Reprinted from
THE PHYSICS OF FLUIDS
Volume 4, Number 11, November, 1961

Low-Frequency Oscillations in Gas Discharges

FRANCIS F. CHEN
Plasma Physics Laboratory,
Princeton University, Princeton, New Jersey
(Received June 26, 1961)

OSCILLATIONS in gas discharges of multifarious types have been observed many times in the past, and the existence of sheath oscillations near the electron plasma frequency has been established by Gabor and associates.[1] On the other hand, low-frequency oscillations ($\sim 10^5$ cps) are also known to exist, and recently attention has been focused on these oscillations both at Stanford[2] (at zero and low magnetic fields) and in our laboratory[3] (at high magnetic fields). Because of the low frequency, these oscillations have usually been attributed to some sort of ion motion, but it has been a mystery why the spectrum should be concentrated in a region so far below the ion plasma frequency. It is the purpose of this note to suggest that whereas the exact mechanism for sustaining high-frequency oscillations at a sheath is unknown, there may be a natural mechanism for generating very low-frequency oscillations near a sheath when the ion temperature is much lower than the electron temperature.

This mechanism is the electron pressure, which causes the ions to stream into the last mean free path before the wall with a velocity equal to or greater than $V_s \equiv (kT_e/M)^{\frac{1}{2}}$, where M is the ion mass. If the ions are cold and collisions infrequent, and if the ions are accelerated by a small electric field in the plasma, they will constitute a monoenergetic stream moving at a velocity V_s relative to the Maxwellian electron distribution. The velocity distribution is shown schematically in Fig. 1. Such a situation is unstable to ion oscillations of all wave numbers $k > 0$, as has been shown by Jackson[4] and Bernstein and Kulsrud[5] and is of marginal stability for $k = 0$. If $\lambda \gg h$, where λ is the ion mean free path and h the Debye length, then the time it takes an ion to travel a distance λ can be much greater than the growth time of the instability, and the oscillations can develop. If the ions have a finite temperature, however, the critical velocity for instability is increased, in a manner computed in reference 4, and the ion stream may be stable. This will be considered further later.

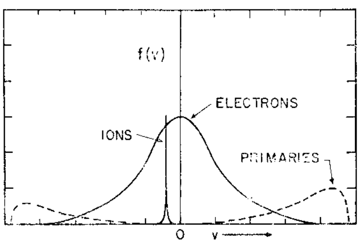

FIG. 1. Schematic of the velocity distributions near a cathode sheath. Zero velocity refers to the laboratory system, and positive velocity is away from the cathode.

The ion waves generated will travel away from the sheath with a velocity V_w in the ion center of mass system, which in turn is traveling toward the sheath with a velocity V_s. If $V_s = V_w$, the waves will not propagate into the plasma. If V_w were slightly greater than V_s, however, the waves would travel slowly into the discharge and appear as very low-frequency oscillations. It is interesting to see what sort of frequency spectrum might be expected in the laboratory system in this case. Let $V_w - V_s = \delta V_s$. The dispersion relation in the ion system is approximately

$$\omega = kV_w(1 + k^2h^2)^{-\frac{1}{2}} = kV_s(1 + \delta)(1 + k^2h^2)^{-\frac{1}{2}}. \quad (1)$$

If the ions are streaming at a velocity V_s, the frequency in the laboratory system is

$$\omega' = \omega - kV_s. \quad (2)$$

If we let $\Omega = \omega/\omega_{pi}$ and $K = kh$, where $\omega_{pi} = V_s/h$, we may solve for K from Eq. (1) and substitute into (2) to obtain

$$\Omega' = \Omega\{1 - [(1 + \delta)^2 - \Omega^2]^{-\frac{1}{2}}\}. \quad (3)$$

Ω' will vanish when $\Omega = 0$ and when $\Omega = \Omega_c = [(1 + \delta)^2 - 1]^{\frac{1}{2}}$. Frequencies higher than Ω_c will not propagate into the plasma, since their phase velocity will be smaller than the drift velocity. For small δ, Ω_c is approximately $(2\delta)^{\frac{1}{2}}$. There will be a maximum frequency Ω' observed in the laboratory. This is found by differentiating (3) with respect to Ω and setting the result to zero; for small δ and Ω, the result is $\Omega'_{\max} \cong (\frac{2}{3}\delta)^{\frac{3}{2}}$.

If the power spectrum in the ion system is $F(\Omega)\, d\Omega$, the spectrum in the laboratory system will be

$$F'(\Omega')\, d\Omega' = F[\Omega(\Omega')]\, |d\Omega/d\Omega'|\, d\Omega'. \quad (4)$$

As shown in Fig. 2(a) for the case $\delta = 0.01$, the observed spectrum is the sum of two branches, the upper branch corresponding to waves whose fre-

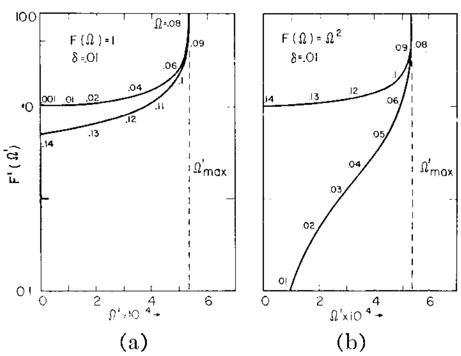

FIG. 2. The frequency spectrum in the laboratory system, plotted logarithmically, for $\delta = 0.01$ for (a) $F(\Omega) = 1$ and (b) $F(\Omega) = \Omega^2$. The numbers on the curves correspond to values of $\Omega(= \omega/\omega_{pi})$ in the ion c.m. system.

quencies appear low because Ω was originally low, and the lower branch corresponding waves whose frequencies are low because Ω was near Ω_c, and therefore their phase velocity is low. In Fig. 2 are shown power spectra both for the case $F(\Omega) = 1$, and for the case $F(\Omega) = \Omega^2$, as would obtain if the energy density were constant in phase space. In Fig. 3 are power spectra, summed over both branches, for various values of δ, for $F(\Omega) = 1$.

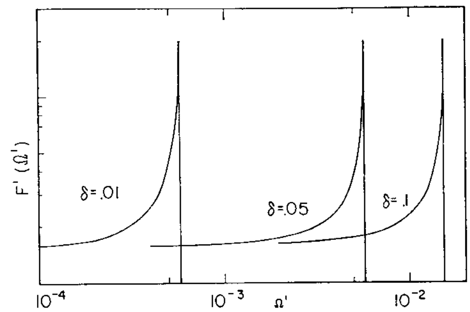

FIG. 3. Frequency spectra for $F(\Omega) = 1$ for three values of δ. The two branches of each spectrum have been added together.

Note that the spectra have a singularity at $\Omega' = \Omega'_{max}$, since the Jacobian in (4) becomes infinite there; however, the singularity is integrable, as it must be. This indicates the possibility of coherent oscillations in the laboratory system although the spectrum in the ion system is smooth.

We have seen that in order for these waves to be observed the ion streaming velocity V_i must be larger than the critical velocity V_c for ion wave instabilities, and that the wave velocity V_w must be larger than V_i. To lowest order, all these velocities are equal to $V_s = (kTe/M)^{\frac{1}{2}}$. We have considered three effects which can make these velocities slightly different from one another. These are (1) the existence of primary electrons, with a velocity distribution like that indicated by the dotted line in Fig. 1; (2) a finite ion temperature; and (3) electron-electron collisions. The detailed discussion of these effects cannot be presented here. To evaluate these effects would be very difficult, and it must in any case be done with reference to a particular experimental situation. To measure these effects would be impossible, because of the extreme accuracy required.

The most serious objection to this proposed mechanism is that ion-neutral collisions would be expected to damp waves of frequencies below the collision frequency so that they cannot propagate from the cathode sheath. However, it must be remembered that because of particle conservation ion streaming cannot occur at the cathode sheath without occurring to some extent also in the body of the plasma. Hence Doppler-shifted ion waves may be generated locally in the body of plasma.

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