle-Avoiding Routing Tree Construction in the λ-Geometry Plane *

Zhe Feng [1], Yu Hu [2], Tong Jing [1], Xianlong Hong [1], Xiaodong Hu [3], Guiying Yan [3]

[1] CST Department
[1] Tsinghua University
Beijing 100084, China
Phone: +86-10-62785564
z-feng04@mails.tsinghua.edu.cn

[2] EE Department
[2] UCLA
Los Angeles, CA 90095, USA
Phone: (310) 267-5407
hu@ee.ucla.edu

[3] Institute of Applied Mathematics
[3] Chinese Academy of Sciences
Beijing 100080, China
Phone: +86-10-62639192
{xdhu, yangy}@amss.ac.cn

ABSTRACT

Routing is one of the important phases in VLSI/ULSI physical design. The obstacle-avoiding rectilinear Steiner minimal tree (OARSMT) construction is an essential part of routing since macro cells, IP blocks, and pre-routed nets are often regarded as obstacles in the routing phase. Efficient OARSMT algorithms can be employed in practical routers iteratively. Recently, IC routing and related researches have been extended from Manhattan architecture (λ_2-geometry) to Y- / X-architecture (λ_3- / λ_4-geometry) to improve the chip performance. This paper presents an $O(n\log n)$ heuristic, λ-OASMT, for obstacle-avoiding Steiner minimal tree construction in the λ-geometry plane. Based on obstacle-avoiding constrained Delaunay triangulation, a full connected tree is constructed and then embedded into λ-OASMT by a novel method called zonal combination. To the best of our knowledge, this is the first work addressing the λ-OASMT problem. Compared with two most recent works on OARSMT problem, λ-OASMT obtains up to 30Kx speedup with an even better quality solution. We have tested randomly generated cases with up to 1K terminals and 10K rectilinear obstacles within 3 seconds on a Sun V880 workstation (755MHz CPU and 4GB memory). The high efficiency and accuracy of λ-OASMT make it extremely practical and useful in the routing phase, as well as interconnect estimation in the process of floorplanning and placement.

* This work was partially supported by National Natural Science Foundation of China (NSFC) under Grant No.60373012 and Specialized Research Fund for the Doctoral Program of Higher Education (SRFDP) of China under Grant No.20050003099

Categories and Subject Descriptors

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.

ISPD'06, April 9–12, 2006, San Jose, CA, U.S.A..

Copyright 2006 ACM 1-58113-000-0/00/0004…$5.00.

B.7.2 [**Integrated Circuit**]: Design Aids

General Terms

Algorithms, Performance, Design, Experimentation, Theory

Keywords

λ-geometry, Obstacle-avoiding, Steiner tree construction, $O(n\log n)$

1. INTRODUCTION

Routing a net, finding a rectilinear Steiner minimal tree (RSMT) for a given terminal set, is a fundamental problem in very / ultra large scale integration (VLSI / ULSI) physical design. In practical routing applications, macro cells, IP blocks, and pre-routed nets are often regarded as obstacles. Thus, obstacle-avoiding rectilinear Steiner minimal tree (OARSMT) construction is often used as an accurate wire length even the delay estimation throughout the process of routing. But it was proved that the RSMT problem is NP-complete [1], which implies that no polynomial-time algorithms can solve the OARSMT problem exactly unless P = NP.

The OARSMT problem has been well studied. The maze routing [2]-[3] and line searching [4]-[5] algorithms were proposed for this problem. Ganley *et al* [6] proposed some heuristics, G3S, G4S, and B4S, for cases with less than 20 terminals. Zhou *et al* proposed an algorithm for a 3-terminal net [7]-[8]. An $O(mn)$ 2-step heuristic [9] was proposed, which works well when the terminal number is less than 7 and the obstacles are convex polygons. There are two recent works. FORst [10] can tackle large scale problem efficiently. An-OARSMan [11] has a good length performance when the terminal number is less than 100.

λ-geometry routing [15] allows along $\lambda > 2$ orientations forming consecutive angles of π / λ. In particular, $\lambda = 2, 3, 4$ and ∞ correspond to Manhattan architecture, Y-architecture, X-architecture, and Euclidean geometry, respectively. Most attention from both academia and industry has been devoted to λ-geometry (especially $\lambda = 3$ and 4) routing recently since the total wire length can be reduced up to 30% compared with Manhattan routing, and the crosstalk can also be reduced [12]-[13].

In this paper, we aim to handle the problem of obstacle-avoiding Steiner minimal tree construction efficiently in the λ-geometry plane (λ-OASMT). The first contribution of this paper is to

propose an $O(n\log n)$ algorithm for OARSMT construction, where n is the sum of the terminal number and obstacle number. The second contribution is that we extend the algorithm to handle obstacle-avoiding Steiner minimal tree construction problem in λ-geometry plane. Based on obstacle-avoiding constrained Delaunay triangulation (OACDT), a full connected tree (FCT) is constructed and then embedded into λ-OASMT by a novel method called zonal combination. To the best of our knowledge, this is the first work addressing the λ-OASMT problem. Compared with two most recent works on the OARSMT problem, λ-OASMT obtains up to 30Kx speedup with an even better quality solution. We tested the randomly generated cases with up to 1K terminals and 10K rectilinear obstacles within 3 seconds on a Sun V880 workstation (755MHz CPU and 4GB memory). The high efficiency and accuracy of λ-OASMT make it extremely practical in the routing phase, as well as interconnect estimation in the process of floorplanning and placement.

The rest of this paper is organized as follows. In Section 2, the outline of the λ-OASMT algorithm is introduced. In Section 3, λ-OASMT is described in detail. Section 4 shows the experimental results and Section 5 concludes the whole paper.

2. OUTLINE OF THE HEURISTIC

The input of the λ-OASMT algorithm is the set of terminals and the set of obstacles shown in Figure.1. The output is λ-OASMT shown in Figure.6. The flow of the algorithm is as follows, which falls into 3 steps.

Step 1: We construct a Delaunay triangulation (DT) [16] based on the set of terminals and corner points [18], which is independent of geometry (see Figure.2). Then, we transform the DT into an OACDT by a novel technique, namely *edge deleting* (see Figure.3).

Note that the edge deleting based OACDT construction is geometry independent, *i.e.* it can handle arbitrary-shape obstacles in the λ-geometry plane (see Figure.7).

Step 2: An obstacle-avoiding minimum spanning tree (OAMST) is generated on OACDT by Prim algorithm [17] (see Figure.4). Then, we delete all the non-terminal leaves of the OAMST, which results in a FCT (see Figure.5). The definition of FCT will be given in Sub-section 3.2. The constructions of OAMST and FCT are independent of geometry.

Step 3: We embed a FCT into a λ-OASMT (see Figure.6). Based on a novel method, namely *zonal combination*, the coordinate space of the current node in FCT is divided into 2λ phases. Then, in each phase, we combine all the geometries into one form to handle.

Figure.1 Terminals and obstacles

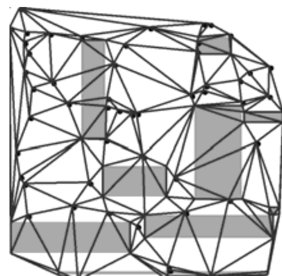


Figure.2 DT

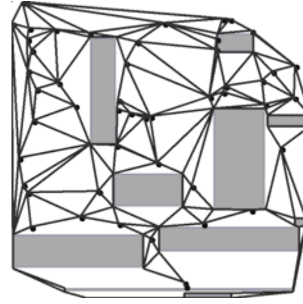


Figure.3 OACDT

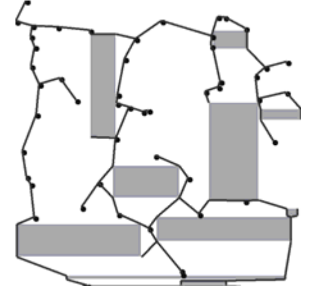
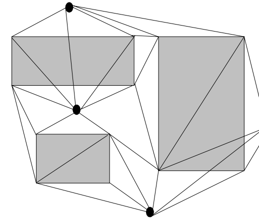


Figure. 4 OAMST

Figure.5 FCT

Figure.6 λ-OASMT



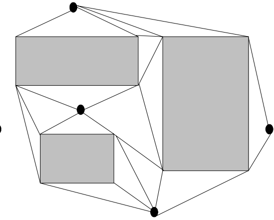


Figure.7(a) DT→OACDT for rectangle

Figure.7(b) DT→OACDT for octagon

3. DETAILED DESCRIPTION OF THE λ-OASMT ALGORITHM

3.1 Step 1: Construct an OACDT

The goal of this step is to construct a graph, on which a tree will be constructed for the use of embedding into a λ-OASMT.

In obstacle-free tree construction scenario, DT is usually used as an initial connection graph, which can be constructed efficiently in $O(n\log n)$ time (n is the number of the terminals) [16] and minimum spanning tree (MST) on DT can be easily transformed into Steiner minimal tree (SMT) since there are many existing algorithms. To tackle obstacles, an obstacle-avoiding DT is needed. In this step, we propose an efficient heuristic to construct an obstacle-avoiding DT, which is defined as follows.

Definition 1 (OACDT): OACDT is a connective constrained DT, which has no edge that intersects with obstacles.

To construct an OACDT, we first generate a DT based on the terminal set and the corner point set, since the graph may not be connective if DT is constructed only based on the terminal set. Then, we delete all the edges that intersect with the obstacles.

If we use a straightforward implementation of checking whether edges of DT intersect with boundaries of obstacles, it will take $C \times n_e \times n_o$ time, where C is maximum edge number of an obstacle, n_e is the number of all DT edges, and n_o is the number of obstacles. Here, we propose a novel technique, namely *edge deleting*, to keep the process of deleting edge running in $O(n\log n)$ time, where n is the sum of terminal number and corner point number. The detailed complexity analysis will be given in Sub-section 3.4.

A. The technique of edge deleting

There are two cases when an edge of DT intersects with a boundary of an obstacle, i) one end point of the edge is also a corner point of an obstacle; ii) both end points of the edge are non-corner points of obstacles.

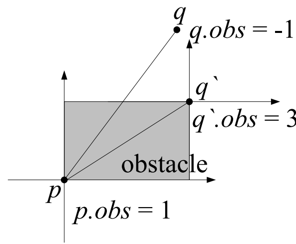


Figure.8 The technique of edge deleting in the 2-geometry

1) Case i

We maintain a domain *obs* for each point, which indicates whether the point is a terminal or a corner point of an obstacle. If the point is a terminal, we set *obs* = -1. If the point is a corner point, we first divide the coordinate space of the point into 2λ phases, and then set the *obs* a value from 1 to 2λ to indicate the phase number that the obstacle is inside. As shown in Figure.8, we first divide the coordinate space of point p into 4 phases for $\lambda = 2$. Then, we find that the obstacle is in the first phase of point p, so we set *p. obs* = 1. Meanwhile, the obstacle is in the third phase of point q\`, so we set *q\`.obs* = 3. We set *q.obs* = -1 since point q is a terminal.

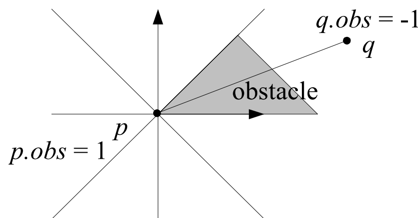


Figure.9 The technique of edge deleting in the 4-geometry

After we set the domain *obs* of each point according to the above definition while inputting terminals and obstacles and we get triples of DT, each edge of a triple will be checked to know whether the edge intersects with the obstacle. If so, the intersecting edge will be deleted. In Figure.8, segment pq is an edge of a DT triple. *p.obs* = 1 means an obstacle is in the first phase of point p. While we check out that the other end point of the edge, notated q, is also in the first phase of point p, it is obvious that edge pq will intersect with the obstacle. Then, edge pq will be deleted. Note that the technique of edge deleting is suitable for arbitrary λ-geometries. Figure.9 shows the case in 4-geometry.

2) Case ii

If both end points of the edge are non-corner points of obstacles, we perform edge deleting by means of checking each obstacle.

Some notations used in this Sub-section are as follows.

$cornerpoint_1$	the start point
$cornerpoint_2$	the end point
current	the current handled point in the routine
last	the point before *current* in the routine
currentline	starting from point *last* to *current*
currentmin	the point which is temporally found to be the nearest to the *currentline* (the nearest means having the least acute angle *currentmin-current-last*)
$candidate_1$, $candidate_2$, ...	candidates as the nearest point to *currentline*, which is the end point of the edge that most possibly intersects with the obstacle.
minline	starting from point *current* to *currentmin*

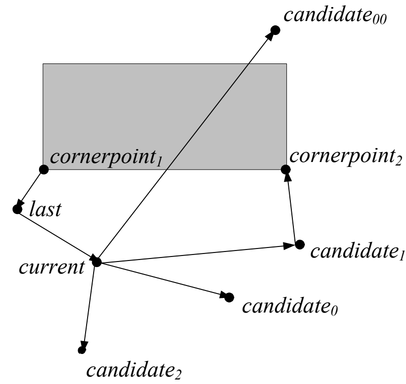


Figure.10 Constructing the shortest path

During the process of checking, we use the method of constructing the shortest path connecting two corner points of each edge of an obstacle, which is outside the obstacle. While constructing the shortest path, we try to find each potential edge forming the path and check whether it intersects with the obstacle. We give up the intersected edges. As in Figure.10, we want to construct the shortest path connecting corner point $cornerpoint_1$ and $cornerpoint_2$. Our strategy is as follows.

It starts from one corner point. Suppose the current handled point is point *current*. We check all the edges connecting to point *current* to get the one with least acute angle *candidate-current-last*. If it does not intersect with the obstacle, we'll select it to form the path. And then the point *candidate* becomes the next one to be handled. But if the edge intersects with the obstacle, it will be given up. We'll select the one with the second / third / … least acute angle *candidate-current-last* to be checked until we find one nonintersecting the obstacle to form the path. Note that constructing the shortest path and detecting the intersection are in turns.

We focus on the following two topics.

a) How to find the edge with the least acute angle *candidate-current-last* among all the edges connecting to the current handled point?

The edge with the least acute angle *candidate-current-last* has such a property. One end point of the edge is point *current*. The other end point is the nearest point to the *currentline* and is on the left side of the *currentline*. That is because as for a closed polygon path along counter-clockwise, the left side of any edge on the path is the interior of the polygon.

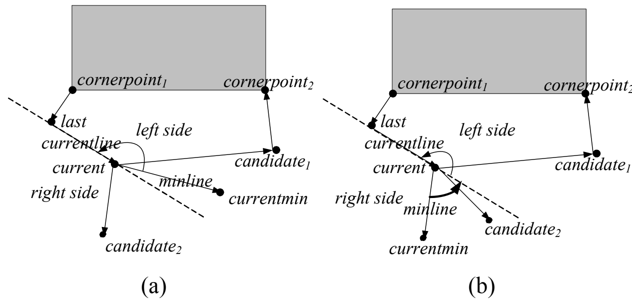


Figure.11 Find the edge with the least acute angle *candidate-current-last*

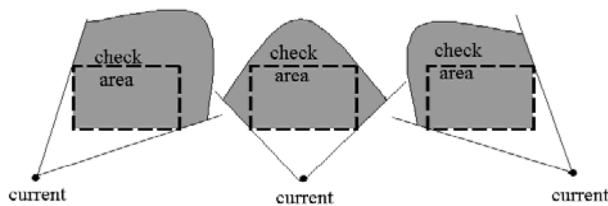


Figure.12 Three situations of checking the intersection

In Figure.11 (a), the edge connecting point *current* and $candidate_1$ is the one to be selected and detected since point $candidate_1$ is the other point besides point *current*, which is the nearest one to the *currentline* and is on the left side of *currentline*.

Figure.11 (a) and Figure.11 (b) show different locations of point *currentmin*, respectively. But in Figure.11 (b), the selecting and detecting method is the same as that in Figure.11 (a). We just need to check all the edges inside the angle of *currentmin-current-last*.

2) How to determine whether an edge intersects with the obstacle?

After selecting the edge, *e.g.* the edge *current*-$candidate_1$ in Figure.11, we check whether it intersects with the obstacle. There are totally 3 situations as shown in Figure.12. If the edge intersects with the obstacle, point $candidate_1$ must be in the check area (the shadow in Figure.12), which can be checked out in constant time.

As a result, all the edges intersecting with obstacles in both cases can be detected and deleted.

In order to keep the connectivity of OACDT, whenever a cut edge [20] is deleted during the process of edge deleting, we connect the separated parts by Detour method [14] with a complexity of $O(n\log n+(e+N)\log t)$, where n is the number of points in the local area, t is the local number of extreme sides of the obstacles, N is the number of searched nodes, and e is the number of the edges of the obstacles. The experimental results show that cut edges are seldom deleted in the algorithm. So the detour work does not dominate the complexity of this step.

3.2 Step 2: Construct full connected tree(FCT)

The goal of this step is to construct a tree connecting all terminals on the OACDT, which should be suitable to be embedded into a λ-OASMT.

Definition 2 (FCT): Full connect tree (FCT) is the tree connecting all terminals and some corner points on the OACDT, and all the leaves are terminals.

We construct a MST on the OACDT to connect all terminals and corner points, which is easy to be implemented since existing algorithms can be used. But we find that some of the corner points are not useful because they are leaves in the tree, which means that such corner points do not span other terminals. So, to get better length performance, we firstly arbitrarily choose a terminal as the root instead of a corner point. Then, we delete all the non-terminal leaves and get the FCT.

The pseudo code of constructing FCT is as follows. We use Prim algorithm to construct a MST connecting all terminals and corner points on the OACDT. Then, we traverse the MST by depth first search (DFS) and delete all the non-terminal leaves (see Figure.4).

3.3 Step 3: Embed FCT into *λ*-OASMT

In this step, we embed FCT into λ-OASMT. A novel method called *zonal combination* is presented to tackle this problem.

The Embedding method [19] is used to transform MST into SMT. In [19], a special shape, L-shape for 2-geometry embedding, was given firstly. Then step by step, the given shape pattern replaced the edges in the tree / graph to shorten the total wire length.

Our *zonal combination* method can unify all the geometries. The coordinate space of the current node is divided into 2λ phases, which sets a shape frame for the corresponding geometry. Also, in each phase, we construct edges according to the geometry.

Moreover, our *zonal combination* method tackles obstacle-avoiding problem by means of bent technique (see Definition 3). The detailed description of the *zonal combination* method is as follows.

The FCT Construction Algorithm
Input: The OACDT G generated in ***Step1***
Output: The FCT T

```
1.Construct OAMST by Prim algorithm on OACDT G
/*set tag, all the nodes with tag will not be deleted*/
2.Tag all the terminals;
3.Push(root, stack);
4.While stack not empty do
    current = Pop(stack);
    If current has tag
    Then
        Do tag current-> parent;
           current = current->parent;
        Till current->parent has tag or current->parent = NULL;
      Push all the child nodes of point current into stack;
/*delete non-terminal leaves nodes*/
5.Push(root, stack);
6.While stack not empty do
    current = Pop(stack);
    If current has not been tagged
    Then
        Delete current;
    Else
     Push all the child nodes of current into stack;
7.Return T;
```

Firstly, we traverse the FCT got from ***Step 2*** and handle all the nodes in FCT one by one. For each node O, there are several child nodes, such as S_1, S_2, S_3 … (see Figure.13). We divide the coordinate space of the current node O into 2λ phases and allocated all the child nodes of node O into the phases according to their relative position to point O.

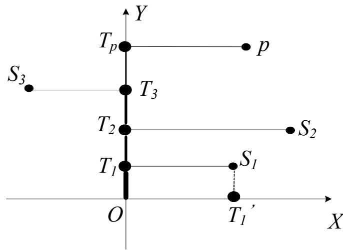


Figure.13 Shared edges

Secondly, phases are handled one by one as follows. We choose the phase including the already constructed edge as a start-up. If there is not any constructed edge, it means that we handle the root. In this case, we will start in the first phase. Then, the rest phases are handled by the counter-clockwise order.

In each phase, all the child nodes are connected to point O by new edges. Especially, since the path connecting the current node O and its parent node p has already been fixed, if we try to share the edges in the path to shorten the total wire length, we should choose the phase including the path as the first phase to deal with. There are following two key issues.

1) How to make full use of shared edges to keep the wire length of the λ-OASMT as short as possible? There are two cases.

a) All the child nodes to be connected to point O are in the same phase, such as point S_1 and S_2 in Figure.13. Point T_1, T_1', T_2, T_3, and T_p are turning points.

We keep to the path $O \rightarrow T_1 \rightarrow S_1$ and $O \rightarrow T_2 \rightarrow S_2$ instead of path $O \rightarrow T_1' \rightarrow S_1$ and $O \rightarrow T_2 \rightarrow S_2$ to share the same edge OT_1. As a result, the way that all the child nodes choose the same path turning direction (e.g., the path turning direction of $O \rightarrow T_1 \rightarrow S_1$ is the same as that of $O \rightarrow T_2 \rightarrow S_2$) will make use of the shared edge.

b) The child nodes are in the neighbor phases, such as point S_1 and point S_3 in Figure.13. We handle the phases in counter-clockwise. That is, the phase that point S_1 is inside will be handled before the one S_3 is inside. Then, if all the child nodes in the S_1-phase choose up-right turning, the child nodes in the S_3-phase will choose up-left turning to share edges and vice versa.

(2) How to get new edges connecting point O and its child nodes avoiding obstacles?

The following concept of bent is helpful to solve this problem.

Definition 3 (Bent): During the process of embedding, if a new generated path intersects with a boundary of an obstacle, the path has to be altered in order to avoid the obstacle. The event is called bent. The new point, added into the path to alter it, is also called bent (see Figure.14). But how to find a bent?

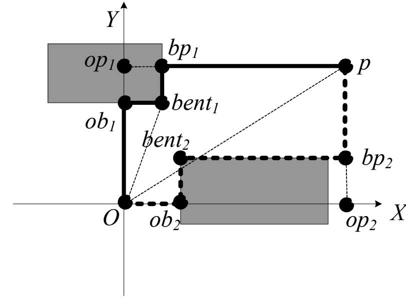


Figure.14 Bent

In Figure.14, O is the current node, while p is the node to be connected to node O. The angle op_2-O-op_1 is the current phase of node O. From key issue 1), we can get the turning. If it chooses the up-right turning ($O \rightarrow op_1 \rightarrow p$) to connect node O and node p, we can find the bent in this way. We check in the rectangle O-op_1-p-op_2 to find whether there is a neighbor of node O or node p happening to be a corner point of an obstacle, and find out the corner number of the obstacle where the point is. In Figure.14, if we find that the neighbor of the node O, $bent_1$, is the right-down corner point of the obstacle, it means that the obstacle is in the rectangle O-op_1-p-op_2 and it needs to be avoided. Thus, $bent_1$ is identified as a bent according to the definition.

After finding the bent, we need to avoid the obstacle. We construct new edges and new nodes to do so. New node ob_1 and bp_1 are generated along the right-down corner of the obstacle, where ob_1 is the intersection of Y-axis and the boundary while bp_1 is the intersection of the horizontal line passing p and the boundary. Thus, when a bent happens, the obstacle-avoiding path

connecting node O and p is made out, i.e. O-ob_1-$bent_1$- bp_1 -p (see Figure.14).

From key issue 1), if it chooses the right-up turning ($O \rightarrow op_2 \rightarrow p$) to connect node O and node p, we can finally make out the obstacle-avoiding path as O-ob_2-$bent_2$-bp_2-p.

The pseudo code is as follows.

```
The Zonal Combination Algorithm
Input:    The FCT T1 generated in Step2
Output: The λ-OASMT T2
-----------------------------------------------------------
Traverse T1 by depth first search (DFS).
  For each node n do
    1. Set 2λ phases for n.
    2. Allocate every child node si of n to phase phi.
    3.Choose the phase ph, which includes the path p
      connecting n and its parent np, as a start-up.
    4. For each phase phi do
      4.1 Decide the direction for all the nodes in the
          phase phi
      4.2 For each node ni in the phase phi do
          4.2.1 Check for bent for node ni
          4.2.2 If  There is a bent
                Then
                    Construct a path matching bent mode,
                    connecting ni and n
                    Combine shared edges between phases
                    Combine shared edges inside phases
                else
                    Construct a path matching non-bent
                    mode, connecting ni and n
                    Combine shared edges between phases
                    Combine shared edges inside phases
  Return T2
```

3.4 The complexity of λ-OASMT algorithm

In ***Step1***, we can generate DT by an $O(n\log n)$ algorithm [16], where n is the sum of the terminal number and the corner point number in obstacles. That is, $n = C_O n_O + n_T$, where n_O is the number of obstacles, n_T is the number of terminals, and C_O is maximum of edges that one obstacle has.

During the process of transforming DT into OACDT, the edge deleting falls into two cases. In case 1), the process of edge deleting is with that of DT construction, which takes $O(n\log n)$ time. In case 2), the time complexity is $C_2 \times n_o$, where n_o is the number of obstacles, $C_2 = C_1 \times C_1' \times C_1''$, C_1 is the maximum number of the points in the shortest path connecting two neighbor corner points, C_1' is the maximum number of edges in the shortest path, $C_1 \times C_1'$ is the period time for constructing the shortest path, and C_1'' is the period time for checking the intersection. As a result, $O(n\log n)$ dominate the time complexity of this part. Therefore, the time complexity of ***Step1*** is $O(n\log n)$.

In ***Step2***, there exist $O(n)$ edges in OACDT. So OAMST can be generated from OACDT by the $O(n\log n)$ Prim algorithm [17]. During the process of deleting non-terminal leaves, the time complexity is dominated by DFS traverse. That is, $C_3 \times n$, where C_3 is the maximum constant period time of the operations that every node is handled by the stack. Therefore, the time complexity of ***Step2*** is $O(n\log n)$.

In ***Step3***, three For-loops dominate the complexity. As for the first loop, n dominates. As for the second one, 2λ dominates. As for the third one, it is a constant C_4, which is the maximum node number in one phase. Therefore, the time complexity of ***Step3*** is $O(n \times 2 \times \lambda \times C_4 \times C_5)$, where C_5 is the period time that pseudo code 4.2.2 performed, i.e. $O(n)$.

All in all, the time complexity of our λ-OASMT algorithm is $O(n\log n)$, where n is the sum of the terminal number and the corner point number in obstacles. We also can say that we get an $O(n\log n)$ algorithm, where n is the sum of the terminal number and obstacle number.

4. EXPERIMENTAL RESULTS

We have implemented the λ-OASMT algorithm in C++ language and performed it on a Sun V880 fire workstation with 755MHz CPU and 4GB memory. We randomly generate some terminals among rectilinear polygon obstacles in a 32767×32767 plane.

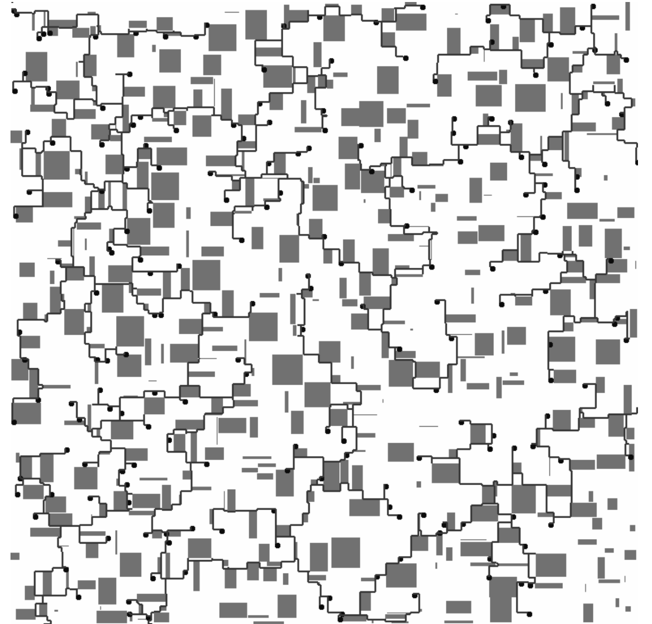


Figure.16 The result of our algorithm of routing 200 terminals in the presence of 500 rectangular obstacles in 2-geometry

Table1 and Table2 show the comparison results with FORst [10] and An-OARSMan [11] on randomly generated cases.

Figure.15 (a)-(c) show the topology differences among 2-OASMT, FORst, and An-OARSMan on the test case with 50 terminals and 10 obstacles.

From Table1, the performance of 2-OASMT is better than that of FORst on larger test cases since FORst divides the problem into several sub-problems (***Step I*** in FORst). Therefore, it just searches in a smaller searching space and loses optimality. However, our algorithm always tries to do in the global. That is, constructing DT, OACDT, OAMST, FCT, and λ-OASMT pay respect to all

obstacles and all terminals. Hence, 2-OASMT can produce better solution for larger problems. For small test cases, FORst and An-OARSMan can produce relative better solutions with more expensive computational time.

From Table2, we can see that λ-OASMT achieves 8249x speedup on FORst / An-OARSMan on average. The speedup increases for larger test cases. We can obtain up to 30Kx speedup for the largest test cases.

To study on the scalability of our algorithm, we test some cases with additional more obstacles, which are more realistic in practical VLSI routing applications, such as the detailed routing, or ECO routing. Table3 shows the results, from which we can see that λ-OASMT consistently runs efficiently for all test cases. Particularly, λ-OASMT can route 1000 terminals in the presence of 10000 rectangular obstacles in 2-geometry within 3 seconds. Figure.16 shows the result of routing 200 terminals in the presence of 500 rectangular obstacles in 2-geometry.

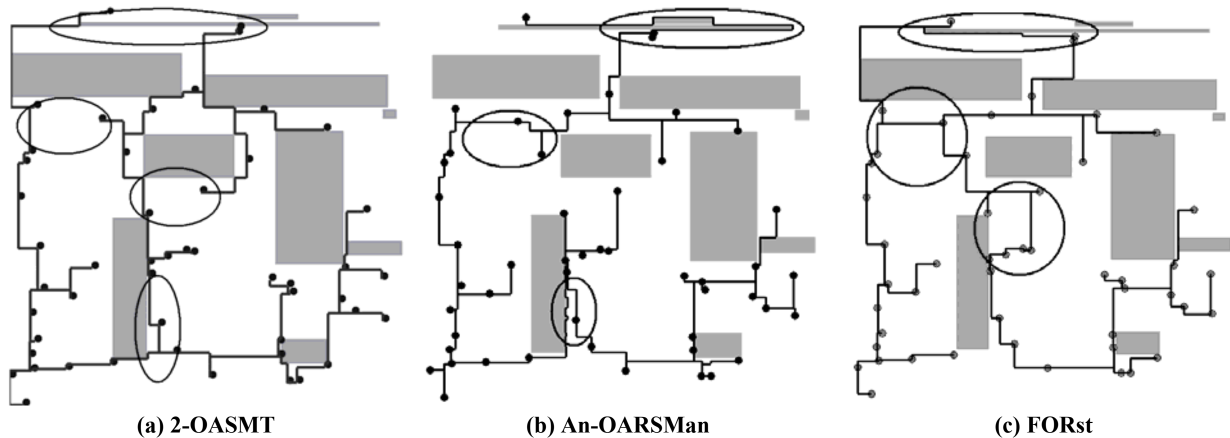


Figure.15 Differences among 2-OASMT, An-OARSMan, and FORst for the test case with 50 terminals among 10 obstacles.

5. CONCLUSIONS

In this paper, we propose an efficient and unified heuristic for λ-OASMT construction. The algorithm is fit for arbitrary interconnect architecture (e.g. Y- / X-Architecture), which is helpful for further shortening the wire length and reducing crosstalk compared with the traditional Manhattan Architecture. The dominate time-complexity of our algorithm is $O(n\log n)$, where n is the sum of the terminal number and obstacle number. Compared with two recent works focusing on OARSMT problem, λ-OASMT obtains up to 30Kx speedup with an even better quality solution. We have tested the randomly generated cases with up to 1K terminals and 10K rectilinear obstacles within 3 seconds on a Sun V880 workstation (755MHz CPU and 4GB memory). The high efficiency and accuracy of λ-OASMT make it is extremely practical and useful in the process of routing, as well as interconnect estimation in the floorplanning and placement phase.

6. REFERENCES

[1] M. R. Garey and D. S. Johnson, "The rectilinear Steiner tree problem is NP-complete", SIAM Journal on Applied Mathematics, 1977, 32: pp.826-834.

[2] C.Y. Lee, "An Algorithm for Connections and Its Application", IRE Trans. on Electronic Computer, 1961, pp.346-365.

[3] F. Rubin, "The Lee Connection Algorithm", IEEE Trans. on Computer, 1974, 23: pp.907-914.

[4] K. Mikami and K. Tabuchi, "A Computer Program for Optimal Routing of Printed Circuit Connectors", IFIPS Proc., 1968, H47, pp.1475-1478.

[5] D.W. Hightower, "A Solution to the Line Routing Problem on the Continous Plane", Proc. of the 6th Design Automation Workshop, 1969, pp.1-24.

[6] J. L. Ganley and J. P. Cohoon, "Routing a multi-terminal critical net: Steiner tree construction in the presence of obstacles", In: Proc. of IEEE ISCAS, London, UK, 1994, pp.113-116.

[7] Z. Zhou, C. D. Jiang, and L. S. Huang, "Finding Obstacle-Avoiding Shortest Path Using Generalized Connection Graph with $O(t)$ Edges", Chinese J. of Software, 2003, 14(2): pp.166-174.

[8] Z. Zhou, C. D. Jiang, and L. S. Huang, "On optimal rectilinear shortest paths and 3-steiner tree routing in presence of obstacles", Journal of Software, 2003, 14(9): pp.1503-1514.

[9] Y. Yang, Q. Zhu, T. Jing, and X.L. Hong, "Rectilinear Steiner Minimal Tree among Obstacles", In: Proc. of IEEE ASICON, Beijing, China, 2003, pp.348-351.

[10] Yu Hu, Zhe Feng, Tong Jing, Xianlong Hong, Yang Yang, Ge Yu, Xiaodong Hu, Guiying Yan, "FORst: A 3-Step Heuristic for Obstacle-Avoiding Rectilinear Steiner Minimal Tree Construction", Journal of Information & Computational Science, 2004, 1(3): pp.107-116.

[11] Yu Hu, Tong Jing, Xianlong Hong, Zhe Feng, Xiaodong Hu, Guiying Yan. "An-OARSMan: Obstacle-Avoiding Routing Tree Construction with Good Length Performance", In: Proc. of IEEE/ACM ASP-DAC, 2005, Shanghai, China, pp.7-12.

[12] X Initiative Home Page. http://www.xinitiative.com, 2001.

[13] Hongyu Chen, Chung-Kuan Cheng, Andrew B. Kahng, Ion Mandoiu, and Qinke Wang, "Estimation of Wirelength Reduction for λ-Geometry vs. Manhattan Placement and Routing", SLIP'03, April 5-6, 2003, Monterey, CA, USA.

[14] S.Q. Zheng, J.S. Lim, and S.S. Iyengar, "Finding obstacle-avoiding shortest paths using implicit connection graphs", IEEE Trans. on Computer Aided Design, 1996, 15(1): pp.103-110.

[15] M. Sarrafzadeh and C. K.Wong, "Hierarchical Steiner tree construction in uniform orientations," IEEE Trans. on Computer-Aided Design, 11(9): pp.1095-1103, 1992.

[16] S. Fortune, "A sweepline algorithm for Voronoi diagrams. Algorithmica", 2(2): pp.153-174, 1987.

[17] R.C. Prim, "Shortest connection networks and some generalizations", Bell System Tech. J. 36(1957): pp.1389-1401.

[18] Martin Zachariasen, "Rectilinear Full Steiner Tree Generation", Technical Report DIKU-TR-97/29, Department of Computer Science University of Copenhagen, 1997.

[19] Janming Ho, G. Vijayan, and C. K. Wong, "A new approach to the rectilinear Steiner tree problem", Proc. of the 26th ACM/IEEE conference on Design Automation, Las Vegas, Nevada, USA, 1989, pp.161-166.

[20] H. Schenck, "Computational Algebraic Geometry", Cambridge University Press, November 2003.

Table 1 Comparison between FORst [10], An-OARSMan [11], 2-OASMT, and 4-OASMT on wire length

Term #	Obs#	Wire Length						
		[10]	[11]	2-OASMT		4-OASMT		
		Wire length	Wire length	Wire length	Improve to [10]/[11] (%)	Wire length	Improve to [10]/[11] [(%)	Improve to 2-OASMT (%)
10	10	29630	27840	30410	-9.23	27279	2.02	10.30
30	10	56880	43090	45640	-5.92	41222	4.34	9.68
50	10	64020	63250	58570	7.34	52432	17.10	10.48
70	10	69450	66310	63340	4.48	57699	12.99	8.91
100	10	84590	82320	83150	-1.01	73090	11.21	12.10
500	100	223415	-	198010	11.37	176497	21.00	10.86
1000	100	285821	-	250570	12.33	222758	22.06	11.10

Table 2 Comparison between FORst [10], An-OARSMan [11], 2-OASMT, and 4-OASMT on running time

Term#	Obs#	Runtime (*s*)					
		[10]	[11]	2-OASMT		4-OASMT	
		Time	Time	Time	Speedup	Time	Speedup
10	10	0.095	0.164	0.002	47x	0.003	32x
30	10	0.584	1.075	0.003	195x	0.003	195x
50	10	0.868	3.504	0.004	217x	0.004	217x
70	10	2.759	10.552	0.004	690x	0.005	552x
100	10	6.312	26.974	0.004	1578x	0.005	1262x
500	100	559.000	-	0.026	21500x	0.033	16939x
1000	100	1240.00	-	0.037	33514x	0.045	27556x
					Average speedup 8249x		

Table 3 Testing on cases with more obstacles

Term#	Obs#	Wire length		Runtime (*s*)	
		2-OASMT	4-OASMT	2-OASMT	4-OASMT
100	500	149725	135454	0.057	0.070
200	500	181470	162762	0.062	0.075
200	800	202741	182056	0.095	0.119
200	1000	214850	193228	0.129	0.154
1000	10000	1723990	1564170	2.823	3.030