HIGH-FREQUENCY DRIFT WAVES EXCITED BY AN ELECTRON BEAM

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I. Introduction

In a recent experiment on period doubling and chaos in a plasma, Jain *et al.*[1] observed an instability occurring in rather unusual conditions. The frequency of about 10 kHz was below that of most instabilities except drift waves, but was above the argon-ion cyclotron frequency Ω_c . The magnetic field of 150 G was sufficient to constrain the electrons but not the ions. The plasma density of about 5×10^8 cm^{-3} was of the same order of magnitude as the density of beam electrons (of about 150 eV) used to ionize the plasma.

Drift waves with ω comparable to Ω_c have been considered by Chen[2], who found that, in the absence of a driving beam, instability occurred only for $\omega < \Omega_c$. In that paper, ion Larmor radius effects were not taken into account, but the results should be correct in the limit $T_i = 0$. In another paper, Chen[3] considered the excitation of drift waves by beams and concluded that it was not possible for weak beams; the dense beam case was not considered. Standard references on drift waves[4] ignore the case of large ion orbits in weak magnetic fields.

In this paper, we study the possibility of drift waves with $\omega > \Omega_c$ being excited by an electron beam in a very low density plasma with weakly magnetized ions. The problem is solved in plane geometry in the local approximation. The method for obtaining the Whittaker function solutions in cylindrical geometry can be found in earlier papers[5] . There are several basic mechanisms which may be operative in this case. One is the interaction of a negative-energy wave on the beam with the parallel velocity of the drift wave. A second is interaction through resonant particles. A third is simply "charge uncovering", which is the effect of replacing plasma electrons with beam electrons which cannot participate in the drift wave because of their large momenta. This effect has been explained[6] in connection with other instabilities. These mechanisms have not yet been sorted out.

II. Calculation of Dispersion Relation

A) Assumptions

We consider a plasma in Cartesian geometry with a uniform magnetic field $\mathbf{B} = B\hat{\mathbf{z}}$. There are three components: 1) ions with $Z = 1$, $T_i = 0$, $n_{oi} = n_o$; 2) plasma electrons with $T_e \neq 0$, $n_{oe} = (1-\alpha)n_o$; and 3) beam electrons with $T_b = 0$, $n_{ob} = \alpha n_o$, and $\mathbf{v}_o = u\hat{\mathbf{z}}$. All species have the same density gradient: $\nabla n_o = n_o'\hat{\mathbf{x}}$. We look for electrostatic oscillations of the form $\phi = \exp i(k_y y + k_z z - \omega t)$ with $k_z^2 << k_y^2$.

B) Equations of motion and continuity

1. Ions. The linearized equation of motion for the cold ion fluid is:

$$M\frac{\partial \mathbf{v}_i}{\partial t} = e(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) \quad . \tag{1}$$

For $\mathbf{E} = -\nabla\phi$, the solution can be written:

$$\begin{aligned} iv_{ix} &= \frac{\Omega_c}{\omega B}\left(\phi' - \frac{\Omega_c}{\omega}k_y\phi\right)\left(1 - \frac{\Omega_c^2}{\omega^2}\right)^{-1} \\ v_{iy} &= \frac{\Omega_c}{\omega B}\left(k_y\phi - \frac{\Omega_c}{\omega}\phi'\right)\left(1 - \frac{\Omega_c^2}{\omega^2}\right)^{-1} \\ v_{iz} &= \frac{\Omega_c}{\omega B}k_z\phi = 0 \quad . \end{aligned} \tag{2}$$

We neglect v_{iz} because k_z is assumed very small.

The linearized ion continuity equation is

$$\frac{\partial n_i}{\partial t} + \mathbf{v}_i \cdot \nabla n_{oi} + n_{oi}\nabla \cdot \mathbf{v}_i = 0 \tag{3}$$

or

$$\frac{n_i}{n_{oi}} = -\frac{1}{\omega}\left[i v_{ix}\frac{n_o'}{n_o} + i v_{ix}' - k_y v_{iy}\right] \quad . \tag{4}$$

Substituting the velocities from Eq. (2), we obtain

$$\frac{n_i}{n_o} = \frac{KT_e/M}{\Omega_c^2 - \omega^2}\,\frac{e}{KT_e}\left[\phi'' + \frac{n_o'}{n_o}\phi' - k_y^2\phi - k_y\frac{\Omega_c}{\omega}\frac{n_o'}{n_o}\phi\right] \quad . \tag{5}$$

2. Plasma electrons. Being the only species with finite temperature and non-negligible collision and Landau damping effects, the plasma electrons follow the equation:

$$mn_e\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -en\,(\mathbf{E} + \mathbf{v_e}\times\mathbf{B}) - KT_e\nabla n_e - mn_e\nu_{eff}\,\mathbf{v_e} \quad , \tag{6}$$

where ν_{eff} is an effective collision frequency to be discussed later. We may safely neglect the inertia and $\mathbf{v}\cdot\nabla\mathbf{v}$ terms on the left-hand side. We may also neglect the dissipative term in the perpendicular motions, since it gives rise only to the slow, classical diffusion of the plasma. Thus, the governing equations are:

$$en\,(\mathbf{E}_{\perp} + \mathbf{v}_{e\perp}\times\mathbf{B}) + KT_e\nabla_{\perp} n_e = 0 \tag{7}$$

$$en\,E_z + KT_e\nabla_z n_e + mn_e\nu_{eff}\,v_{ez} = 0 \quad . \tag{8}$$

In equilibrium, all terms in Eq. (8) vanish, but Eq. (7) gives the diamagnetic drift:

$$\mathbf{v}_{oe} = \frac{KT_e}{eB}\,\frac{\nabla n_{oe}\times\hat{z}}{n_{oe}} \equiv \mathbf{v}_D \quad . \tag{9}$$

Since $\nabla n_{oe}/n_{oe} = \nabla n_o/n_o$, we have

$$\mathbf{v}_D = \frac{KT_e}{eB}\,\frac{n_o'}{n_o}\,\hat{y} \quad . \tag{10}$$

In first order, Eqs. (7) and (8) are solved in the usual way[7], and the resulting velocities are inserted into the equation of continuity. The derivatives of n_e and ϕ cancel out as long as the electron mass is neglected, and one obtains the modified electron Boltzmann relation:

$$\frac{n_e}{n_{oe}} = \frac{e\phi}{KT_e} \frac{\omega_* + ib\sigma_{\|}}{\omega + ib\sigma_{\|}} \quad . \tag{11}$$

This result is exactly the same[8] as in the usual drift wave, but here n_{oe} is no longer equal to n_o . The new quantities are defined as follows:

$$\omega_* \equiv k_y v_D = -k_y \frac{KT_e}{eB} \frac{n_o'}{n_o} \tag{12}$$

$$b \equiv k_y^2 a_i^2 \ , \quad a_i^2 \equiv KT_e / M\Omega_c^2 \tag{13}$$

$$\sigma_{\|} \equiv (k_z^2 / k_y^2)(\omega_c / \nu_{eff})\Omega_c \tag{14}$$

$$\omega_s \equiv b\sigma_{\|} = k_z^2 (KT_e / m\nu_{eff}) = D_{\|e} k_z^2 \quad . \tag{15}$$

When collisions are the dominant mechanism that inhibits electron motion along **B** , then ν_{eff} is either ν_{ei} (for fully ionized plasmas) or ν_{eo} (for weakly ionized plasmas). Then ω_s or $b\sigma_{\|}$ can be recognized as the reciprocal of the time for an electron to diffuse a parallel wavelength. When collisions are unimportant, ν_{eff} can be replaced by an effective collision rate due to Landau damping. It can be seen from Eq. (13) that the usual Boltzman relation $n_1 / n_o = e\phi / KT_e$ is recovered in the limit $\sigma_{\|} \rightarrow \infty$.

3. Electron beam. The linearized equation for a cold beam with uniform velocity $\mathbf{v}_o$ is:

$$m \left[\frac{\partial \mathbf{v}_b}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{v}_b \right] = -e(\mathbf{E} + \mathbf{v}_b \times \mathbf{B}) \quad . \tag{16}$$

Defining

$$\omega_b \equiv \omega - k_z u \quad , \tag{17}$$

we may write the solution as:

$$\left.\begin{aligned} iv_{bx} &= -\frac{\omega_c}{\omega_b B}\,(\phi' + \frac{\omega_c}{\omega_b}k_y\phi)\,(1 - \frac{\omega_c^2}{\omega_b^2})^{-1} \\ v_{by} &= -\frac{\omega_c}{\omega_b B}\,(k_y\phi + \frac{\omega_c}{\omega_b}\,\phi')\,(1 - \frac{\omega_c^2}{\omega_b^2})^{-1} \\ v_{bz} &= -\frac{\omega_c}{\omega_b B}\,k_z\phi \quad . \end{aligned}\right\} \tag{18}$$

In the limit of small m but finite $m \times u$, these become:

$$iv_{bx} = k_y\phi / B \quad , \qquad v_{by} = \phi' / B \tag{19}$$

$$v_{bz} = e\phi / mu \quad , \tag{20}$$

where we have approximated ω_b by $-k_z u$, since $\omega << k_z u$.

The equation of continuity for the beam is:

$$\frac{\partial n_b}{\partial t} + \mathbf{v}_{ob} \cdot \nabla n_b + n_{ob} \nabla \cdot \mathbf{v}_b + \mathbf{v}_b \cdot \nabla n_{ob} = 0 \quad , \tag{21}$$

or

$$\omega_b \frac{n_b}{n_{ob}} + iv_{bx} - k_y v_{by} - k_z v_{bz} + iv_{bx}\frac{n_o'}{n_o} = 0 \quad . \tag{22}$$

Substituting the velocities from Eqs. (19) and (20), we find:

$$\frac{n_b}{n_{ob}} = \frac{e\phi}{KT_e}\left[\frac{\omega_*}{\omega_b} - \frac{k_z^2 v_{th}^2}{2\,\omega_b^2}\right] \quad , \tag{23}$$

where

$$v_{th}^2 \equiv 2KT_e / m \quad . \tag{24}$$

C) Dispersion relation

The quasineutrality condition $n_i = n_e + n_b$, together with the definitions $n_i = n_o$, $n_{ob} = \alpha n_o$, $n_{oc} = (1-\alpha) n_o$ and the densities given by Eqs. (5), (11), and (23), yields the dispersion relation

$$\frac{b\omega - \omega_*}{\dfrac{\omega^2}{\Omega_c^2} - 1} - (1-\alpha)\omega \frac{\omega_* + ib\sigma_{\parallel}}{\omega + ib\sigma_{\parallel}} = \alpha\omega \left[\frac{\omega_*}{\omega - k_z u} - \frac{k_z^2 v_{th}^2}{2(\omega - k_z u)^2} \right] . \tag{25}$$

1. Limit of no beam. When $\alpha = 0$, this reduces to:

$$(\omega + ib\sigma_{\parallel})(b - \frac{\omega_*}{\omega}) = (\frac{\omega^2}{\Omega_c^2} - 1)(\omega_* + ib\sigma_{\parallel}) . \tag{26}$$

a) When $\omega^2 << \Omega_c^2$, this further reduces to the familiar drift wave equation

$$\omega^2 + i\sigma_{\parallel}(\omega - \omega_*) = 0 , \tag{27}$$

which has the asymptotic $(\sigma_{\parallel} >> \omega_*)$ growth rate γ given by

$$\omega = \omega_* + i\gamma = \omega_* + i\omega_*^2/\sigma_{\parallel} . \tag{28}$$

The growth rate for resistive drift waves is found by using Eq. (14) for $\sigma_{\parallel}$, with $\nu_{eff} = \nu_{ei} = n_o e^2 \eta / m$, where η is the resistivity. One then obtains

$$\sigma_{\parallel} = \frac{k_z^2}{k_y^2} \frac{v_A^2}{c^2} \frac{4\pi}{\eta} , \qquad \gamma = \frac{\omega_*^2}{4\pi} \frac{k_y^2}{k_z^2} \frac{\eta c^2}{v_A^2} , \tag{29}$$

where $v_A^2 = c^2 B^2 / 4\pi n_o M$ is the square of the Alfvén speed.

Collisionless drift waves can be obtained from Eq. (27) by substituting for ν_{eff} the effective collision frequency due to Landau damping:

$$\nu_{eff} = \nu_{LD} = \frac{\sqrt{\pi}}{2} k_z v_{th} e^{-\zeta^2} \quad , \tag{30}$$

where $\zeta = \omega / k_z v_{th}$. Eq. (30) is derived in the Appendix. The asymptotic growth rate found from Eqs. (28), (14), and (30) can be written

$$\gamma = \frac{\omega_*^2}{\omega_c \Omega_c} \frac{k_y^2}{k_z^2} \nu_{LD} = \sqrt{\pi}\, \omega_*^2 \frac{b e^{-\zeta^2}}{k_z v_{th}} \quad , \tag{31}$$

which agrees with the standard universal instability growth rate[4,9].

b) When ω is of the order of Ω_c , Eq. (26) can be written:

$$b^{-1} \omega_* \omega^3 - \Omega_c^2 \omega^2 + i\sigma_{\|} [\, \omega^3 - (1+b)\Omega_c^2 \omega + \Omega_c^2 \omega_* \,] = 0 \quad . \tag{32}$$

We point out two limits. When the plasma is uniform and $\sigma_{\|}$ is large enough that the electron Boltzmann relation is satisfied, the first two terms in the square bracket give

$$\omega^2 = (1+b)\Omega_c^2 \quad , \quad \text{or} \quad \omega^2 = \Omega_c^2 + k_y^2 c_s^2 \quad . \tag{33}$$

This is just the electrostatic ion cyclotron wave. When the magnetic field is so weak that $b >> 1$, as in the experiment by Jain *et al.*[1] , setting the square bracket to zero shows that $\omega = \omega_D$, where

$$\omega_D \equiv \omega_* / b \quad . \tag{34}$$

The asymptotic solution for $\sigma_{\|} >> \omega_*$ is then:

$$\omega = \omega_D \left(1 + \frac{\omega_D^2}{b\Omega_c^2} \right) + \frac{i}{\sigma_{\|}} \frac{\omega_D^2}{b\,\Omega_c^2} (\Omega_c^2 - \omega_D^2) \quad . \tag{35}$$

It is seen that there is no instability for $\omega > \Omega_c$ in this case.

2. Effect of driving beam. When none of the approximations above can be made, Eq. (25) is a sixth degree equation for ω. Fortunately, the condition $\omega << |k_z u|$ is satisfied by several orders of magnitude; and, furthermore, the last term in Eq. (25) is only a 5% correction to the preceding term in the experiment under consideration. The square bracket in Eq. (25) can then be replaced by $-\omega_* / k_z u$, and Eq. (25) reduces to the following cubic equation:

$$(1-\alpha)\omega^3 + (\alpha - \frac{b\omega}{\omega_*})\Omega_c^2 \omega - \alpha \frac{\omega^2}{k_z u}(\omega^2 - \Omega_c^2) =$$

$$= ib\sigma_{\parallel}\Omega_c^2 \left[\frac{b\omega}{\omega_*} - 1 - \frac{\omega}{\omega_*}(\frac{\omega^2}{\Omega_c^2} - 1)(1 - \alpha - \frac{\alpha\omega_*}{k_z u}) \right] . \tag{36}$$

For large $\sigma_{\parallel}$, the square bracket is approximately zero, so that again $\omega \simeq \omega_* / b$ for large b. Recalling the definitions $\omega_D \equiv \omega_* / b$ and $\omega_s \equiv b\sigma_{\parallel}$, we can write the asymptotic solution as:

$$\omega = \omega_D + \frac{\omega_D^2}{\omega_*}\left[\frac{\omega_D^2}{\Omega_c^2} - 1\right]\left[1 + \alpha\left[1 + \frac{\omega_*}{k_z u}\right]\right] +$$

$$\frac{i\omega_D}{\omega_s}(1-\alpha)\left[\frac{\omega_D^2}{\Omega_c^2} - 1\right]\left[\frac{\alpha}{1-\alpha}\frac{\omega_D}{k_z u} - 1\right] . \tag{37}$$

From this we see that if $\omega \simeq \omega_D > \Omega_c$, there is instability only for $\alpha \simeq 1$, since we have already assumed $\omega_D / k_z u << 1$. To see if instability occurs more readily for intermediate values of $\sigma_{\parallel}$ [smaller than the asymptotic value assumed in Eq. (37)], we must solve the cubic equation (36) or the 6th degree equation (25). Unfortunately, Eq. (36) is also cubic in k_z, since $\sigma_{\parallel} \propto k_z^2$, so that the spatial growth rate is no easier to compute. It is not likely that the exact solution of Eq. (36) will yield instability for $\omega > \Omega_c$, in contradiction to Eq. (37).

There are several features of Eq. (37), however, that are suggestive of the experimental observations. The imaginary part of ω is sensitive to variations in Ω_c, u, or α which could make one of the two parenthetical factors at the end change sign. The real part of ω varies as $\omega_* / b \propto 1 / k_y$, so that higher azimuthal mode numbers will produce subharmonics. The frequency also varies linearly with B (the correction term being smaller by $1/b$), which does not agree with observations; but one has to solve Eq. (36) properly to be sure.

The direction of ion motion in response to an electric field changes sign at $\omega = \Omega_c$, as seen in Eq. (2). In normal drift waves, where $\omega < \Omega_c$, the ion response amplifies the E-field; and there is instability. In high-frequency drift waves, where $\omega > \Omega_c$, the ion motion is stabilizing; and it takes a very strong excitation mechanism to overcome the basic stability of the plasma. Even if this mode is unstable, it is hard to understand why the plasma does not choose to oscillate at a frequency lower than Ω_c. Note that the curvature of the magnetic field is concave, so that the centrifugal force of the beam cannot drive the gravitational instability. The only conclusion we can draw at the present time is that, if indeed Ω_c is below ω, then the instability is not a plasma instability but a gas-discharge instability involving such processes as ionization and relaxation.

Appendix

When resonant-particle effects, as well as collisions, are important in limiting the free motion of electrons along **B**, one can nonetheless derive an effective resistivity η_{eff} or collision frequency ν_{eff} which takes both Landau and collisional damping into account. Instead of using Eq. (8) to describe the parallel electron motion, one can use the linearized Boltzmann equation with a Krook collision term:

$$\frac{\partial f_1}{\partial t} + v\frac{\partial f_1}{\partial z} - \frac{e}{m}E\frac{\partial f_o}{\partial v} = \left(\frac{n_1}{n_o}f_o - f_1\right)\nu \ , \qquad \text{(A1)}$$

where ν is the total collision frequency against ions and neutrals, and v and E stand for v_z and E_z. Fourier analyzing, taking a Maxwellian for f_o, and defining

$$\zeta \equiv \frac{\omega + i\nu}{k_z v_{th}} \ , \qquad \zeta_o \equiv \frac{\omega}{k_z v_{th}} \ , \qquad \text{(A2)}$$

we find

$$f_1 = \frac{ieE}{m}\left[\frac{f_o'(v)}{\omega - kv + i\nu} - \frac{i\nu}{kv_{th}^2}Z'(\zeta_o)\frac{f_o(v)}{\omega - kv + i\nu}\right] , \qquad \text{(A3)}$$

where $Z'(\zeta)$ is the derivative of the plasma dispersion function [10]. The current j_z can be found by integrating Eq. (A3):

$$j_z = -e\int_{-\infty}^{\infty} v f_1(v)\,dv \ . \qquad \text{(A4)}$$

The integrals can be evaluated with the help of the identity

$$Z'(\zeta) = -2[1 + \zeta Z(\zeta)] \ . \qquad \text{(A5)}$$

The result in mks is:

$$\frac{j_z}{E_z} = i\omega\varepsilon_o \frac{\omega_p^2}{\omega^2} \zeta_o Z'(\zeta)\,[\,\zeta + \frac{i\nu}{2\omega} \zeta_o Z'(\zeta_o)\,] \qquad \text{(A6)}$$

a) Limit of no Landau damping $(\zeta \rightarrow \infty)$

We note the asymptotic expansion for $\zeta >> 1$:

$$Z(\zeta) = -\frac{1}{\zeta} - \frac{1}{2\zeta^3} + \ldots + i\sqrt{\pi}\, e^{-\zeta^2} \qquad \text{(A7)}$$

$$\zeta^2 Z'(\zeta) = 1 + \frac{3}{2}\frac{1}{\zeta^2} + \ldots - 2i\sqrt{\pi}\, \zeta^3 e^{-\zeta^2} \quad . \qquad \text{(A8)}$$

Thus when $\zeta \rightarrow \infty$, Z' approaches 0 and $\zeta^2 Z'$ approaches 1. We then have

$$\frac{j_z}{E_z} = i\omega\varepsilon_o \frac{\omega_p^2}{\omega^2} (1 - \frac{i\nu}{\omega}) \quad .$$

Taking the reciprocal and expanding $(1 - i\nu/\omega)^{-1}$, we obtain

$$\frac{E_z}{j_z} = \frac{-i}{\omega\varepsilon_o} \frac{\omega^2}{\omega_p^2} (1 + \frac{i\nu}{\omega}) \quad . \qquad \text{(A9)}$$

The real part of this is the resistivity:

$$\mathrm{Re}\,(\frac{E_z}{j_z}) = \eta = \frac{\nu}{\varepsilon_o \omega_p^2} = \frac{m\nu}{ne^2} \quad , \qquad \text{(A10)}$$

which is the usual relation.

b) Limit of Large Landau damping $(\zeta \rightarrow 0)$.

We now use the power series expansion of $Z(\zeta)$ valid for $\zeta << 1$:

$$Z(\zeta) = -2\zeta + \frac{4}{3}\zeta^3 + \ldots + i\sqrt{\pi}\, e^{-\zeta^2} \qquad \text{(A11)}$$

$$Z'(\zeta) = -2 + 4\zeta^2 + \ldots - 2i\sqrt{\pi}\,\zeta e^{-\zeta^2} \tag{A12}$$

In this limit, Z' approaches -2, and the square bracket in Eq. (A6) becomes, simply, ζ_o. For the factor $Z'(\zeta)$ in Eq. (A6), we retain the Landau term and write

$$\frac{j_z}{E_z} = i\omega\varepsilon_o \frac{\omega_p^2}{\omega^2} \zeta_o^2 (-2)(1 + i\sqrt{\pi}\,\zeta e^{-\zeta^2}) \quad . \tag{A13}$$

Taking the reciprocal, expanding the last ( ), using the definition of ζ_o, and taking the real part, we obtain

$$\mathrm{Re}\left(\frac{E_z}{j_z}\right) = \eta_{eff} = \frac{\sqrt{\pi}}{2} \frac{m}{ne^2} k_z v_{th} e^{-\zeta^2} \quad . \tag{A14}$$

Here it is not necessary to distinguish between ζ and ζ_o. Comparing with Eq. (A10), we see that the effective collision rate due to Landau damping is

$$\nu_{eff} = \nu_{LD} = \frac{\sqrt{\pi}}{2} k_z v_{th} e^{-\zeta^2} \quad . \tag{A15}$$

For the intermediate case of comparable collisional and Landau damping, it can be seen that these two processes interfere with each other. The solution can be obtained only by using the actual Z-function in Eq. (A6).

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