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Decay of a plasma created between negatively biased walls

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When a slab plasma is created between negatively biased walls with a vacuum gap between the plasma and the wall, the rate of decay of the density is governed by the space-charge limit to the ion current. This limit is modified by the ion velocity gained in a presheath. It is shown that the sheath criterion implies eventual stagnation of an initially fast-moving sheath, followed by a slower motion on a different time scale.

I. STATEMENT OF THE PROBLEM

We consider the one-dimensional situation shown in Fig. 1, in which a plasma of thickness $2a$ is suddenly created between parallel plates separated by $2b$. A highly transparent grid is imagined to exist at the midplane $x = 0$, serving as a potential reference ($\phi = 0$) and as an electron sink to preserve charge neutrality. Ions are drawn to the highly negative walls, which have a potential ϕ_0 such that $-e\phi_0 \gg KT$. Electrons are Maxwellian at temperature T, and ions are assumed to be born with zero temperature. Collisions are neglected.

A similar problem was treated by the author almost 20 years ago,[1] when the concern was the stability of the ion stream in the sheath. The present paper deals with the motion of the sheath itself in a configuration specifically excluded from the previous work; namely, one in which the plasma does not fill the space between the walls. In the laboratory such a situation could occur, for instance, if the plasma were ionized by a collimated beam of ultraviolet light or of fast electrons as in a double-discharge gas laser. The ionization could even be accidental, as in UV-induced breakdown between high voltage electrodes. The positive electrode, in practice, would probably not be a real grid but would be located above or below the plasma. Because of the large electron mobility, such an electrode would fix the potential of the plasma almost as well as the fictional grid drawn in Fig. 1. The grid allows us to reduce the problem to one dimension and thus bring out the main features of the sheath-edge motion without obscuring them in algebraic detail.

For definiteness, suppose that KT is a few eV and $|e\phi_0|$ a few kV, and let the initial plasma density n_0 be of order 10^{10}–10^{12} cm^{-3}, with dimensions a and b of the order of centimeters, as is typical of gas discharges. The Debye length λ_D is then 10^{-2}–10^{-3} cm, several orders smaller than the plasma dimensions. Furthermore, the transit time of ions across the vacuum gap is much shorter than the decay time of the plasma, and the decay will evolve through a series of quasi-steady states in which the electron density follows the Boltzmann relation

$$n_e = n_0 \exp(e\phi/KT), \tag{1}$$

and the ion velocity and the potential ϕ have adjusted themselves to the steady-state values dictated by space charge.

To treat this problem at the simplest level, one can consider the plasma to be a perfect conductor emitting ions, neglecting both the structure of the sheath and the redistribution of density within the plasma. The flux of ions is then given by the Child–Langmuir formula for space-charge-limited flow[2]:

$$\Gamma = \frac{1}{9\pi}\left(\frac{2}{eM}\right)^{1/2}\frac{|\phi_0|^{3/2}}{s^2} \quad \text{(cgs esu)}, \tag{2}$$

where s is the distance between the plasma edge and the wall. This flux is supplied by the retrograde motion of the plasma edge, which exposes successive layers of ions to the accelerating field as excess electrons are rapidly removed by thermal motion to the anode grid. Setting $\Gamma = n_0(ds/dt)$, we obtain

$$s^2\frac{ds}{dt} = \frac{1}{9\pi n_0}\left(\frac{2}{eM}\right)^{1/2}|\phi_0|^{3/2}. \tag{3}$$

Thus,

$$s^3 = s_0^3 + \frac{1}{3\pi n_0}\left(\frac{2}{eM}\right)^{1/2}|\phi_0|^{3/2}t, \tag{4}$$

where $s_0 = b - a$ is the value of s at $t = 0$. The plasma is lost in a time

$$t = 3\pi n_0(eM/2)^{1/2}(b^3 - s_0^3)|\phi_0|^{-3/2}. \tag{5}$$

This time is longer than the transit time of each ion by a factor of approximately $\Omega_p\tau$, where Ω_p is the ion plasma frequency and τ is the transit time of an ion with the maximum energy $|e\phi_0|$.

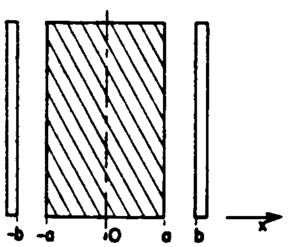

FIG. 1. Geometry of the idealized problem.

II. FORMULATION USING THE BOHM SHEATH CRITERION

At the next level of approximation, we must account for the structure of the sheath at the plasma edge. There are three distinct regions: (1) a plasma region where quasineutrality holds ($n_i \simeq n_e$), (2) a Debye sheath where the electron density falls from n_i to a negligible fraction of n_i, and (3) a

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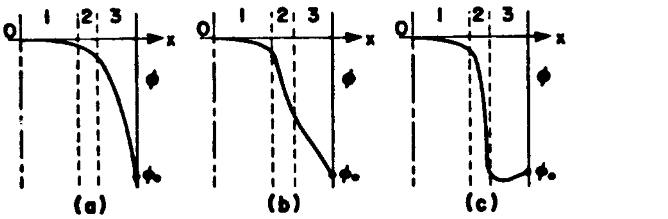

FIG. 2. Possible shapes for potential variation in the plasma (1), Debye sheath (2), and Child–Langmuir sheath (3). The thickness of region (2) has been exaggerated for clarity.

Langmuir sheath filled only with streaming ions. Regions (1) and (3) have a macroscopic scalelength L on the order of a or s, while region (2) has a much smaller scalelength on the order of λ_D. Though the thickness of the Debye sheath is entirely negligible, the conditions required for its existence—i.e., for a smooth transition between plasma and near-vacuum—have an important influence on the way the plasma decays.

Figure 2 shows three possible ways in which the solution $\phi(x)$ for each region can join together. In (a), the potential and its derivative vary monotonically through the three regions. In (b), the usual solution for a wall sheath is matched to the usual solution for a space-charge-limited diode, but with a change in the sign of the curvature. This cannot occur because a layer of negative charge would be required at the plasma edge. Once the Bohm sheath criterion[2] has been satisfied at the sheath edge, the electron density cannot exceed the ion density anywhere in the sheath. In (c), we show a nonmonotonic potential. The same argument can be used to eliminate this possibility; in addition, any small amount of dissipation would cause ions to be trapped in the trough and raise the potential there. We therefore consider only potential shapes of type (a).

In quasisteady state, the directed velocity of ions entering the Debye sheath at the boundary between regions (1) and (2) must exceed the Bohm velocity[2]

$$v_B \equiv (KT_e/M)^{1/2}. \tag{6}$$

This velocity gives the ion motion sufficient rigidity for n_i to exceed n_e as the potential ϕ falls, as is required to give $\phi''(x)$ the right sign. A presheath potential drop of $\frac{1}{2}KT_e$ or larger must therefore exist in the plasma. Indeed, it is well known that in the collisionless case the quasineutral plasma solution breaks down[1] (becomes double-valued) at $|e\phi| = 0.854KT$, where the ions have maximum energy $1.3v_B$. The ion distribution function $f_i(\phi,v)$ in this case can be shown[1] to be highly peaked at $v = (2e\phi/M)^{1/2}$ and to be independent of the spatial distribution of ionization sources, though the profile of $\phi(x)$ does, of course, depend on these sources. Our procedure is to define the sheath edge at $e\phi = \frac{1}{2}KT$ and approximate $f_i(v)$ there by a monoenergetic stream with $v = v_B$. The value of $|e\phi|$ rises to a few times KT within a few Debye lengths. Since $|e\phi_0| \gg KT$ is assumed, we may neglect the Debye sheath altogether and match the Child–Langmuir solution directly to the plasma solution at the sheath edge.

To do this we must modify Eq. (2) to account for the fact that ions enter the Langmuir sheath with a velocity $v_0 > v_B$. We omit the details, which are straightforward. Let the Child–Langmuir sheath be matched with zero slope to the plasma solution at a plane where $n = n_s, v_i = v_0, \Gamma = n_s v_0$. Then we find

$$\Gamma = \frac{(2/M)^{1/2}}{9\pi e^2}\,\frac{[(e|\phi_0| + \frac{1}{2}Mv_0^2)^{1/2} - (\frac{1}{2}Mv_0^2)^{1/2}][(e|\phi_0| + \frac{1}{2}Mv_0^2)^{1/2} + 2(\frac{1}{2}Mv_0^2)^{1/2}]^2}{s^2}, \tag{7}$$

which reduces to Eq. (2) when $v_0 = 0$. For $v_0 = v_B$ and $KT \ll |e\phi_0|$, this becomes

$$\Gamma = \frac{1}{9\pi}\left(\frac{2}{eM}\right)^{1/2}\frac{|\phi_0|^{3/2}}{s^2}\left[1 + 3\left(\frac{\frac{1}{2}KT}{|e\phi_0|}\right)^{1/2}\right]. \tag{8}$$

For the potential in the Langmuir sheath to span a range $\simeq|\phi_0|$ in a distance s, the ion flux Γ must have the value given by Eq. (2) or (8). The flux provided by the presheath is

$$\Gamma_s = n_s v_B = n_0 \exp(-1/2)(KT/M)^{1/2}, \tag{9}$$

n_0 being the midplane density. If Γ_s is larger than Γ, there is insufficient voltage to draw the Bohm current across the vacuum gap, and a monotonic potential of the type shown in Fig. 2(a) cannot occur. In the usual case, Γ_s will be smaller than Γ; then the deficit in flux must be made up by a retrograde motion of the sheath edge, giving an additional flux $n_s(ds/dt)$. Thus we have

$$\Gamma = n_s\left(v_B + \frac{ds}{dt}\right) \simeq 0.6n_0\left(v_B + \frac{ds}{dt}\right). \tag{10}$$

The midplane density is determined by the loss of ions to the sheath at the rate $\Gamma_s = n_s v_B$. The number of ions per cm^2 between the midplane and the sheath edge is $N = \beta n_0(b - s)$, where β is a profile-dependent form factor, somewhat less than unity, such that $\bar{n} = \beta n_0$. Since $\Gamma_s = -dN/dt$, we have

$$\beta\frac{d}{dt}[n_0(b - s)] = -0.6n_0 v_B. \tag{11}$$

III. APPROXIMATE SOLUTION

Equations (8), (10), and (11) are coupled differential equations governing the sheath motion and density decay. A complicated analysis can be avoided by realizing that the decay proceeds in two distinct stages. In the first stage, N is so large that it can supply the flux Γ without suffering any appreciable decrease. Then n_0 is approximately constant, and Eqs. (8) and (10) give

$$s^2\left(v_B + \frac{ds}{dt}\right) = C, \tag{12}$$

where

$$C = \frac{(2/eM)^{1/2}}{5.4\pi n_0} |\phi_0|^{3/2} \left[1 + 3\left(\frac{\frac{1}{2}KT}{e|\phi_0|} \right)^{1/2} \right]. \tag{13}$$

It is seen that the sheath stagnates ($ds/dt = 0$) at

$$s \equiv s_1 = (C/v_B)^{1/2}. \tag{14}$$

In principle, one can find the time to reach stagnation, t_1, by integrating Eq. (12):

$$t_1 = \int_{s_0}^{s_1} \frac{s^2 ds}{C - v_B s^2} = \frac{1}{v_B} \int_{s_0}^{s_1} \frac{s^2\, ds}{s_1^2 - s^2}$$

$$= \frac{1}{v_B} \left(\frac{s_1}{2} \ln \frac{s_1 + s}{s_1 - s} \right)_{s_0}^{s_1}. \tag{15}$$

This expression diverges, indicating that the constant n_0 assumption fails before s_1 is reached. However, the time t_1 can be estimated by dividing the distance $s_1 - s_0$ by the velocity $\dot{s} = Cs_0^{-2} - v_B$.

When $s = s_1$, the flux Γ is provided by the Bohm flux Γ_s, and the plasma slowly decays while maintaining an almost constant profile. The sheath edge must remain sharp as long as $\lambda_D \ll a$ holds. The loss rate is initially given by Eq. (8) with $s \simeq s_1$. As n_0 falls, however, Γ_s falls below Γ, and a slow motion of the sheath edge must occur to make up the deficit. If the plasma volume is large enough, the motion will be much slower in the second stage than in the first stage, and the term $-ds/dt$ in Eq. (11) can be neglected. We then have

$$n_0^{-1}\left(\frac{dn_0}{dt}\right) = -\frac{0.6 v_B}{\beta a_1}, \tag{16}$$

where $a_1 = b - s_1$. The density decays exponentially with the time constant

$$\tau = \beta a_1/0.6 v_B \simeq 4a_1/3v_B. \tag{17}$$

In the final stage, the plasma will collapse rapidly when n_0 and a reach such small values that the sheath edge must again move rapidly to supply the flux Γ. In other words, the plasma then cannot shield out the applied electric field because it contains an insufficient number of Debye lengths.

IV. EFFECTS NEGLECTED

To treat this problem more accurately would require solving for the distribution function $f_i(v)$, where the velocity v depends on $\phi(x_0,t_0) - \phi(x,t)$; i.e., the potential difference the ion has traversed since its birth. Because of the time dependence, $f_i(v)$ will show an explicit dependence on x, as contrasted with the steady-state case.[1] It is unlikely that this difficult numerical calculation would greatly alter the time scales found in our approximate analysis.

It should be pointed out that the quasisteady transport assumed here cannot be set up without an initial state of turmoil. Suppose that a uniform plasma with warm electrons and cold ions is suddently created between $x = -a$ and $x = a$ in the presence of a vacuum electric field. Ions will be drawn toward the negative walls, and electrons will be repelled; but at $x = a + \epsilon$ the electron density will exceed the ion density, causing the potential $\phi(x)$ to curve upward instead of downward. The potential will then flop around, perhaps generating acoustic shocks and solitons, until the ions have been redistributed in the plasma to form a presheath. Only after the Bohm criterion is satisfied at the sheath edge can the quasisteady flow proceed.

[1] F. F. Chen, Nuovo Cimento **26**, 698 (1962).

[2] F. F. Chen, in *Introduction to Plasma Physics* (Plenum, New York, 1974), Sec. 8.2.