Improved High-Rate Space-Time Trellis Codes via Orthogonality and Set Partitioning

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Abstract

In this paper, we propose a new technique for designing an improved high-rate space-time code. The proposed code construction is based on a typical concatenation of a space-time block code and an outer TCM encoder. However, unlike the existing STBC-TCM schemes which are rate-lossy, the proposed designs enable full-rate transmissions, which is possible by expanding the cardinality of the available orthogonal space-time signal points before concatenating with the outer TCM encoder. The classic set partitioning technique is then employed to realize large coding gains. We present several design examples of the new full-rate space-time codes for a system with 2 transmit antennas. Simulation results show that the new space-time codes considerably outperform the original ST-TCM designs. For example, the new 4-state 2-bits/sybol QPSK space-time code performs even better than the original 32-state design, while performance of the new 32-state QPSK code is only 1.5 dB away from the outage probability limit. Decoding complexity of the proposed M-TCM construction is made relatively low by exploiting signal orthogonality.

1. Introduction

Space-time trellis coded modulation (ST-TCM) has emerged as a promising method for enhancing performance of a digital communication system in wireless fading channels. Since the original ST-TCM design was introduced by Tarokh, Seshadri, and Calderbank (TSC) [1], there has been extensive research aimed at improving the performance of the original TSC ST-TCM designs. Because the original TSC ST-TCM codes were handcrafted, they are not optimum designs. In recent years, much research has been completed which proposed code constructions or performed systematic searches for different convolutional encoders [2, 3, 4, 5, 6, 7]. This work provided coding advantages over the original TSC designs, however, only marginal performance gains were obtained.

The previously proposed ST-TCM never really used the revolutionary idea of Ungerboeck [8] that was constellation expansion and set partitioning. In this paper we intend to tie in this idea of Ungerboeck to the design of ST-TCM. A common design approach that enables set partitioning is to concatenate the orthogonal space-time block codes [9, 10] with an outer TCM or M-TCM encoder (STBC-TCM), e.g. [11, 12, 13, 14, 15]. By using the orthogonal space-time block code as an inner code, the ST-TCM design problem is basically transformed into an equivalent one for the classic TCM scheme. This is advantageous since it allows one to adopt the standard design methods such as the classic set partitioning [16], for designing a good ST-TCM. In fact, the existing TCM designs for the AWGN channels can be optimally used as an outer code [11]. Nonetheless, this design approach cannot realize a full-rate ST-TCM since the inner block code is full rate and the outer TCM encoder must have redundancy.

In order to obtain a high-rate space-time code with the STBC-TCM construction, the cardinality of the inner space-time block code must be enlarged. This idea was independently explored in [17] and has shown significant performance improvement. In the sequel, we propose a technique to form an expanded orthogonal space-time signal set. A techniques is proposed to take the original Alamouti code and generate the required constellation subsets. The constellation subsets of space-time block code are generated by applying unitary transformations to the Alamouti code. Once the expanded orthogonal signal set is formed, the design of a good space-time code is mostly analogous to the classic TCM design technique. The classic set partitioning method can be directly adopted for partitioning of signals within each block-code subset, and the STBC-TCM construct can be guaranteed to achieve full-diversity by a simple design rule that restricts transition branches leaving from (or arriving to) each state to be labeled by codewords from the same block-code subset. This design rule is analogous to the original design rule proposed in the original TSC paper [1].

In this work, several new full-rate space-time codes for a system with 2 transmit antennas are derived based on the proposed technique. Simulation results show that the new space-time codes considerably outperform the original ST-TCM counterparts. For example, the new 4-state 2-bits/sybol QPSK space-time code achieves even better performance than the original TSC 32-state design, while the new 32-state QPSK code performs about 1.5 dB away from the outage probability limit. Moreover, decoding complexity of the proposed expanded-STBC-TCM construction is reasonably low because codeword orthogonality can be exploited to simplify computation for the optimal demodulator.

1.1. System Model

We consider a space-time wireless communication system with L_t antennas at the transmitter. The transmitter encodes k_c information bits into $N_c L_t$ symbols corresponding to the edge in the trellis of a space-time code with 2^{ν_c} states. The encoded symbols are divided into L_t streams, each of which is linearly modulated and simultaneously transmitted via each antenna.

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The rate of this space-time code is defined as $R_c = k_c/N_c$ bits/symbol. The symbols are chosen from a constellation with an average energy of R_c/L_t. The channel between a transmit and a receive antenna is modeled as a frequency non-selective flat Rayleigh fading process. For brevity of this presentation, we restrict our attention to designing space-time codes for independent quasi-static fading, i.e., the channel is constant during a frame of data but fades independently from frame to frame and from antenna to antenna. Also, the design will be optimized for a system with a single antenna at the receiver, nonetheless, such a design also performs well in general. For convenience, we assume here that the receiver is equipped with only one antenna.

At the receiver, assuming ideal timing information, an $N_f \times 1$ vector of matched filter output is formed for maximum likelihood sequence decoding, with N_f denoting the frame length. This observation vector can be expressed in a simple matrix form as

$$\vec{Q} = \sqrt{E_b}\mathbf{D}\vec{C} + \vec{N} \tag{1}$$

where E_b defines the bit energy per receive antenna, $\mathbf{D}$ is an $N_f \times L_t$ transmitted codeword matrix formed as

$$\mathbf{D} = \begin{bmatrix} D_1(1) & D_2(1) & \dots & D_{L_t}(1) \\ D_1(2) & D_2(2) & \dots & D_{L_t}(2) \\ \vdots & \vdots & \ddots & \vdots \\ D_1(N_f) & D_2(N_f) & \dots & D_{L_t}(N_f) \end{bmatrix} \tag{2}$$

with $D_i(k)$ being the symbol transmitted at time k, $\vec{C}$ is an $L_t \times 1$ channel vector whose i^{th} element, $C(i)$, denotes the complex coefficient of the channel between the i^{th} transmit antenna and the receiver, and $\vec{N}$ is the additive white Gaussian noise (AWGN) with a covariance matrix $N_0\mathbf{I}_{N_f}$. With the quasi-static fading assumption, the channel distortion coefficients are independent zero-mean Guassian random variables with a unit variance, and the signal-to-noise ratio (SNR) per receive antenna is computed as $\gamma_b = E_b/N_0$. The receiver is assumed to have perfect knowledge of channel state information (CSI).

1.2. Space-Time Code Design Criteria

The standard design criteria for constructing good space-time codes [18, 1] are based on an analysis of the pairwise error probability (PWEP), i.e., the probability that the optimum codeword decoder makes an erroneous decision in favor of a given codeword ($\mathbf{D} = \mathbf{d}_\beta$) over the transmitted one ($\mathbf{D} = \mathbf{d}_\alpha$). For the maximum likelihood decoder with perfect CSI in quasi-static fading, the PWEP is asymptotically bounded by [19]

$$P(\alpha, \beta) \leq \frac{\begin{pmatrix} 2\Delta_H - 1 \\ \Delta_H - 1 \end{pmatrix}}{\gamma_b^{\Delta_H} \prod_{i=1}^{\Delta_H} \lambda_i} \tag{3}$$

where λ_i for $i = 1, 2, \dots, \Delta_H$ are the nonzero eigenvalues of the signal matrix $\mathbf{C}_s$ which is defined as

$$\mathbf{C}_s = \mathbf{E}(\alpha, \beta)^H \mathbf{E}(\alpha, \beta) \tag{4}$$

with $\mathbf{E}(\alpha, \beta) = (\mathbf{d}_\alpha - \mathbf{d}_\beta)$ being the codeword difference matrix, Δ_H is the rank of $\mathbf{E}(\alpha, \beta)$ which may be referred to as the effective Hamming distance. It is apparent from (3) that to minimize the error probability, it is desirable to maximize Δ_H and the product measure $\Delta_p = \prod_{i=1}^{\Delta_H} \lambda_i$. The minimum values of the effective Hamming distance $\Delta_H(\min)$ and the product measure $\Delta_p(\min)$ over all possible pairwise error events define the diversity advantage and the coding advantage of a space-time code, respectively. If $\Delta_H(\min)$ equals to the total number of transmit antennas, the space-time code is said to achieve full diversity (assuming the single-receiver case). In summary, the standard design goal is to maximize $\Delta_H(\min)$ and $\Delta_p(\min)$.

2. New Space-Time Code Construction

The proposed code construction is based on the concatenated STBC-TCM scheme in which an M-TCM encoder with multiplicity N is an outer code and the $N \times L_t$ orthogonal space-time block code is an inner code. Traditionally, this concatenated scheme outputs a rate-lossy space-time code. However, a full-rate space-time code is possible by expanding the cardinality of the orthogonal space-time signal set before concatenating it with an outer encoder. Once the expanded orthogonal signal set is formed, the standard set partitioning method can be used to partition codewords in each block-code subset. A simple design rule can then be posed on the outer encoder to guarantee that the resulting space-time code achieves full-diversity. In the sequel, we formulate the expanded orthogonal space-time block code, describe partitioning of this signal set, and present several design examples of the new full-rate space-time codes.

2.1. Expanded Orthogonal Space-Time Block Code

The goal here is to expand the cardinality of the orthogonal space-time signal set, in order that sufficient signal points are available to allow a full-rate STBC-TCM design. Note that we do not require that the Hamming distance achieves the maximum possible value for every codeword pair in the entire signal set, but it is desirable that this holds for codewords in each given signal subset.

To do this, first the standard space-time block code is used as a building subset. A distinct block-code subset can then be generated from the original signal subset by applying certain unitary transformations, i.e., if $\mathbf{\Omega}(\vec{s})$ is an $N \times L_t$ orthogonal block code for an input $\vec{s}$, another orthogonal block code $\tilde{\mathbf{\Omega}}(\vec{s})$ can be formed by

$$\tilde{\mathbf{\Omega}}(\vec{s}) = \mathbf{H}\,\mathbf{\Omega}(\vec{s})\,\mathbf{G} \tag{5}$$

where $\mathbf{H}$ and $\mathbf{G}$ are some $N \times N$ and $L_t \times L_t$ diagonal unitary matrices, respectively.

For example, we derive the expanded signal set for the 2-transmit antenna case. We start with the 2×2 Alamouti signals,

$$\mathbf{\Omega}(s_0, s_1) = \begin{bmatrix} s_0 & s_1 \\ -s_1^* & s_0^* \end{bmatrix} \tag{6}$$

as the building subset. For the 8-PSK alphabet, we can generate additional isolated signal subsets by using (5) with $\mathbf{H}$ being an identity matrix and the generator matrix $\mathbf{G}$ drawn from the following set:

$$\mathbf{G} \in \left\{ \begin{matrix} \begin{bmatrix} 1 & 0 \\ 0 & \pm 1 \end{bmatrix}, & \begin{bmatrix} 1 & 0 \\ 0 & \pm j \end{bmatrix}, \\ \begin{bmatrix} 1 & 0 \\ 0 & \pm e^{j\frac{\pi}{4}} \end{bmatrix}, & \begin{bmatrix} 1 & 0 \\ 0 & \pm e^{j\frac{3\pi}{4}} \end{bmatrix} \end{matrix} \right\} \tag{7}$$

For the BPSK and QPSK alphabets, only the first two and the first four generators in (7) are valid, respectively.

In this work, we will limit our attention to the 2-antenna transmitter case. We found that the first four generators are sufficient for obtaining a good space-time code. The space-time block codes resulting from the four generators shall be denoted correspondingly by

$$\mathbf{A}(s_0, s_1) = \begin{bmatrix} s_0 & s_1 \\ -s_1^* & s_0^* \end{bmatrix} \tag{8}$$

$$\mathbf{B}(s_0, s_1) = \begin{bmatrix} s_0 & -s_1 \\ -s_1^* & -s_0^* \end{bmatrix} \tag{9}$$

$$\mathbf{J}(s_0, s_1) = \begin{bmatrix} s_0 & js_1 \\ -s_1^* & js_0^* \end{bmatrix} \tag{10}$$

$$\mathbf{K}(s_0, s_1) = \begin{bmatrix} s_0 & -js_1 \\ -s_1^* & -js_0^* \end{bmatrix}. \tag{11}$$

2.2. Set Partitioning

For this case ($L_t = 2$), a distance (determinant) measure of a given pair of signals from each block code:

$$\langle \mathbf{A}(\alpha_0, \alpha_1), \mathbf{A}(\beta_0, \beta_1) \rangle, \langle \mathbf{B}(\alpha_0, \alpha_1), \mathbf{B}(\beta_0, \beta_1) \rangle,$$

$$\langle \mathbf{J}(\alpha_0, \alpha_1), \mathbf{J}(\beta_0, \beta_1) \rangle, \text{ or } \langle \mathbf{K}(\alpha_0, \alpha_1), \mathbf{K}(\beta_0, \beta_1) \rangle$$

is uniformly given by

$$d^2(\alpha, \beta) = |\alpha_0 - \beta_0|^2 + |\alpha_1 - \beta_1|^2 \tag{12}$$

which is conveniently equivalent to the squared Euclidean distance of the input labels. That is, a subset of signals from a given block code shall be partitioned based on the squared Euclidean distance of the input labels. The classic set partitioning technique [16] was basically provided to do exactly this. Therefore, we can directly adopt the standard technique when partition each block code subset. Note that the relationship between (12) and the product measure of the concatenated code can be obtained via (4).

In summary, the expanded signal set can be partitioned as shown in Figure 1, with the lower-level partitions defined analogously to [16].

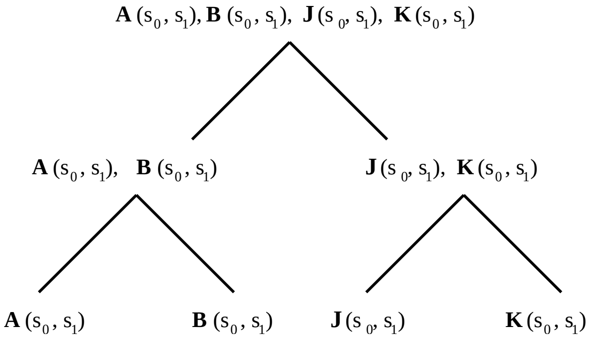

Figure 1: Higher-level partitions

2.3. Design Examples

First of all, we derive a design rule to guarantee that the proposed STBC-TCM construction results in a space-time code with full-diversity. Due to the fact that each of the four partitions, $\mathbf{A}(s_0, s_1)$, $\mathbf{B}(s_0, s_1)$, $\mathbf{J}(s_0, s_1)$, or $\mathbf{K}(s_0, s_1)$, is a full rate, full-diversity space-time code, we can arrive at the following simple design rule: "*transition branches leaving from (or merging to) each state are uniquely labeled with codewords from the same partition.*" Note that this design rule implies that parallel transition branches are permissible. This is advantageous because the TCM designs with parallel transition branches typically achieve better performance when encoder memory is small.

In this paper, we present several design examples of the new full-rate QPSK and 8-PSK space-time codes for a 2-antenna transmitter. In this case, the M-TCM with 2 multiplicity is used as an outer encoder, and thus 16 and 64 transition branches leaving from or merging to each state are needed to achieve the desired code rate of 2 bits/symbol and 3 bits/symbols, respectively.

For the 4-state and the 8-state QPSK codes, we design the outer M-TCM encoder to have 8 parallel transition branches. We use the standard method to divide each of the four block code subsets into 2 partitions with a cardinality of 8, e.g,

$$\begin{aligned} \mathbf{A}_0 &\equiv \{\, \mathbf{A}(\pm 1, \pm 1),\ \mathbf{A}(\pm j, \pm j) \,\} \\ \mathbf{A}_1 &\equiv \{\, \mathbf{A}(\pm 1, \pm j),\ \mathbf{A}(\pm j, \pm 1) \,\} \\ \mathbf{B}_0 &\equiv \{\, \mathbf{B}(\pm 1, \pm 1),\ \mathbf{B}(\pm j, \pm j) \,\} \\ \mathbf{B}_1 &\equiv \{\, \mathbf{B}(\pm 1, \pm j),\ \mathbf{B}(\pm j, \pm 1) \,\} \end{aligned}$$

and similarly for $\mathbf{J}_0$, $\mathbf{J}_1$, $\mathbf{K}_0$, and $\mathbf{K}_1$. The proposed simple design rule is then used to construct the STBC-TCM codes that achieve full-diversity. Following the standard technique, the coding gain is enlarged by maximizing the "distance" among the transition branches leaving from and merging to a given state. The new 4-state and 8-state 2-bits/symbol QPSK space-time codes are depicted in Figure 2 and 3, respectively. Note that for the 4-state encoder, it is sufficient to use only 2 block code subsets, but it is better to use all four subsets for a larger encoder.

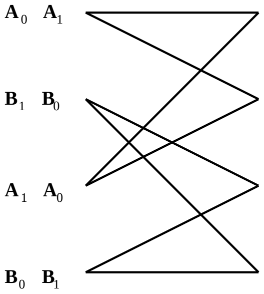

Figure 2: New 4-state 2-bits/symbol QPSK code

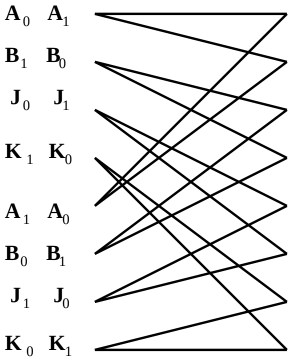

Figure 3: New 8-state 2-bits/symbol QPSK code

The proposed 4-state and 8-state QPSK space-time codes achieve full diversity and have a minimum product measure

$\Delta_p(\min) = 16$, which is better than that achieved by the original TSC ST-TCM with 32 states. The large coding advantage results from the exploitation of signal partitioning and symmetries. Simulation results in the sequel verify that the new 4-state and 8-state space-time codes outperform the original 32-state space-time code.

For an encoder with larger memory, we may permit less parallel branches to improve the coding gain. For the 32-state QPSK code, the M-TCM encoder has 2 parallel branches, and each block code is divided into 8 sub-partitions, e.g.,

$$\begin{aligned}
\mathbf{A}_{00} &\equiv \{\mathbf{A}(1,1), \mathbf{A}(-1,-1)\}\,, & \mathbf{A}_{01} &\equiv \{\mathbf{A}(1,-1), \mathbf{A}(-1,1)\} \\
\mathbf{A}_{02} &\equiv \{\mathbf{A}(j,j), \mathbf{A}(-j,-j)\}\,, & \mathbf{A}_{03} &\equiv \{\mathbf{A}(j,-j), \mathbf{A}(-j,j)\} \\
\mathbf{A}_{10} &\equiv \{\mathbf{A}(1,j), \mathbf{A}(-1,-j)\}\,, & \mathbf{A}_{11} &\equiv \{\mathbf{A}(1,-j), \mathbf{A}(-1,j)\} \\
\mathbf{A}_{12} &\equiv \{\mathbf{A}(j,-1), \mathbf{A}(-j,1)\}\,, & \mathbf{A}_{13} &\equiv \{\mathbf{A}(j,1), \mathbf{A}(-j,-1)\}
\end{aligned}$$

Similar construction hold for $\mathbf{B}_{0i}, \mathbf{B}_{1i}, \mathbf{J}_{0i}, \mathbf{J}_{1i}, \mathbf{K}_{0i}, \mathbf{K}_{1i}$ for i = 0, 1, 2, and 3. The new 32-state 2-bits/symbol QPSK space-time code is depicted in Figure 4. Simulation results in the sequel show the performance of this code is about 1.5 dB away from the outage probability limit.

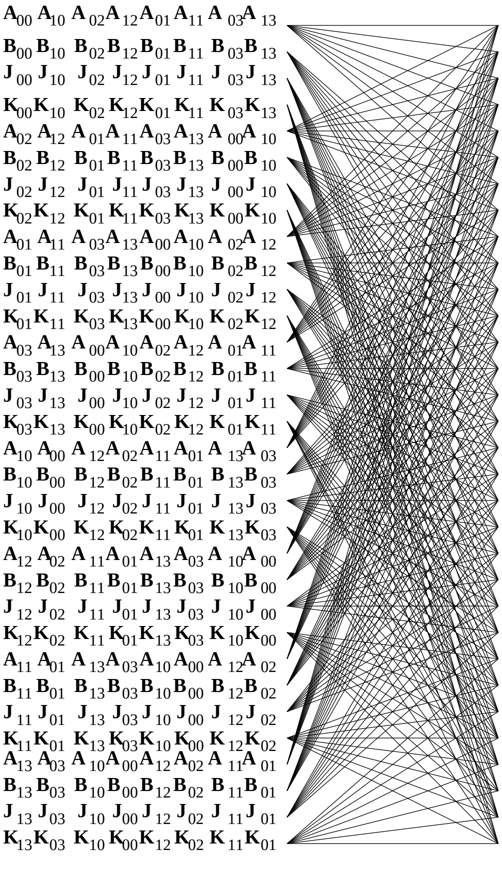

Figure 4: New 32-state 2-bits/symbol QPSK code

These same design concepts can now be used for 8PSK modulation. For brevity of this presentation, we will only include one example 8PSK code, an 8-state 3-bits/symbol 8PSK space-time code. In this case, we partition each subset of block code into 4 sub-partitions each with a cardinality of 16, e.g., (defining $\theta = e^{j\frac{\pi}{4}}$ and $\varphi = e^{j\frac{3\pi}{4}}$)

$$\begin{aligned}
\mathbf{A}_{p0} &\equiv \{\mathbf{A}(\pm 1, \pm 1), \mathbf{A}(\pm j, \pm j), \mathbf{A}(\pm 1, \pm j), \mathbf{A}(\pm j, \pm 1)\} \\
\mathbf{A}_{p1} &\equiv \{\mathbf{A}(\pm \theta, \pm \theta), \mathbf{A}(\pm \varphi, \pm \varphi), \mathbf{A}(\pm \theta, \pm \varphi), \mathbf{A}(\pm \varphi, \pm \theta)\} \\
\mathbf{A}_{p2} &\equiv \{\mathbf{A}(\pm 1, \pm \theta), \mathbf{A}(\pm j, \pm \varphi), \mathbf{A}(\pm 1, \pm \varphi), \mathbf{A}(\pm j, \pm \theta)\} \\
\mathbf{A}_{p3} &\equiv \{\mathbf{A}(\pm \theta, \pm j), \mathbf{A}(\pm \varphi, \pm 1), \mathbf{A}(\pm \theta, \pm 1), \mathbf{A}(\pm \varphi, \pm j)\}
\end{aligned}$$

and similarly for $\mathbf{B}_{pi}, \mathbf{J}_{pi}, \mathbf{K}_{pi}$ for i = 0, 1, 2, and 3. The new 8-state 8-PSK code is shown in Figure 5.

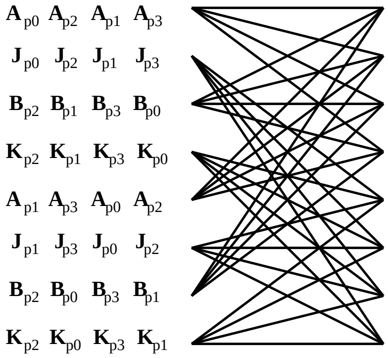

Figure 5: New 8-state 3-bits/symbol 8-PSK code

3. Decoding and Complexity Consideration

In practice, a drawback of an M-TCM encoder is it can produce a greater decoding complexity than a single dimensional TCM. In the proposed design, this disadvantage is suppressed because the orthogonality between antennas of the transmitted signals and the MPSK constellation can be exploited to reduce computational complexity.

For the maximum likelihood sequence decoder with perfect CSI in the case of $L_t = 2$, the branch metric at the k^{th} decoding interval (at time $2k$ and $2k+1$) for a transition labeled with a transmitted matrix

$$\mathbf{d}(k) = \begin{bmatrix} d_1(2k) & d_2(2k) \\ d_1(2k+1) & d_2(2k+1) \end{bmatrix}$$

given the observations at this interval are $q(2k)$ and $q(2k+1)$ and the channel gain realizations are $\vec{c} = [c(1)\ c(2)]^T$ is given as

$$M(\mathbf{d}(k)) = \sum_{i=2k}^{2k+1} \left| q(i) - \sum_{l=1}^{2} d_l(i) c(l) \right|^2 . \qquad (13)$$

Completing the square and using the orthogonality and the unit magnitude of MPSK gives

$$M(\mathbf{d}(k)) = -\sum_{i=2k}^{2k+1} \sum_{l=1}^{2} \Re\left[q^*(i) d_l(i) c(l)\right] + C \qquad (14)$$

where C is a constant which is not a function of $\mathbf{d}(k)$.

Since the branch metric has the form in (14), a low complexity decoder can be implemented. Sufficient statistics now can be shown to be

$$\begin{aligned}
\hat{s}_{0A}(k) &= c(1)q^*(2k) + c^*(2)q(2k+1)\\
\hat{s}_{1A}(k) &= c(2)q^*(2k) - c^*(1)q(2k+1)\\
\hat{s}_{0B}(k) &= c(1)q^*(2k) - c^*(2)q(2k+1)\\
\hat{s}_{1B}(k) &= -c(2)q^*(2k) - c^*(1)q(2k+1)\\
\hat{s}_{0J}(k) &= c(1)q^*(2k) + jc^*(2)q(2k+1)\\
\hat{s}_{1J}(k) &= jc(2)q^*(2k) - c^*(1)q(2k+1)\\
\hat{s}_{0K}(k) &= c(1)q^*(2k) - jc^*(2)q(2k+1)\\
\hat{s}_{1K}(k) &= -jc(2)q^*(2k) - c^*(1)q(2k+1).
\end{aligned}$$

The decoder can then compute the branch metric for a transition labeled with a codeword $\mathbf{A}(s_0, s_1)$, $\mathbf{B}(s_0, s_1)$, $\mathbf{J}(s_0, s_1)$, or $\mathbf{K}(s_0, s_1)$ by one of the following expressions:

$$\begin{aligned}
f(\mathbf{A}(s_0, s_1), k) &= -\Re\{s_0\hat{s}_{0A}(k)\} - \Re\{s_1\hat{s}_{1A}(k)\}\\
f(\mathbf{B}(s_0, s_1), k) &= -\Re\{s_0\hat{s}_{0B}(k)\} - \Re\{s_1\hat{s}_{1B}(k)\}\\
f(\mathbf{J}(s_0, s_1), k) &= -\Re\{s_0\hat{s}_{0J}(k)\} - \Re\{s_1\hat{s}_{1J}(k)\}\\
f(\mathbf{K}(s_0, s_1), k) &= -\Re\{s_0\hat{s}_{0K}(k)\} - \Re\{s_1\hat{s}_{1K}(k)\}
\end{aligned}$$

correspondingly. Once the branch metrics are computed, the Viterbi algorithm is applied, as usual, to search for the path with the lowest accumulated metric.

Note that this branch metric computation is simpler because several terms were neglected or cancelled out by the orthogonality and constant-envelope signal conditions. For a greater efficiency, the decoder may cache any common computation results for multiple usages. It may also exploit the fact that a given symbol may be a complex value with either a zero-real part or a zero-imaginary part, as well as the linearity of the metric function, e.g., $f(\mathbf{A}(s_0, s_1), k) = -f(\mathbf{A}(-s_0, -s_1), k)$.

For example, Table 1 shows a coarse estimate of decoding complexity of the original 2-bit/symbol QPSK ST-TCM with a generic decoder and the new QPSK ST-TCM with the efficient decoder, in terms of the number of arithmetic operations per symbol. With the efficient implementation, it can be seen that the decoding complexity of the new codes is comparable to or better than that of the single dimensional ST-TCM designs.

QPSK	4 states	8 states	32 states
Original ST-TCM	200	260	680
New ST-TCM	120	220	680

Table 1: Decoding complexity (operations per symbol)

4. Simulation Results and Discussions

In this section, we evaluate and compare performance of the new and the original TSC space-time codes for $L_t = 2$ via frame error rate simulations. A frame equals to 128 channel symbols in the simulations.

Figures 6 and 7 depict performance of the proposed 2-bits/symbol QPSK space-time codes vs. the existing ST-TCM designs. Note that the best-known ST-TCM designs were the Yan and Blum (YB) codes [3], based on the analysis of the distance spectrum [20]. It can be seen that the best-known 4-state design provides only a fractional performance gain over the original 4-state TSC design, while the proposed 4-state QPSK code achieves a large 2-dB performance advantage. More impressively, the proposed 4-state QPSK code even outperforms the original 32-state TSC design and the performance is only about 2.5 dB away from the outage probability. This level of performance is comparable to that of the original 64-state ST-TCM and a novel space-time turbo-TCM scheme such as [21]. It is quite intriguing that we can achieve this level of performance by such a simple design. The performance gain is more pronounced with the new 32-state code whose performance is only 1.5 dB away from the outage probability limit. On the other hand, the 32-state YB code perform roughly the same as the new 8-state code.

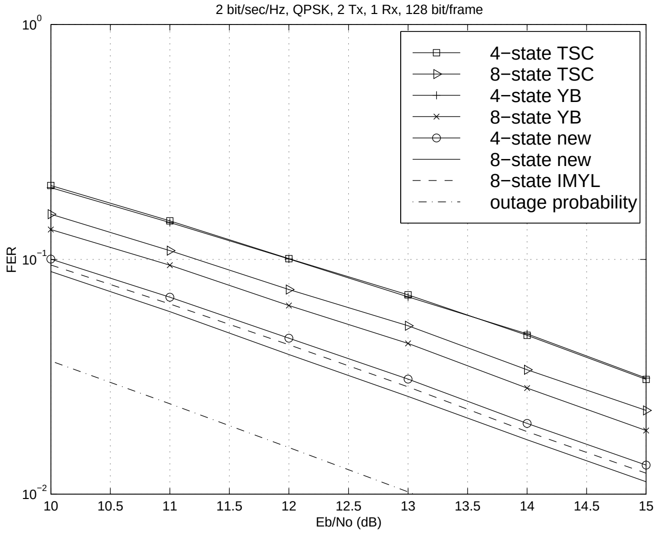

Figure 6: Performance of the 2-bits/symbol QPSK space-time codes

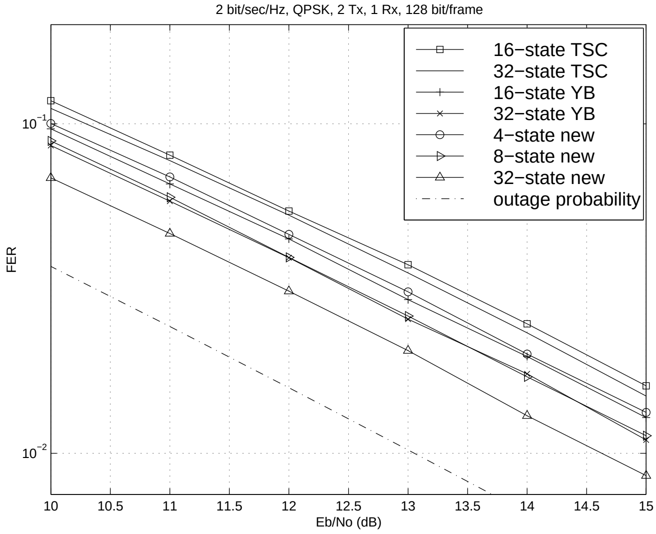

Figure 7: Performance of the 2-bits/symbol QPSK space-time codes

Note that, we also include a comparison of performance of the proposed codes and the IMYL code [17] which was obtained independently from a similar design concept. It is worthwhile to distinguish [17] from the proposed work. In [17], only 2 block code subsets: $\mathbf{A}(s_0, s_1)$ and $\mathbf{B}(s_0, s_1)$ were proposed in their designs, while we used 4 signal subsets in this work. Using more signal points can typically give better designs. We can

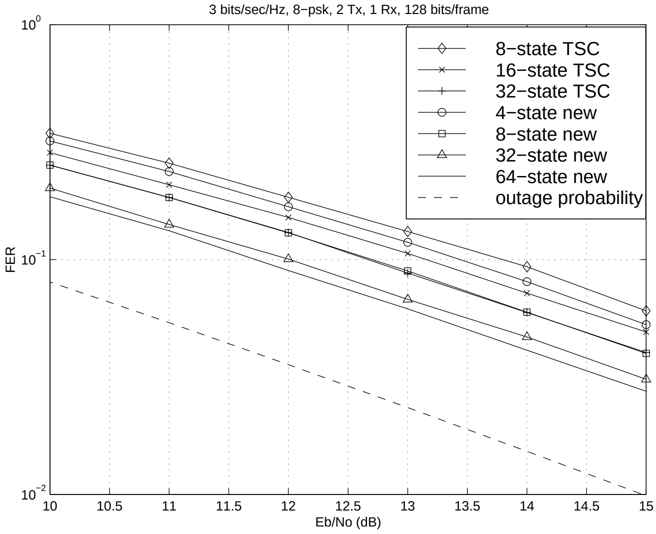

Figure 8: Performance of the 3-bits/symbol 8-PSK space-time codes

see that the proposed 8-state QPSK code performs better than the 8-state IMYL code. In [17], only the 8-state and 16-state codes were derived, while we also unveil the new 4-state design which impressively outperforms the original 32-state TSC design, as well as the new 32-state design whose performance is only 1.5-dB away from the fundamental limit. Note that we did not include performance of the 16-state IMYL code since it performed virtually the same as the 8-state one. We did not show performance of the new 16-state code due to the similar reason. We also derived the several new 3 bits/symbol 8-PSK space-time codes. Moreover, we propose to exploit orthogonality of the signals to derive an efficient decoder that has comparable complexity with a generic decoder for the traditional ST-TCM design. The fact that the decoding complexity can be kept relatively low nicely justifies the proposed STBC-TCM construction.

In addition, we show performance of several new improved 3-bits/symbol 8-PSK space-time codes in Figure 8. It is noted that the new 8-state code achieves similar performance as that of the original 32-state TSC design, and further performance improvement can be achieved by a larger encoder.

5. Conclusions

In this paper, we proposed a novel technique for designing a high-rate space-time code that achieved a significant performance gain over the original ST-TCM designs. The proposed 4-state 2-bits/sybol QPSK space-time code outperformed the original 4-state ST-TCM design by a considerably-wide margin and it performed even better than the original 32-state design, while the new 32-state QPSK code achieved a superior performance level that was merely 1.5 dB away from the outage probability limit. The major performance improvement was due to an exploitation of the set partitioning concept, which is essential to the design of a good trellis code, but was not possible in the original ST-TCM designs. Additionally, signal orthogonality was exploited to keep decoding complexity of the proposed STBC-TCM construction relatively low. The ability to achieve excellent performance with relatively low decoding complexity at high transmission rate made the proposed design technique particularly attractive.

6. References

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