Interaction of Electron Beams with Inhomogeneous Plasmas

Francis F. Chen
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey
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In experiments on the interaction of electron beams with a plasma,[1-3] it is not uncommon to find large amplitude low-frequency oscillations superimposed on the radio-frequency oscillations under study. We have investigated the possibility that the low-frequency phenomena are drift waves excited by the beam and have found that the transverse motion of the ions makes such an interaction impossible. In Fig. 1 the solid curves show the dispersion relation for drift waves in a low-density inhomogeneous plasma in a uniform magnetic field.[4] The wave traveling in the direction of the ion pressure-gradient drift v_{di} is damped, and that traveling in the direction of the electron pressure-gradient drift v_{de} is unstable when the resistivity η is finite and there exists a small component $k_\parallel$ of $\mathbf{k}$ along the magnetic field $B\hat{\mathbf{z}}$. The resistive overstability goes over continuously[5] into the ordinary universal instability when $\eta \to 0$. The growth rate of the electron-drift mode is ordinarily small but is possibly increased by an electron beam if a resonance occurs between the parallel phase velocity of the drift wave in the plasma and the velocity of a space-charge wave[6] in the beam. The dashed lines on Fig. 1 show the dispersion relation for the latter at fixed $k_\perp$ and varying $k_\parallel$. If the beam plasma frequency ω_b is right, an intersection with the electron drift mode can occur. The wave on the beam traveling with $k_\perp$ in the opposite direction intersects the ion-drift mode over a larger range of ω_b and can possibly reverse the damping of that mode. Since drift waves are characterized by $\omega/k_\parallel \ll v_{the}$ (the electron thermal velocity) and since we consider energetic beams with directed velocity $v_0 \gg v_{the}$, resonance requires that a fast backward wave exist in the beam; this requires a large value of ω_b and a rather high perveance beam.

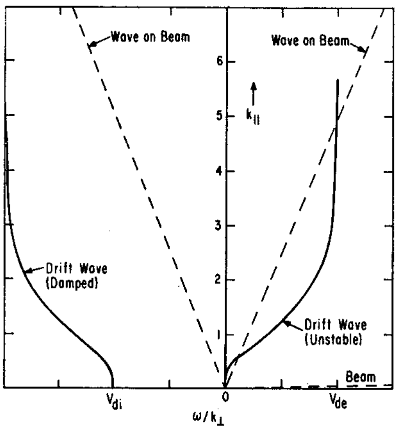

Fig. 1. Dispersion curves of $\omega/k_\perp$ vs $k_\parallel$ for fixed $k_\perp$ for resistive drift waves (solid curves) and for space-charge waves on an electron beam (dashed lines). The slope of the latter is proportional to the beam density. The ordinate is in units of $ak_\perp^{3/2}(n_e'e\eta/B)^{1/2}$.

Consider a system of four components: (1) plasma electrons with temperature KT_e, density n_e, and a uniform density gradient n_e'/n_e in the x direction; (2) plasma ions with temperature $\lambda_i KT_e$ and density $n_i = n_e$; (3) beam electrons with temperature $\lambda_b KT_e$, directed velocity $v_0\hat{\mathbf{z}}$, and uniform density n_b; (4) a uniform distribution $n_p = n_b$ of ions at $T_i = \lambda_i T_e$ to neutralize the beam. We linearize the equations of motion and of continuity for each species under electrostatic perturbations of the form $\exp i(k_\perp y + k_\parallel z - \omega t)$ and insert the perturbed densities into Poisson's equation. Finite ion Larmor radius effects are retained via the viscosity tensor.[4] We assume $k_\parallel/k_\perp$ sufficiently small that ion motions parallel to $\mathbf{B}$ may be neglected and $\Omega \equiv \omega/\omega_{ci} \ll 1$ (drift-wave approximations). The motion of plasma electrons along $\mathbf{B}$ is assumed to be controlled by resistivity alone; whereas, the parallel motion of beam electrons is controlled by inertia alone. We then obtain the following dispersion relation:

$$\frac{h^2}{a^2}\kappa^2\left[1 + \frac{\omega_b^2}{\omega_{ce}^2}\left(\frac{M}{m}\sin^2\theta + \frac{\tilde{\omega}^2\sin^2\theta - (1-\omega^2)\cos^2\theta}{\tilde{\omega}^2(1-\tilde{\omega}^2) + \lambda_b(m/M)\kappa^2[\tilde{\omega}^2\sin^2\theta - (1-\tilde{\omega}^2)\cos^2\theta]}\right)\right]$$
$$= \frac{\delta\kappa_\perp - i\kappa_\perp^2 Y}{\Omega + i\kappa_\perp^2 Y} - \frac{\kappa_\perp^2\Omega + (1-\lambda_i\kappa_\perp^2)\,\delta\kappa_\perp}{\Omega} \qquad (1)$$

where $h^2 = KT_e/4\pi n_e e^2$, $a^2 = KT_e/M\omega_{ci}^2$, $\omega_b^2 = 4\pi n_b e^2/m$, $\kappa^2 = \kappa_\parallel^2 + \kappa_\perp^2$, $\kappa_\perp = k_\perp a$, $\tan\theta = k_\perp/k_\parallel$, $\delta = an_e'/n_e$, $Y = (n_e e\eta/B)^{-1}\theta^2$, and $\tilde{\omega} = (\omega - k_\parallel v_0)/\omega_{ce}$. The right-hand side of Eq. (1), when set equal to 0, gives the dispersion relation for resistive drift waves,[4] shown in Fig. 1 for $\lambda_i = 1$. The left-hand side, when set equal to 0, is the dispersion equation for the electron beam and the nondrifting ions which neutralize it. For $\lambda_b \ll (v_0/v_{the})^2$, it can be shown that the term proportional to λ_b is negligible; hence, we neglect it. The square bracket can then be recognized as the usual cold beam-plus-ion dispersion relation

$$1 = \omega_{pi}^2\left(\frac{\cos^2\theta}{\omega^2} + \frac{\sin^2\theta}{\omega^2 - \omega_{ci}^2}\right) + \omega_{pe}^2\left[\frac{\cos^2\theta}{(\omega - k_\parallel v_0)^2} + \frac{\sin^2\theta}{(\omega - k_\parallel v_0)^2 - \omega_{ce}^2}\right] \qquad (2)$$

in the limit $\Omega^2 \ll 1$ and $\theta^2 \ll \Omega^2$. These approximations eliminate from Eq. (1) acoustic waves and the lower hybrid frequency, which appear in Eq. (2).

With $\lambda_b = 0$, Eq. (1) is seen to be a quartic equation for ω with complex coefficients. However, since $k_\parallel v_0 \gg \omega$, we may set $\tilde{\omega} \approx -k_\parallel v_0/\omega_{ce}$ in the left-hand side; whereupon, it becomes independent of ω, and the equation is only quadratic. One can then convince oneself that, for $h^2 \ll a^2$ and $n_{b0} \ll n_{e0}$, the left-hand side (the beam) has very little effect on the right-hand side (the drift waves). To be sure the higher degree terms have no effect, we have actually computed the solutions of the complex quartic with 20-digit accuracy. For parameters corresponding to the $m = 3$ mode of an 0.2-eV potassium plasma 1.5 cm in radius, with $n_e = 3 \times 10^{11}$ cm^{-3} and $B = 4000$ G, and a 1-keV beam of various densities up to $n_b = 2.5 \times 10^{10}$ cm^{-3}, we find that the imaginary part of the drift waves is affected by the beam less than 1 per cent in all cases, even when n_b has the value corresponding to synchronism between the drift wave and the beam space-charge wave. We, therefore, conclude that it is not possible to excite drift waves by a fast electron beam. This result is in agreement with the kinetic theory results of Kuleshov and Rukhadze[7] and of Arsenin,[8] which show that the destabilizing effect comes only in the Landau (resonant particle) term, which is exponentially small when $v_0 \gg v_{the}$.

Furthermore, we can show that no excitation can be expected for arbitrarily large values of n_b/n_e. Consider the square bracket of Eq. (1), which gives the dispersion equation for an electron beam passing through a neutralizing background of ions at rest. For $\lambda_b = 0$, this can be written

$$\tilde{\omega}^4 - (1 + p^2)\tilde{\omega}^2 + p^2 \cos^2 \theta = 0, \qquad (3)$$

$$p^{-2} = (M/m) \sin^2 \theta + (\omega_{ce}^2/\omega_b^2).$$

The roots are $\tilde{\omega}^2 \approx 1 + p^2 \sin^2 \theta$ and $\tilde{\omega}^2 \approx p^2 \cos^2 \theta$. For $\omega_b^2/\omega_{ce}^2 \ll m/M$, $p \approx \omega_b/\omega_{ce}$, the usual waves in the beam are recovered: $(\omega - k_\parallel v_0)^2 \approx \omega_{ce}^2 + \omega_b^2 \sin^2 \theta$ (upper hybrid), and $(\omega - k_\parallel v_0)^2 \approx \omega_b^2 \cos^2 \theta$ (Langmuir oscillation). However, if $\omega_b^2/\omega_{ce}^2 \gg m/M$, $p^{-2} \approx (M/m) \sin^2 \theta$, we obtain $(\omega - k_\parallel v_0)^2 \approx \omega_{ce}^2$ and $(\omega - k_\parallel v_0)^2 \approx \omega_{ci}\omega_{ce} \cos^2 \theta$, which is a modified lower hybrid frequency in the beam frame. For $B > 1000$ G or so, it can be shown that the cyclotron or upper hybrid wave is too fast for synchronism with a drift wave in the plasma. If it were not for the ion term $(M/m) \sin^2 \theta$, synchronism could be achieved with the space-charge wave by making ω_b and, hence p, sufficiently large. However, the presence of the ion term, which can be traced to the ion motion in the y direction due to finite inertia and hence finite compressibility, puts an upper limit on p. This has the effect of limiting the wave velocity in the beam frame to a velocity considerably smaller than v_0. The presence of a compressible ion fluid makes it impossible for fast backward waves in the beam to exist such that $\omega \ll \omega_{ci}$ in the laboratory; therefore, synchronism with a drift wave cannot be achieved for any beam density. The low-frequency phenomena mentioned at the outset must be caused by another effect, probably the centrifugal instability of a plasma rotating in a radial electric field.

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