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In order to check the accuracy of electron temperature measurements by the Langmuir probe method in a plasma with large fluctuations in space potential, a method has been used which employs a floating probe in addition to the usual biased probe. The results show that the oscillations do not noticeably affect temperatures found by the standard method, so that it is not usually necessary to use this two-probe time-resolved method even in the presence of fairly large oscillations. Furthermore, the usefulness of this method in the case of an inhomogeneous plasma is illustrated.

I. INTRODUCTION

IN many types of gas discharges, particularly in a strong magnetic field, Langmuir probe measurements are complicated by the presence of large-amplitude, low frequency oscillations in local plasma density and potential. This difficulty can be overcome by using a fast-sweeping voltage supply to obtain the probe characteristic in a time short compared with the period of the oscillations. A much simpler method, however, is to use two probes, one of which measures the probe current at a fixed potential, while the other measures the instantaneous floating potential. If kT_e is constant, which we shall assume in this paper, the floating probe will indicate the instantaneous space potential relative to the fixed probe, and a probe characteristic which is not averaged over the fluctuations in potential can be displayed on an x-y oscilloscope.

We shall use this method to verify that ordinary dc measurements of electron temperature are not greatly affected by the oscillations and to reveal certain properties of an inhomogeneous plasma. The classical method of double probes[1] is unsatisfactory for two reasons: (1) only a small number of electrons in the tail of the velocity distribution is sampled, and (2) the frequency response is limited because the entire probe circuitry is floating and there is, therefore, considerable stray capacitance to ground.

The electron temperature is measured by the slope of the $\ln I_e$-V curve in the transition region of the probe characteristic. The electron flux density I_e for a Maxwellian distribution is given by

$$I_e = n(kT_e/2\pi m)^{\frac{1}{2}} \exp(eV/kT_e), \qquad (1)$$

where n is the plasma density and V the probe potential relative to the plasma potential; V is negative in this region. Hence the slope of the $\ln I_e$-V curve is just e/kT_e. Since I_e is linearly proportional to n, it is clear that fluctuations in n will not affect the average value of I_e. On the other hand, fluctuations in V will increase the value of I_e because of the nonlinearity of Eq. (1). It is well known, however, that the slope of the *time-averaged* $\ln I_e$-V curve is unaffected by oscillations in V as long as the electron distribution is Maxwellian and the excursions in V do not extend into the saturation regions of the probe characteristic. This can be seen by adding a time-varying part $\tilde{n}$ and $\tilde{V}$ to n and V, so that the time average of Eq. (1) is given by

$$\langle I_e\rangle = n(kT_e/2\pi m)^{\frac{1}{2}} \times\exp(eV/kT_e)\langle(1+\tilde{n}/n)\exp(e\tilde{V}/kT_e)\rangle. \qquad (2)$$

As was neatly pointed out by Garscadden and Emeleus,[2] the time-averaged quantity is independent of V, so that $d\ln I_e/dV$ is unaffected regardless of the amplitudes, shapes, and phases of the fluctuations.

In the particular case $\tilde{n}=0$, $\tilde{V}=\delta V\cos\omega t$, the average current is given by

$$\langle I_e\rangle = I_{e0}\langle\exp(e\delta V\cos\omega t/kT_e)\rangle = I_{e0}I_0(e\delta V/kT_e), \qquad (3)$$

a result first given by Kojima, Takayama, and Shimauchi.[3] Here I_0 is a Bessel function. Experimental verification of Eq. (3) has been provided by numerous authors.[2,4–7] The effect of excursions into the saturation electron and ion current regions has been treated, respectively, by Garscadden and Emeleus[2] and Boschi and Magistrelli.[6] Cross-correlations between $\tilde{n}$ and $\tilde{V}$ have been treated by Crawford.[7] In all these investigations, oscillations in $\tilde{V}$ were produced by imposing an oscillating potential on the probe. In the present work, we study the effect on apparent electron temperature of actual large-amplitude density and space potential fluctuations in an unstable plasma.

II. EXPERIMENTAL ARRANGEMENT

A schematic of the apparatus is shown in Fig. 1. The two Langmuir probes, 0.38 mm in diameter by 2.3 mm long, are spaced about 2 mm apart and are normally aligned along the same line of force; that is, rotated 90°

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[1] E. O. Johnson and L. Malter, Phys. Rev. **80**, 58 (1950).

[2] A. Garscadden and K. G. Emeleus, Proc. Phys. Soc. (London) **79**, 535 (1962).

[3] S. Kojima, K. Takayama, and A. Shimauchi, J. Phys. Soc. Japan **8**, 55 (1953).

[4] K. Takayama, H. Ikegami, and S. Miyazaki, Phys. Rev. Letters **5**, 238 (1960).

[5] M. Sugawara and Y. Hatta, Inst. Plasma Phys. (Japan) Report IPPJ-4 (1963).

[6] A. Boschi and F. Magistrelli, Nuovo Cimento **29**, 487 (1963).

[7] F. W. Crawford, J. Appl. Phys. **34**, 1897 (1963).

from the position shown on the schematic. This alignment is made by floating both probes and observing the difference in the ac component of the floating potentials by means of a differential amplifier. The probe shaft is rotated until the difference signal is a minimum. Since the correlation length of the oscillations along the magnetic field is much larger than the probe spacing, this difference signal can be made much smaller than the individual signals. Thus both probes are immersed in plasma of approximately the same space potential.

One probe is then biased to a steady but variable voltage by the power supply shown on the right of Fig. 1. The current to the biased probe is measured by a 100-Ω resistor and a Tektronix plug-in differential amplifier. A high rejection ratio is necessary because the 100-Ω resistor may be biased to a comparatively high voltage. The current signal is then fed to the vertical axis of a Tektronix type 536 x-y oscilloscope.

The other probe is allowed to float, and measures the ac component of the space potential. The instantaneous potential difference between the floating probe and the biased probe is then displayed on the horizontal axis of the oscilloscope through another plug-in differential amplifier.

Although high rejection ratio is not necessary here, one has to worry about frequency response. The output impedance of the floating probe, as determined by varying the terminating resistor, is of the order of several thousand ohms. Since it is not easy to reduce the capacity to ground of the probe lead below several tens of picofarads, response to 10 Mc is not easily achievable. Fortunately, the measured frequency spectrum of the oscillations shows that the dominant fluctuations are well below 1 Mc. Still, in order to extend frequency response to 1 Mc it was necessary to minimize stray capacity by using a high impedance 'scope probe placed close to the Langmuir probe. A 'scope probe was also used on the biased probe merely to balance the gains on the inputs to the horizontal amplifier.

To see how the time-resolved probe characteristic is obtained, we note that the floating probe is held at a constant dc potential by the plasma, while the biased probe is held at a zero ac potential by the power supply. Thus the ac part of the potential difference between the two probes comes from the floating probe, and the dc part from the biased probe. These parts are then added in the proper phase by the differential amplifier. As the plasma potential oscillates, the spot on the 'scope automatically traces out a portion of the probe characteristic. Then if the power supply is varied, the entire probe characteristic is traced out with no nonlinear averaging of the probe current.

In practice the floating probe voltage is not connected directly to the 'scope but is applied through a 1-μF blocking capacitor. Aside from shifting the horizontal position of the spot on the 'scope, this has the effect of maintaining a constant probe output impedance by ensuring that the load line of the terminating resistor is always drawn through the same region of the probe curve. The 100-kΩ terminating resistor serves to attenuate capacitive pickup through the probe shield, to discharge the 1-μF capacitor, and to make the load line fairly constant with frequency by paralleling the stray capacitance. The battery shown in series with the bias supply is to enable the voltage to be swept through zero from negative to positive values, and the 4-kΩ bleeder on the bias supply enables it to deliver negative current.

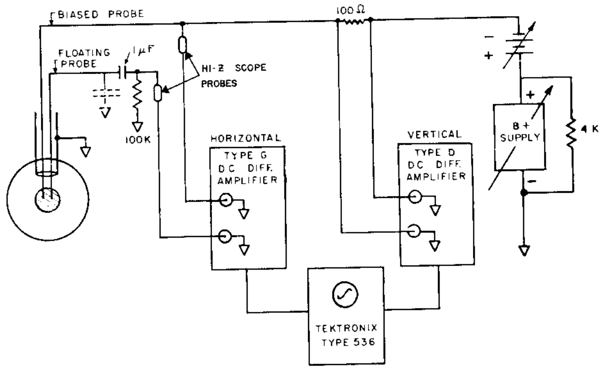

FIG. 1. Block diagram of double-probe circuit for obtaining time-resolved probe characteristics.

The plasma was produced by a hot-cathode reflex discharge[8] in a magnetic field of 4000 G and had a density of order 10^{12} cm^{-3}. A continuous spectrum of oscillations below 1 Mc in both density and potential is observed in this discharge. These oscillations are believed to be caused by drift instabilities[9] driven by the radial electric field.[10] The spectrum is typical of that in many other types of discharges in strong magnetic fields. The amplitude of the potential fluctuations is of order kT_e/e; both the amplitude and kT_e are functions of radial position.

III. MEASUREMENT OF ELECTRON TEMPERATURE

A typical probe characteristic obtained by this method is shown in Fig. 2. In (a) the probe bias was set near the floating potential, and a part of the probe characteristic was swept out by the oscillations of plasma potential. Note that the oscillations are so large that the excursions extend into the region of saturation ion current. In (b) the probe bias was slowly varied during the exposure so that the entire characteristic is traced out.

The trace is rather broad and ill-defined for several reasons. The largest part of the fuzziness is due to fluctuations in density, which merely broaden the trace in the

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[8] R. Bingham, F. F. Chen, and W. Harries, Princeton Plasma Physics Laboratory Report MATT-63 (1962).

[9] F. F. Chen, Princeton Plasma Physics Laboratory Report MATT-249 (1964).

[10] F. F. Chen, Phys. Rev. Letters **8**, 234 (1962).

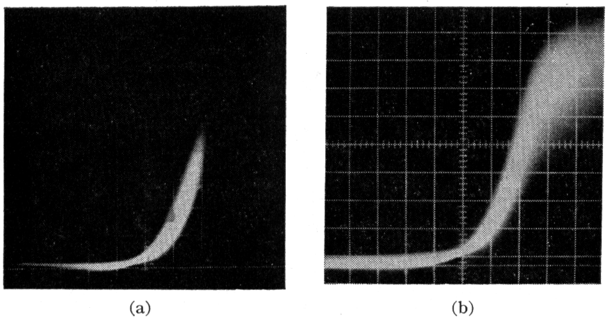


FIG. 2. Typical time-resolved probe characteristics. In (a), the dc probe bias is fixed; in (b) it is varied to sweep out the entire curve. The horizontal gain was 5 V/div.

vertical direction symmetrically around the average value, and, therefore, should not affect the measurement of kT_e. The remainder of the fuzziness is probably due to inaccuracies of the method: (1) difference in space potential at the two probes, (2) fluctuation of the electron temperature, or (3) inadequate frequency response of the circuitry or the probe sheaths. The extent of these inaccuracies is indicated by the narrowness of the trace at the floating potential; if density fluctuations were the only effect, the trace should be very narrow at the floating potential where the current is zero regardless of density. A check on this was made by aligning the probes perpendicular to the magnetic field. In this orientation, the probe separation is a sizable fraction of the correlation length across the field; hence the two probes are not in regions of identical space potential. It was then seen that the trace was considerably broadened at the floating potential.

The electron temperature is found from such oscillograms by drawing a line by eye through the middle of the trace and measuring the electron current point by point from a baseline drawn by extrapolating the saturation ion current. The points are then plotted on semilogarithmic paper, as shown in Fig. 3. In spite of the fuzziness of the trace, the reading error is quite small, and the temperatures found are reproducible to ±10%, which is the order of accuracy one is concerned with in this paper. The largest reading error in Fig. 3 is shown by the error bar on the first point. There is, however, a larger error due to an uncertainty in the position of the baseline, which will greatly affect the points at low currents. The circles in Fig. 3 are the points originally plotted. Since they form a continuous curve, it was obvious that the baseline was drawn too high. By adding a constant to each point, the curve can be straightened out, as indicated by the crosses. The slight rise at the low current end may indicate a high velocity component in the electron distribution, which would not be unexpected in the discharge. Only when a baseline correction is made in this manner do the results appear reasonable and reproducible.

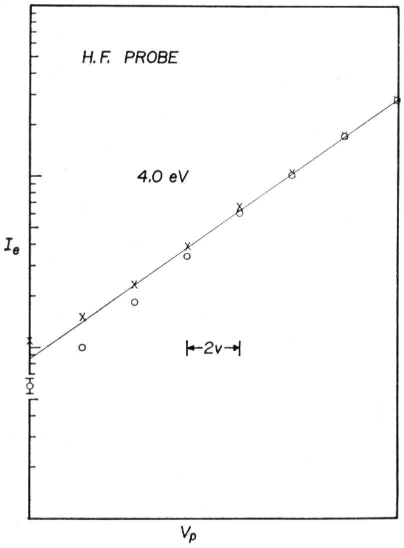


FIG. 3. Semilog plot of time-resolved probe characteristic.

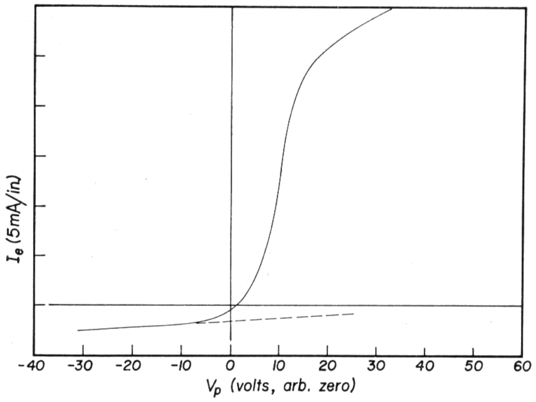


FIG. 4. Typical dc probe characteristic taken with an x-y recorder.

Figure 4 shows a probe characteristic taken on an x-y recorder by the usual dc method; that is, by filtering out the fluctuations in probe current and plotting against the probe potential relative to ground rather than to the plasma potential. The baseline is more accurately determined in this case, and usually no correction is necessary. The electron temperature is determined by the usual semilogarithmic plot.

Results on measurements of kT_e made by these two methods are shown in Fig. 5. Since the density, temperature, and hash amplitude varied with radius, a comparison of the two methods could be made under a variety of conditions by taking a radial scan. Each point in Fig. 5 is an average of several probe curves. The fluctuation in kT_e between runs under the same conditions was less than ±10%. Direct-current measurements were made immediately before and after each high frequency measurement to check that there was no drift in the discharge conditions.

It is seen that the two methods give temperatures that agree within about ±10%. There is no consistent tendency for the dc method to yield either a higher or a lower temperature. Thus temperatures measured by the dc method are fairly reliable, and it will not usually be necessary to use time-resolved measurements even when the potential fluctuations are of the order of kT_e or larger.

IV. APPLICATION TO AN INHOMOGENEOUS DISCHARGE

In Sec. III we were concerned with electron temperature measurements, and the details of the discharge were unimportant. In this section we shall see what the time-resolved probe method can show about the detailed structure of our particular discharge. In taking the self-swept probe traces at different probe biases and radial positions, we have found many strange and puzzling shapes, which may be understood if the inhomogeneity of the discharge is taken into account.

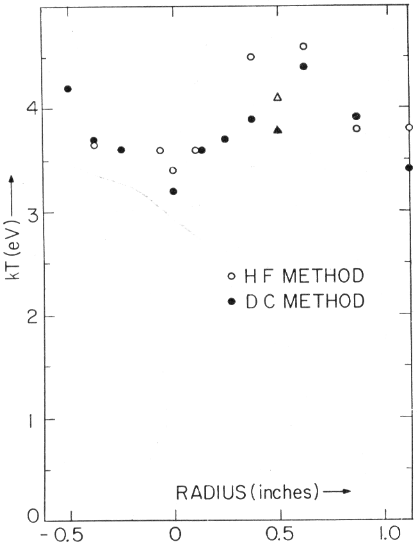
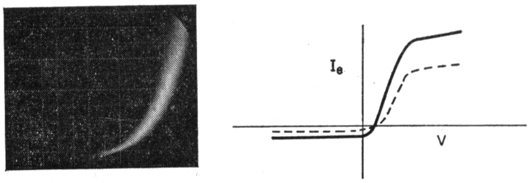


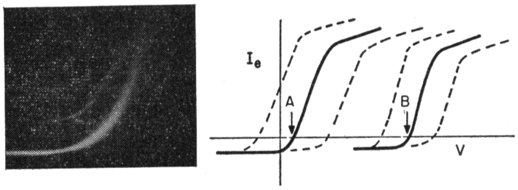
Fig. 5. Comparison of electron temperature measurements made with the high frequency method and the dc method at different radii in the discharge. The triangular points have a large error in radius.

As shown by streak photographs,[8] the discharge of the reflex arc is inhomogeneous and rotating, so that the probe will alternately sample different regions of plasma as the arc column rotates. If there are only two discrete regions, one would expect the experimentally observed probe characteristic to be a superposition of two different probe curves. This has actually been observed, as shown in Fig. 6.

If the two regions of plasma have the same floating potential but different densities, the probe will alternately sample the two curves shown on the upper right of Fig. 6. In the oscillogram on the upper left, this shows up as a "fishtail" effect on the saturation ion portion of the characteristic. Apparently the hash is too great for this to be seen in the electron portion of the characteristic.



SAME FLOATING POTENTIAL, DIFFERENT DENSITIES



GREATLY DIFFERENT FLOATING POTENTIALS

Fig. 6. Effects observed with the time-resolved double-probe method in an inhomogeneous plasma.

In case the floating potentials of the two regions of plasma are greatly different, the two probe curves can actually be separated. This is illustrated in the bottom half of Fig. 6. Suppose the two probe curves are so far apart in floating potential that they never overlap, even when they oscillate between the positions shown by the dotted lines. Then when the probe is biased at point A, the oscillations will trace out curve A when the probe is in the corresponding region of the plasma. However, when the probe is in the region corresponding to curve B, the difference between probe potential and floating potential is so large that the spot moves off the face of the 'scope, and curve B is not observed. When the probe is biased at point B curve A is similarly not observed. Thus the two curves can be viewed separately, as is shown on the left lower oscillogram, which is a double exposure with a vertical shift between the two curves. One curve is brighter than the other because the probe spends more time in one region than the other. The temperatures given by the two curves are 4.4 and 3.2 eV, a difference larger than the experimental error. The average temperature agrees well with that given by the dc method.

Finally, the shape of the trace with the dc probe bias fixed to draw saturation electron or ion current can yield information on the correlation between density and potential. If this correlation is small, the trace will be an elliptical blob with a vertical axis. On the other hand, if the density has a tendency to be high when the potential is low or vice versa, the ellipse will be tilted.

We have shown by a new technique that dc measurements of kT_e are accurate to within the experimental error of 10% even in the presence of space potential fluctuations so large that Eq. (2) is not valid. Furthermore, this technique can give information on the structure of an inhomogeneous discharge. Further details may be found in an earlier report.[11] We are indebted to Dr. R. Bingham for help with the experiment.

We wish to point out, in conclusion, that the use of a small floating probe near the main current-collecting probe is advisable even when there are no oscillations, since this serves as a check on the perturbation of the plasma caused by the probe. This is particularly important in a magnetic field.

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[11] F. F. Chen, Princeton Plasma Physics Laboratory Report MATT-62 (1961).