# Physical mechanisms for hot-electron stabilization of low-frequency interchange modes

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The stabilization of low-frequency background and hot-electron interchange modes is studied by using a relatively simple multi-fluid model. Physical pictures for the stabilization mechanisms such as the "charge-uncovering" effect are given and compared with the stabilization of interchange modes by the finite Larmor radius (FLR) effect.

## I. INTRODUCTION

Numerous calculations on the stability characteristics of tandem mirrors and bumpy tori have been carried out. The relevant physics has common features for both the tandem-mirror and bumpy-torus cases. Both of these configurations rely on a hot-electron population, although their roles are different. The stability theories[1–3] predict that the dominant instabilities in those configurations which include a hot-electron component are classified into five possible types: a low-frequency, hot-electron interchange mode, which is stable for sufficiently large values of the ion density; a high-frequency, hot-electron interchange mode, which is also stable for high enough ion density; a magnetic compressional mode, which is unstable if the ion density is too high; a background plasma interchange mode, which is stable either if the hot-electron diamagnetic well is deep enough or if the uncanceled part of the $E\times B$ drift (depending on the hot electron to core density fraction) is large enough; and a background pressure driven interchange with hot-electron dynamics, which is stabilized for sufficiently small background pressure.

A large experimental effort is currently aimed at correlating fluctuation behavior with the hot-electron component in the Nagoya Bumpy Torus (NBT) and the ELMO Bumpy Torus (EBT) experiments. These observations[4,5] generally indicate that the system seems to be stable against the background interchange mode even if the beta value of hot electrons is not enough to create a diamagnetic well. For the plasma parameters of the present experiment, the core plasma beta $\beta_c$ is less than 0.1%, and the hot-electron beta value $\beta_h$ is $\simeq$3%–6%. The purpose of the present study is to clarify the physical picture for the stabilization of the low-frequency background and hot-electron interchange modes by means of a relatively simple multi-fluid model.

## II. DERIVATION OF THE DISPERSION RELATION

To simplify the analysis, we use the following model: (a) slab geometry; (b) background densities varying only in the $x$ direction, perpendicular to a uniform magnetic field $\mathbf{B}_0 = B_0\hat{z}$; (c) fluid plasma composed of three components—core electrons, ions, and hot electrons; (d) low plasma beta (electrostatic approximation); and (e) field curvature introduced only through an artificial gravity. The model configuration is shown conceptually in Fig. 1(a). There we consider a plasma boundary lying in the $y$-$z$ plane, density gradients for three components $\nabla n_{0j}$ in the $x$ direction, and gravitational fields $\mathbf{g}_j$ ($j$ refers to the particle species) in the $-x$ direction. In the equilibrium state, we have

$$m_j n_{0j}(\mathbf{v}_{0j}\cdot\nabla)\mathbf{v}_{0j} = q_j n_{0j}\mathbf{v}_{0j}\times\mathbf{B}_0 + m_j n_{0j}\mathbf{g}_j, \tag{1}$$

where we assume that $\mathbf{g}_j$ is constant, so that $(\mathbf{v}_{0j}\cdot\nabla)\mathbf{v}_{0j}$ vanishes. Taking the cross product of Eq. (1) with $\mathbf{B}_0$, we find

$$\mathbf{v}_{0j} = \frac{m_j}{q_j}\frac{\mathbf{g}_j\times\mathbf{B}_0}{B_0^2}, \tag{2}$$

where $\mathbf{g}_j = -\hat{x}v_\parallel^2/R_c$, $R_c$ being the radius of magnetic curvature. Since we consider $\mathbf{g}_j$ to be a curvature driven gravitational force, Eq. (2) represents a curvature drift velocity.

Note that the pressure terms $\nabla p_j$ have been omitted from the equations of motion, though the temperatures must clearly be finite to produce the curvature drifts. This omission is possible because we consider only flute modes with $k_\parallel = 0$, which have growth rates unaffected by drift-wave excitation. However, the real part of the frequency in what follows will be missing a term of order $\omega_{*j} \equiv k(\kappa T_\parallel/eB_0)|\nabla n_0/n_0|$, which, compared with $kv_{0j}$ as given by Eq. (2), is larger by a factor $\simeq(T_\perp/T_\parallel)(R_c/a)$. Thus our results on $\mathrm{Re}(\omega)$ are accurate only for $T_\perp/T_\parallel$ much smaller than the aspect ratio, and , in practice, can be applied to experiments only after corrections for both $\mathbf{E}_0\times\mathbf{B}_0$ rotation and drift-wave effects.

If a ripple develops in the interface as the result of random fluctuations [Fig. 1(b)], the drifts $\mathbf{v}_{0j}$ will cause a charge to build up on the sides of the ripple, and an electric field will develop which changes sign as one goes from crest to trough in the perturbation. As can be seen from Fig. 1(b), the $\mathbf{E}_1\times\mathbf{B}_0$ drift is upward in those regions where the surface has moved upward, and downward where it has moved downward. Whether the perturbation grows or not depends on whether these $\mathbf{E}_1\times\mathbf{B}_0$ drifts are in phase, as shown in Fig. 1(b), or out of phase with the density ripple.

To study the effect of the hot-electron component on stability, we use the usual linearized wave analysis for waves propagating in the $y$ direction with $\mathbf{k} = k\hat{y}$. The perturbed equations of motion are

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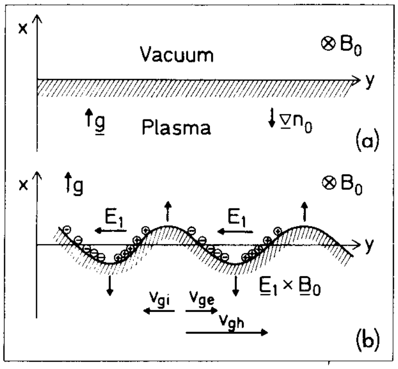


FIG. 1. (a) Configuration used as the theoretical model. (b) Physical mechanism of the interchange instability.

$$m_j n_{0j}\left(\frac{\partial}{\partial t}\mathbf{v}_{1j} + (\mathbf{v}_{0j}\cdot\nabla)\mathbf{v}_{1j}\right) = q_j n_{0j}(\mathbf{E}_1 + \mathbf{v}_{1j}\times\mathbf{B}_0), \tag{3}$$

where the information on $\mathbf{g}_j$ is contained in $\mathbf{v}_{0j}$. For perturbations of the form $\exp[i(ky-\omega t)]$, we have

$$m_j(\omega - \mathbf{k}\cdot\mathbf{v}_{0j})\mathbf{v}_{1j} = iq_j(\mathbf{E}_1 + \mathbf{v}_{1j}\times\mathbf{B}_0). \tag{4}$$

Solving for $\mathbf{v}_{1j}$ to order $(\omega - kv_{0j})^2/\omega_{cj}^2$, we obtain

$$v_{1jx} = \frac{E_{1y}}{B_0}\left(1 + \frac{(\omega - kv_{0j})^2}{\omega_{cj}^2}\right),$$

$$v_{1jy} = -i\frac{E_{1y}}{B_0}\frac{\omega - kv_{0j}}{\omega_{cj}}, \tag{5}$$

where $\omega_{cj}$ is the signed cyclotron frequency $q_jB_0/m_j$, and $E_{1x}$ vanishes by virtue of the "local" approximation $E_{1x} = -\partial\phi_1/\partial x = 0$. It is found from Eq. (5) that the $x$ component of $\mathbf{v}_1$ is an $\mathbf{E}_1\times\mathbf{B}_0$ drift which is independent of particle species when $|\omega - kv_{0j}|^2 \ll \omega_{cj}^2$ because we neglected finite Larmor radius (FLR) effects in the present model, and that the $y$ component of $\mathbf{v}_{1j}$ is a polarization drift in the particle frame. Note that the hot-electron polarization drift may be comparable to that of ions if $T_h/T_i \simeq m_i/m_e$.

The perturbed equation of continuity is

$$(\omega - \mathbf{k}\cdot\mathbf{v}_{0j})n_{1j} = -in'_{0j}v_{1jx} + n_{0j}\mathbf{k}\cdot\mathbf{v}_{1j}, \tag{6}$$

which, when combined with Eq. (5), gives

$$n_{1j} = -ikn_{0j}\frac{E_{1y}}{B_0}\left(\frac{n'_{0j}}{kn_{0j}} + \frac{(\omega - kv_{0j})}{\omega_{cj}}\right)(\omega - kv_{0j})^{-1}. \tag{7}$$

Assuming the $Z=1$ quasineutrality condition $n_{1i} = n_{1e} + n_{1h}$, we have the following cubic dispersion relation:

$$\frac{1}{\omega + \Omega_i} - \frac{1-\alpha}{\omega - \Omega_e} - \frac{\alpha}{\omega - \Omega_h} - \frac{kL_n}{\Omega_c} = 0, \tag{8}$$

where

$$\nabla n_0 = (-n_0/L_n)\hat{x}, \quad L_n > 0,$$
$$\mathbf{v}_{0i} = -\hat{y}\kappa T_i/eB_0R_c, \quad R_c > 0,$$
$$\mathbf{v}_{0e} = \hat{y}\kappa T_e/eB_0R_c,$$
$$\mathbf{v}_{0h} = \hat{y}\kappa T_h/eB_0R_c,$$
$$\Omega_i = -kv_{0i}, \quad \Omega_i > 0,$$
$$\Omega_e = kv_{0e}, \quad \Omega_e > 0,$$
$$\Omega_h = kv_{0h}, \quad \Omega_h > 0,$$
$$\alpha = n_{0h}/n_{0i} = n_{0h}/n_0,$$
$$\Omega_c = eB_0/m_i.$$

If we define $\overline{\Omega}_c = \Omega_c/kL_n$, Eq. (8) reduces to

$$\omega^3 + A\omega^2 + B\omega + C = 0, \tag{9}$$

where

$$A = \Omega_i - \Omega_e - \Omega_h, \tag{10}$$

$$B = \Omega_h(\Omega_e - \Omega_i) - \Omega_e\Omega_i + \overline{\Omega}_c[(1-\alpha)\Omega_e + \Omega_i + \alpha\Omega_h], \tag{11}$$

$$C = \Omega_i\Omega_e\Omega_h - \overline{\Omega}_c[\Omega_e\Omega_h + (1-\alpha)\Omega_i\Omega_h + \alpha\Omega_i\Omega_e]. \tag{12}$$

If we then assume the frequency ordering

$$\Omega_h \gg \Omega_i,$$
$$\alpha\Omega_h = o(\Omega_i) = o(\Omega_e),$$

Eqs. (10)–(12) can then be written

$$A \simeq -\Omega_h, \tag{13}$$

$$B \simeq \Omega_h(\Omega_e - \Omega_i) + \overline{\Omega}_c(\Omega_e + \Omega_i + \alpha\Omega_h), \tag{14}$$

$$C \simeq \Omega_i\Omega_e\Omega_h - \overline{\Omega}_c\Omega_h(\Omega_c + \Omega_i). \tag{15}$$

## III. STABILIZATION OF INTERCHANGE MODES

We now consider the stabilization of low-frequency, hot-electron interchange modes and background interchange modes caused by the hot-electron component.

### A. Hot-electron interchange mode

Let $\omega = o(\Omega_h)$ and $\overline{\Omega}_c \ll \Omega_h^2/\Omega_i$. Then $C$ in Eq. (15) is at most of order $\Omega_h^2$ and can be dropped. We now have

$$\omega^2 + A\omega + B = 0, \tag{16}$$

where

$$A = -\Omega_h, \tag{17}$$

$$B = \overline{\Omega}_c(\Omega_e + \Omega_i + \alpha\Omega_h). \tag{18}$$

The solutuion of Eq. (16) is

$$\omega = \tfrac{1}{2}\Omega_h \pm [\tfrac{1}{4}\Omega_h^2 - \overline{\Omega}_c(\Omega_e + \Omega_i + \alpha\Omega_h)]^{1/2}. \tag{19}$$

The frequency is $\mathrm{Re}(\omega) = kv_{0h}/2$, which is the average of $kv_{0h}$ and $kv_{0i}$ in the limit $\Omega_h \gg \Omega_i$. At this frequency, the charge buildup in the wave frame [Fig. 1(b)] is equally shared by the hot electrons of density $\alpha n_0$ and the neutralizing ions, also of density $\alpha n_0$. The cold ions and electrons of density $(1-\alpha)n_0$ drift with essentially the same velocity in this frame and can be made to contribute equally to the charge buildup by a small shift in $\mathrm{Re}(\omega)$, or order $\Omega_i$, which has been neglected in this limit. The asymptotic growth rate is

$$\gamma = -i\,\mathrm{Im}(\omega) \simeq [\overline{\Omega}_c(\alpha\Omega_h + \Omega_i + \Omega_e)]^{1/2},$$

or

$$\gamma^2 = \frac{\Omega_c}{L_n}(\alpha v_{0h} - v_{0i} + v_{0e})$$

$$= \frac{\kappa T_i + \kappa T_e + \alpha\kappa T_h}{MR_c}\left|\frac{n_0'}{n_0}\right|. \quad (20)$$

Thus the instability is driven by the total pressure. Stabilization occurs at

$$\tfrac{1}{4}\Omega_h^2 = \gamma^2 \simeq \alpha\Omega_h\bar{\Omega}_c,$$

for $\alpha\Omega_h \gtrsim \Omega_i,\Omega_e$. Thus, stability requires

$$\alpha < \frac{1}{4}\frac{\Omega_h}{\bar{\Omega}_c} = \frac{1}{4}\frac{kv_{0h}}{\Omega_c}kL_n, \quad (21)$$

which is in agreement with the standard result, Eq. (107) of Ref. 1. As in the stabilization of an ordinary flute mode by ion inertia,[6] the mechanism is the polarization drift of the ions, which in this case has been increased by large Re($\omega$) [cf. $v_{1iy}$ in Eq. (5)]. The divergence of $\mathbf{v}_{1i}$ cancels the charge build-up caused by $v_{0h}$.

### B. Background interchange mode

Now let $\omega = o(\Omega_i) = o(\Omega_e)$. The $\omega^3$ term can then be neglected, and we have

$$\omega^2 + \bar{B}\omega + \bar{C} = 0, \quad (22)$$

where

$$\bar{B} = \Omega_i - \Omega_e - \bar{\Omega}_c[\alpha + (\Omega_e + \Omega_i)/\Omega_h], \quad (23)$$

$$\bar{C} = -\Omega_i\Omega_e + \bar{\Omega}_c(\Omega_e + \Omega_i). \quad (24)$$

The solution is

$$\omega = \tfrac{1}{2}\{\Omega_e - \Omega_i + \bar{\Omega}_c[\alpha + (\Omega_e + \Omega_i)/\Omega_h]\}$$
$$\pm (\tfrac{1}{4}\{\Omega_e - \Omega_i + \bar{\Omega}_c[\alpha + (\Omega_c + \Omega_i)/\Omega_h]\}^2$$
$$+ \Omega_i\Omega_e - \bar{\Omega}_c(\Omega_e + \Omega_i))^{1/2}. \quad (25)$$

If

$$1 \gg \alpha \gg \Omega_i/\Omega_h,$$

as is the usual case, $\omega$ simplifies to

$$\omega = \tfrac{1}{2}(\Omega_e - \Omega_i + \alpha\bar{\Omega}_c) \pm [\tfrac{1}{4}(\Omega_e - \Omega_i + \alpha\bar{\Omega}_c)^2$$
$$+ \Omega_i\Omega_e - \bar{\Omega}_c(\Omega_e + \Omega_i)]^{1/2}. \quad (26)$$

Note that the only destabilizing term does not contain $\alpha$. The excitation of an interchange mode by the presence of hot electrons does not occur in this low-frequency case. The entire stabilizing effect of $\alpha$ is to decrease the number of cold electrons. In fact, we can derive the same equation by neglecting the hot-electron term in Eq. (8) altogether:

$$\frac{1}{\omega + \Omega_i} - \frac{1-\alpha}{\omega - \Omega_e} - \frac{kL_n}{\Omega_c} = 0,$$

which gives

$$\omega^2 + \bar{B}\omega + \bar{C} = 0, \quad (27)$$

where

$$\bar{B} = \Omega_i - \Omega_e - \alpha\bar{\Omega}_c, \quad (28)$$

$$\bar{C} = -\Omega_i\Omega_e + \bar{\Omega}_c(\Omega_e + \Omega_i). \quad (29)$$

Thus, we recover the same solution [Eq. (26)] as before. The frequency is

$$R_e(\omega) = \tfrac{1}{2}(\Omega_e - \Omega_i + \alpha\bar{\Omega}_c), \quad (30)$$

and the asymptotic growth rate is

$$\gamma = [\bar{\Omega}_c(\Omega_e + \Omega_i)]^{1/2} = \left(\frac{\kappa T_i + \kappa T_e}{MR_c}\left|\frac{n_0'}{n_0}\right|\right)^{1/2}. \quad (31)$$

The last term ($\alpha\bar{\Omega}_c$) in Eq. (28) or Eq. (30) arises from the small noncancellation of ion and electron $E\times B$ drifts caused by the replacement of cold electrons by hot ones. The frequency shift caused by this term plays an important role in the stabilization of interchange modes, the physical mechanism of which will be discussed in detail later. We note that the hot electrons with drift velocity $v_{0h}$ play no role in either Re($\omega$) or $\gamma$. This is because, in this low-frequency mode, the hot electrons stream by so fast in the wave frame that their orbits are undeflected by $\mathbf{E}_1$, and, furthermore, the space charge caused by their rippled density [Fig. 1(b)] appears as a high-frequency oscillation which the slow mode cannot follow.

We can rewrite the discriminant of Eq. (26) as

$$\tfrac{1}{4}(\Omega_e + \Omega_i + \alpha\bar{\Omega}_c)^2 - \alpha\bar{\Omega}_c\Omega_i - \bar{\Omega}_c(\Omega_e + \Omega_i)$$
$$\simeq \tfrac{1}{4}(\Omega_e + \Omega_i + \alpha\bar{\Omega}_c)^2 - \bar{\Omega}_c(\Omega_e + \Omega_i), \quad \text{for } \alpha \ll 1. \quad (32)$$

Thus the stabilization occurs, if $\alpha\bar{\Omega}_c \gg \Omega_i$, for

$$\alpha^2 > 4(\Omega_e + \Omega_i)\frac{kL_n}{\Omega_c} = 4\frac{kL_n}{\Omega_c}kv_{0h}\frac{T_e + T_i}{T_h}. \quad (33)$$

This is the standard result, Eq. (111) of Ref. 1 when $2T_c = T_e + T_i$. To obtain this, we assumed $\alpha\bar{\Omega}_c \gg \Omega_i$, while the stability requirement is

$$\alpha\bar{\Omega}_c > 2[\bar{\Omega}_c(\Omega_c + \Omega_i)]^{1/2} \simeq 2\sqrt{2}(\bar{\Omega}_c\Omega_i)^{1/2},$$

which is usually larger than $\Omega_i$. Therefore, if $\alpha$ is large enough to stabilize, it is large enough to make the necessary approximation. When $\alpha\bar{\Omega}_c \gg \Omega_i$, we have simply

$$\omega = \tfrac{1}{2}\alpha\bar{\Omega}_c \pm [(\tfrac{1}{2}\alpha\bar{\Omega}_c)^2 - \Omega_c(\bar{\Omega}_e + \Omega_i)]^{1/2}. \quad (26')$$

Equation (33) is the stability criterion for the low-frequency background interchange mode caused by the so-called "change-uncovering" effect discussed by Berk *et al.*[1] The criterion (33) gives a lower limit on the fraction of hot-electron density for stabilization. The stabilization is caused by the presence of $n_h$ rather than $\beta_h$ (diamagnetic well). We shall show that the mechanism can be traced to the inequality of ion and electron $\mathbf{E}_1\times\mathbf{B}_0$ drifts in a manner similar to the FLR effect.

## IV. PHYSICAL PICTURE OF THE CHARGE-UNCOVERING EFFECT IN THE BACKGROUND INTERCHANGE MODE

The charge separations given by the equation of continuity (6) are:

$$\begin{array}{ccc} [1] & [2] & [3] \end{array}$$

$$\frac{\partial n_i}{\partial t} = -ikv_{0i}n_i - ikn_{0i}v_{iy} + \frac{n_{0i}}{L_n}v_{ix}, \quad (34)$$

$$\frac{\partial n_e}{\partial t} = -ikv_{0e}n_e - ikn_{0e}v_{ey} + \frac{n_{0e}}{L_n}v_{ex}, \quad (35)$$

$$\frac{\partial n_h}{\partial t} = -ikv_{0h}n_h - ikn_{0h}v_{hy} + \frac{n_{0h}}{L_n}v_{hx}, \quad (36)$$

where, from Eq. (5),

$$v_{jx} \simeq \frac{E_y}{B_0}\left(1 + \frac{(\omega - kv_{0j})^2}{\omega_{cj}^2}\right), \tag{37}$$

$$v_{jy} \simeq \mp i \frac{E_y}{B_0} \frac{\omega - kv_{0j}}{|\omega_{cj}|}. \tag{38}$$

The terms [1] represent the charge buildup due to the zero-order drifts when plasma is rippled; the terms [2] represent the charge buildup due to polarization drifts; and the terms [3] represent the charge buildup due to $\mathbf{E}_1 \times \mathbf{B}_0$ drifts, if unequal. Consider first the $n_h$ equation. Once we assume $\omega \ll \Omega_h$, the left-hand side (lhs) of Eq. (36) can be neglected, and $n_h$ is given by

$$n_h = \underset{②}{-\alpha n_0 \frac{kv_{ny}}{\Omega_h}} \underset{③}{- i\alpha n_0 \frac{v_{hx}}{L_n \Omega_h}}. \tag{39}$$

Compare the polarization term ② with the ion term [2]:

$$\left|\frac{v_{ph}}{v_{pi}}\right| = \alpha \left|\frac{v_{hy}/\Omega_h}{v_{iy}/\omega}\right| = \alpha \frac{\Omega_c}{\omega_c|} \frac{\omega}{\Omega_i} \simeq \frac{m_e}{m_i}\alpha, \tag{40}$$

and also compare the $\mathbf{E}_1 \times \mathbf{B}_0$ term ③ with the difference $n_0 \Delta v_E$ between $n_{0i} v_{ix}$ and $n_{0e} v_{ex}$:

$$\left|\frac{n_{0h} v_{Eh}}{n_0 \Delta v_E}\right| = \left|\frac{(\alpha v_{hx})/(L_n \Omega_h)}{(\alpha v_{ex})/(L_n \omega)}\right| = \frac{[1 + \Omega_h^2/\omega_c^2]/\Omega_h}{(1/\omega)} \simeq \frac{\omega}{\Omega_h}, \tag{41}$$

provided $\Omega_h^2/\omega_c^2 \ll 1$. Thus, the charge separation due to the hot electrons is negligible relative to the other effects, and the hot component can be neglected altogether. This is because the hot electrons go by so fast that they do not have time to fluctuate in density. Thus, the effect is to remove a fraction of the core electrons from the wave.

Now consider the ion and electron equations alone. In the inertial terms, the electron $v_{ey}$ is down by $\Omega_c/\omega_c = m_e/m_i$, and as usual the ion polarization drift is the important one. In the ion $\mathbf{E} \times \mathbf{B}$ term, we have neglected FLR corrections and assumed $\omega^2 \ll \Omega_c^2$, so that

$$v_{ix} = E_y/B_0 = v_{ex}.$$

However, there are now fewer electrons, so there is a net charge accumulation due to $v_E$:

$$\frac{\partial n_+}{\partial t} = (n_{0i} - n_{0e}) \frac{E_y/B_0}{L_n} = \frac{\alpha}{L_n} n_0 \frac{E_y}{B_0}. \tag{42}$$

This is exactly like FLR stabilization[7], which has been discussed by one of the authors (FFC),[6] where the reduction of the ion $\mathbf{E} \times \mathbf{B}$ drift

$$v_{Ei} = v_{ix} = (E_y/B_0)(1 - \tfrac{1}{4}k^2\rho_i^2), \tag{43}$$

relative to the electron $\mathbf{E} \times \mathbf{B}$ drift $v_{Ee} = E_y/B_0$ causes a charge imbalance given by

$$\frac{\partial n_+}{\partial t} = \frac{n_0}{L_n} \frac{E_y}{B_0} \frac{1}{2} k^2 \rho_i^2. \tag{44}$$

Here $\rho_i$ is the ion Larmor radius $(2\kappa T_i/m_i)^{1/2}/\Omega_c$. The effect is most easily seen from a particle description. Figure 2 shows the situation when the FLR term dominates over the inertial term in an ordinary $g$ mode. In (a) we show the phase relationships among $n_1$, $\phi_1$, $E_{1y}$, and $v_{Ej}$. At the left, we see that $\phi$ and $n_1$ are 90° out of phase due to the charge-forming mechanism of Fig. 1. Since $v_{Ei} < v_{Ee}$ in the particle picture, the ions drift more slowly than the electrons. This difference, coupled with the density gradient $\nabla n_0$, creates a space charge $\sigma$, which is 180° out of phase with $n_1$. The space charge $\sigma$ has the effect of shifting the original space-charge distribution in the $-y$ direction. When the FLR effect is large, $\phi$ is shifted so that it is 180° out of phase with $n_1$, as shown in Fig. 2(b). The corresponding $\mathbf{E} \times \mathbf{B}$ drifts are also shifted, so that the new space charge $\sigma_{\rm FLR}$ due to $v_{ix} - v_{ex}$ is exactly out of phase with the space charge $\sigma_g$ due to the gravitational drifts. When $\sigma_{\rm FLR}$ and $\sigma_g$ are exactly equal and opposite, the interchange mode is at marginal stability.

Figure 3 shows the stabilization mechanism in a hot-electron plasma when the $\alpha$ effect dominates. In (a) we show the phase relationships when stabilization is weak. In this case, it is immaterial whether or not $v_{Ei}$ is slightly less than $v_{Ee}$. The fact that there are fewer core electrons (by a factor $1 - \alpha$) means that the $v_E$ drifts along $\nabla n_0$ cause a space-charge buildup $\sigma$, phased as shown. The hot electrons stream by so fast at the velocity $v_{0h}$ that their $\mathbf{E} \times \mathbf{B}$ drifts are completely out of synch with the wave; the low-frequency part of the hot-electron $\sigma$, as shown in Eq. (41), is relatively small. The space charge $\sigma$ [Fig. 3(a)] is now in phase with $n_1$, in contrast to that in Fig. 2(a), where it is 180° out of phase with $n_1$. Thus, the hot-electron stabilization mechanism, though similar to the FLR mechanism, has the opposite sign.

When the $\alpha$ effect is large, the space charge $\sigma_\alpha$ due to $v_{Ei} - (1 - \alpha)v_{Ee}$ overcomes the space charge $\sigma_g$ due to the gravitational drifts. The pattern of drifts and space charges is then shifted to that shown in Fig. 3(b). When the new $\sigma_\alpha$ and $\sigma_g$ are equal and opposite, the interchange mode is at marginal stability due to hot electron component. Owing to the change of sign noted earlier, $n_1$ and $\phi_1$ are now in phase (as in a drift wave), a situation opposite to that in FLR stabilization.

By measuring the phase of $\phi_1$ relative to $n_1$, the experimentalists can determine whether FLR or the hot-electron charge uncovering effect dominates the stabilization of a flute mode. There is also a difference in $k$ dependence. According to Eq. (33), small-$k$ modes are easier to stabilize by the $\alpha$ effect, while large-$k$ modes are easily stabilized by the FLR effect.

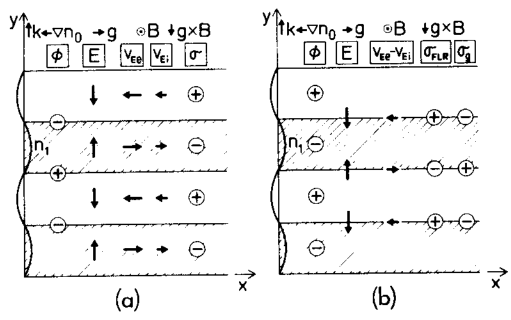


FIG. 2. Phase relationships in an unstabilized (a) and FLR stabilized (b) background interchange mode.

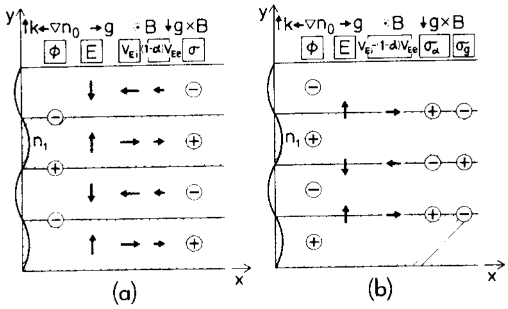

FIG. 3. Phase relationships in an unstabilized (a) and hot-electron stabilized (b) background interchange mode.

## V. DISCUSSION

We have studied the stabilization of low–frequency background and hot-electron interchange modes due to a hot-electron component. Although the present fluid model is highly idealized compared with the Vlasov model,[1–3] it provides useful information for understanding stabilization mechanisms such as the "charge-uncovering" effect. In particular, we have discovered that there are similarities between the FLR and hot-electron stabilization mechanisms.

The analogy to FLR stabilization can also be made quantitative. One of us has previously shown[8] that the RKR[7] result on FLR stabilization can be obtained from the fluid equations with magnetic viscosity. When translated to the notation of the present paper, Eq. (70) of Ref. 8 (with the misprint "4" corrected) reads

$$\omega = \tfrac{1}{2}(\Omega_e - \Omega_i - \tfrac{1}{2}k^2\rho_i^2\overline{\Omega}_c) \pm [\tfrac{1}{4}(\Omega_e + \Omega_i + \tfrac{1}{2}k^2\rho_i^2\overline{\Omega}_c)^2 - \overline{\Omega}_c(\Omega_e + \Omega_i)]^{1/2}. \quad (45)$$

According to Eq. (44), the FLR-induced ion space charge is proportional to $\frac{1}{2}k^2\rho_i^2 n_0$, while that caused by the replacement of cold electrons by hot ones is proportional to $-\alpha n_0$. The analogy between FLR and $\alpha$ stabilization can be made by replacing $\frac{1}{2}k^2\rho_i^2$ in Eq. (45) by $-\alpha$, the minus sign being due to the opposite phase of the two effects. Equation (45) then becomes

$$\omega = \tfrac{1}{2}(\Omega_e - \Omega_i + \alpha\overline{\Omega}_c) \pm [\tfrac{1}{4}(\Omega_e + \Omega_i - \alpha\overline{\Omega}_c)^2 - \overline{\Omega}_c(\Omega_e + \Omega_i)]^{1/2}. \quad (46)$$

Comparing this with Eq. (26), we see that the real parts are in exact agreement, with $\alpha\overline{\Omega}_c$ being the frequency shift due to the noncancellation of ion and electron $\mathbf{E}\times\mathbf{B}$ drifts. The stabilizing terms exactly agree when $\alpha^2\Omega_c^2$ is large compared with the term $(\Omega_e + \Omega_i)$ caused by ion inertia. When the FLR or $\alpha$ effect does not dominate, there is a difference in the stabilizing terms of Eqs. (26) and (46) because of the interference between inertial and $\alpha$ stabilization. As can be seen from Figs. 2(a) and 3(a), the FLR and $\alpha$ effects are 180° out of phase with each other, while the ion polarization drifts $v_{iy}$ [Eq. (38)] have the same phase on both diagrams.

We have restricted our discussion to instabilities in a sufficiently low beta limit ($\beta \ll 1$). Accordingly, we cannot discuss the beta limit such as the Nelson–Van Dam–Lee (NVL) stability boundary, which is related to an enhanced compressibility associated with compressional Alfvén waves. Also, the importance of the ambipolar field on low-frequency modes such as drift waves[9] and electrostatic flute modes[10] has been discussed recently. The present study should be extendable to a more complete discussion including those effects.


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