DESIGN AND IMPLEMENTATION OF BUTLER MATRIX

-Simultaneus beam formation

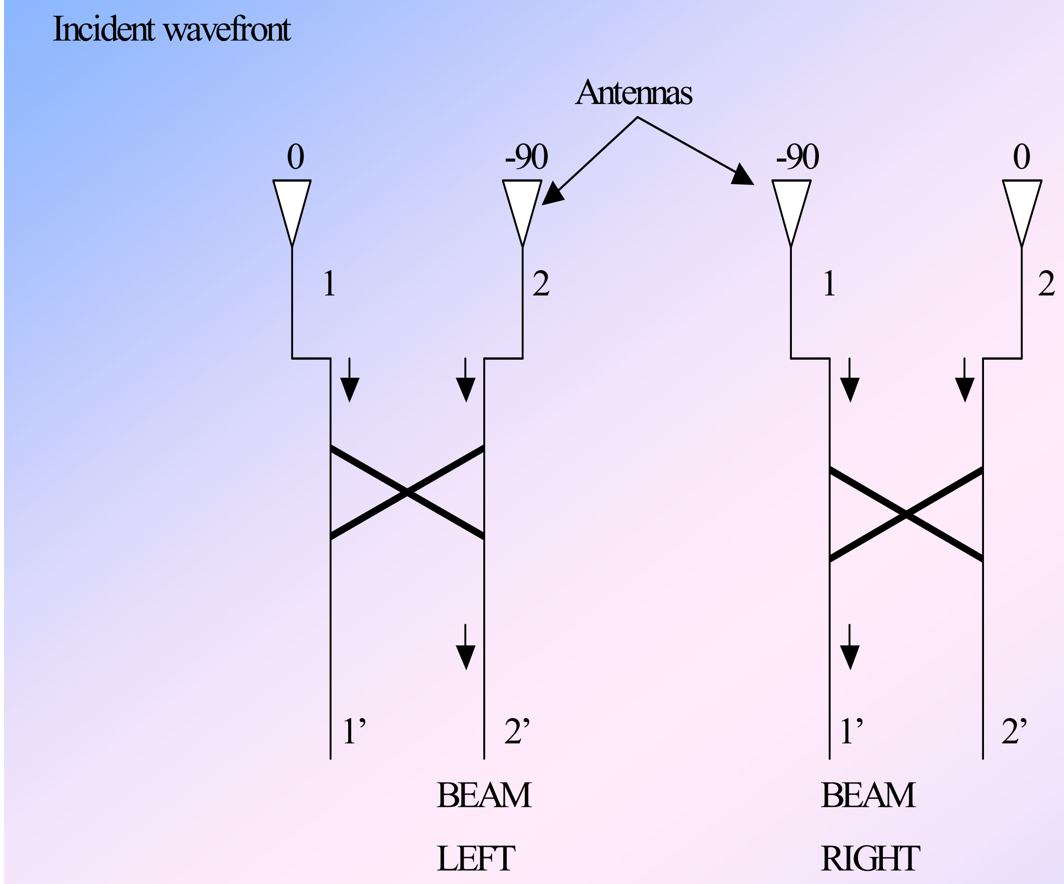
-Harish Rajagopalan

- ☞ Objective
- ☞ Elements of Butler matrix
- ☞ Study of 2x2, 4x4 and 8x8 butler matrices
- ☞ Testing and results for 4x4
- ☞ Conclusion and future scope

Antenna Beam Scanning Methods

- Mechanical Scanning
- Electronic Scanning
  - Phase Scanning
    - Series Feed (Blass matrix)
    - Parallel Feed (Butler matrix)
  - Frequency Scanning

Principle of Butler matrix



$I_1 = A_1 e^{j0}$ ------ (1)

$I_2 = A_2 e^{j\pi/2}$ ------(2)

and $A_1 = A_2$ -------(3)

$I_1' = I_1 + I_2 e^{j\pi/2}$

or $I_1' = A_1 e^{j0} + A_2 e^{j\pi}$ -------(4)

$I_2' = I_1 e^{j\pi/2} + I_2,$

$I_2' = A_1 e^{j\pi/2} + A_2 e^{j\pi/2},$

Or $I_2' = (A_1 + A_2) e^{j\pi/2}$ -------(5)

A_1 and A_2 → amplitudes of antenna currents

I_1 and I_2 → Input antenna currents

I_1' and I_2' → output antenna currents

Characteristics of Butler matrix

-Number of beams = Number of antenna elements =N

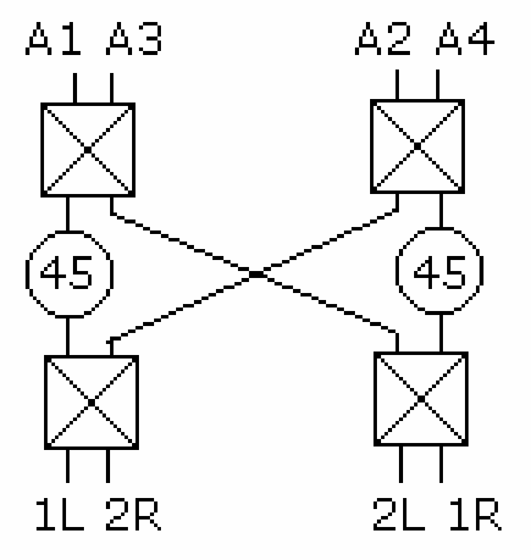
-Number of Hybrid rings = N/2log$_2$N

-Number of Phase shifters = N/2 (log$_2$N-1)

-Low insertion loss

-Uniform antenna array illumination

4 x 4 Butler matrix



$1L = A1 \angle 45^0 + A2 \angle 90^0 + A3 \angle 135^0 + A4 \angle 180^0$

$2R = A1 \angle 135^0 + A2 \angle 0^0 + A3 \angle 225^0 + A4 \angle 90^0$

$2L = A1 \angle 90^0 + A2 \angle 225^0 + A3 \angle 0^0 + A4 \angle 135^0$

$1R = A1 \angle 180^0 + A2 \angle 135^0 + A3 \angle 90^0 + A4 \angle 45^0$

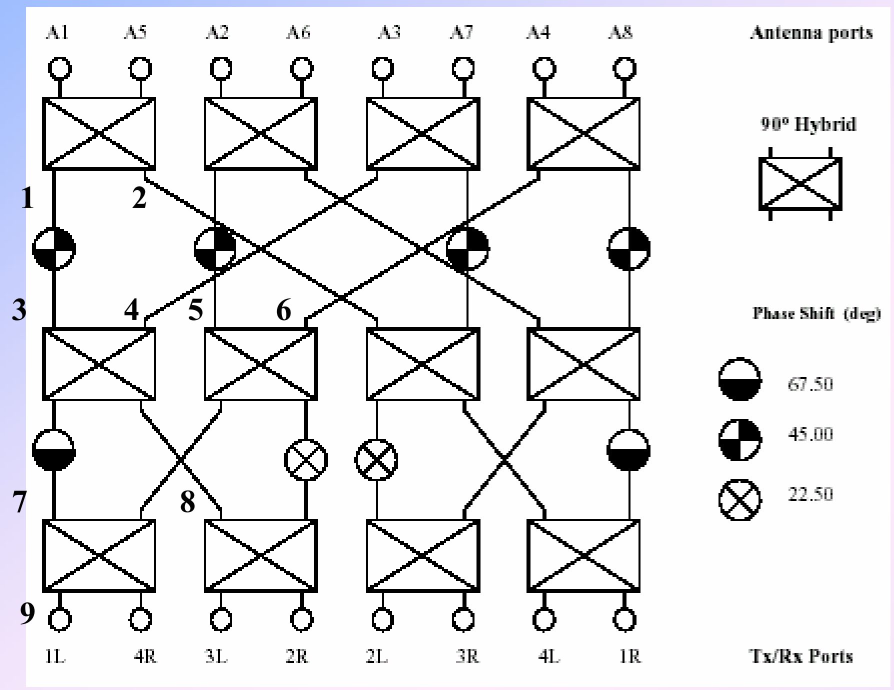
Phase matrix of 4 x 4 Butler matrix

$$\begin{bmatrix} 2R \\ 1R \\ 1L \\ 2L \end{bmatrix} = \begin{bmatrix} e^{-j3/4\pi} & e^{-j0\pi} & e^{-j5/4\pi} & e^{-j\pi/2} \\ e^{-j\pi} & e^{-j3/4\pi} & e^{-j\pi/2} & e^{-j\pi/4} \\ e^{-j\pi/4} & e^{-j\pi/2} & e^{-j3/4\pi} & e^{-j\pi} \\ e^{-j\pi/2} & e^{-j5/4\pi} & e^{-j0\pi} & e^{-j3/4\pi} \end{bmatrix} \begin{bmatrix} A1 \\ A2 \\ A3 \\ A4 \end{bmatrix}$$

$$\begin{bmatrix} 2R \\ 1R \\ 1L \\ 2L \end{bmatrix} = \begin{bmatrix} 0 & -135 & 90 & -45 \\ 0 & -45 & -90 & -135 \\ 0 & 45 & 90 & 135 \\ 0 & 135 & -90 & 45 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix}$$

Phase progression

8 x 8 Butler matrix



Equation for 1L port.

At point 1: $A1 + A5\angle 90$

At point 2: $A2 + A6\angle 90$

At point 3: $A1\angle 45 + A5\angle 135$

At point 4: $A3 + A7\angle 90$

At point 5: $A2\angle 45 + A6\angle 135$

At point 6: $A4 + A8\angle 90$

At point 7: $A1\angle 45 + A5\angle 90 + A3\angle 90$

$+A7\angle 180$

At point 8: $A2\angle 45 + A6\angle 135 + A4\angle 90$
$+A8\angle 180$

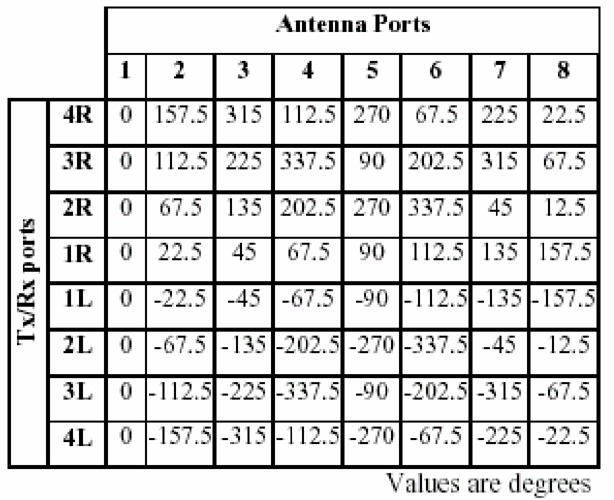
At point 9: $A1\angle 112.5 + A5\angle 157.5$
$+A3\angle 157.5 + A7\angle 247.5$

At port 1L: $A1\angle 112.5 + A5\angle 157.5 + A3\angle 157.5 + A7\angle 247.5 + A2\angle 135 + A6\angle 225 + A4\angle 180 + A8\angle 270$

Phase matrix of 8 x 8 Butler matrix

$$
\begin{bmatrix} 4R \\ 3R \\ 2R \\ 1R \\ 1L \\ 2L \\ 3L \\ 4L \end{bmatrix}
=
\begin{bmatrix}
e^{-j9\frac{\pi}{8}} & e^{-j2\frac{\pi}{8}} & e^{-j11\frac{\pi}{8}} & e^{-j4\frac{\pi}{8}} & e^{-j13\frac{\pi}{8}} & e^{-j6\frac{\pi}{8}} & e^{-j15\frac{\pi}{8}} & e^{-j8\frac{\pi}{8}} \\
e^{-j9\frac{\pi}{8}} & e^{-j4\frac{\pi}{8}} & e^{-j15\frac{\pi}{8}} & e^{-j10\frac{\pi}{8}} & e^{-j5\frac{\pi}{8}} & e^{0} & e^{-j11\frac{\pi}{8}} & e^{-j6\frac{\pi}{8}} \\
e^{-j10\frac{\pi}{8}} & e^{-j7\frac{\pi}{8}} & e^{-j4\frac{\pi}{8}} & e^{-j\frac{\pi}{8}} & e^{-j14\frac{\pi}{8}} & e^{-j11\frac{\pi}{8}} & e^{-j8\frac{\pi}{8}} & e^{-j5\frac{\pi}{8}} \\
e^{-j12\frac{\pi}{8}} & e^{-j11\frac{\pi}{8}} & e^{-j10\frac{\pi}{8}} & e^{-j9\frac{\pi}{8}} & e^{-j8\frac{\pi}{8}} & e^{-j7\frac{\pi}{8}} & e^{-j6\frac{\pi}{8}} & e^{-j5\frac{\pi}{8}} \\
e^{-j5\frac{\pi}{8}} & e^{-j6\frac{\pi}{8}} & e^{-j7\frac{\pi}{8}} & e^{-j8\frac{\pi}{8}} & e^{-j9\frac{\pi}{8}} & e^{-j10\frac{\pi}{8}} & e^{-j11\frac{\pi}{8}} & e^{-j12\frac{\pi}{8}} \\
e^{-j5\frac{\pi}{8}} & e^{-j8\frac{\pi}{8}} & e^{-j11\frac{\pi}{8}} & e^{-j14\frac{\pi}{8}} & e^{-j\frac{\pi}{8}} & e^{-j4\frac{\pi}{8}} & e^{-j7\frac{\pi}{8}} & e^{-j10\frac{\pi}{8}} \\
e^{-j6\frac{\pi}{8}} & e^{-j11\frac{\pi}{8}} & e^{-j16\frac{\pi}{8}} & e^{-j5\frac{\pi}{8}} & e^{-j10\frac{\pi}{8}} & e^{-j15\frac{\pi}{8}} & e^{-j4\frac{\pi}{8}} & e^{-j9\frac{\pi}{8}} \\
e^{-j8\frac{\pi}{8}} & e^{-j15\frac{\pi}{8}} & e^{-j6\frac{\pi}{8}} & e^{-j13\frac{\pi}{8}} & e^{-j4\frac{\pi}{8}} & e^{-j11\frac{\pi}{8}} & e^{-j2\frac{\pi}{8}} & e^{-j9\frac{\pi}{8}}
\end{bmatrix}
\cdot
\begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \end{bmatrix}
$$

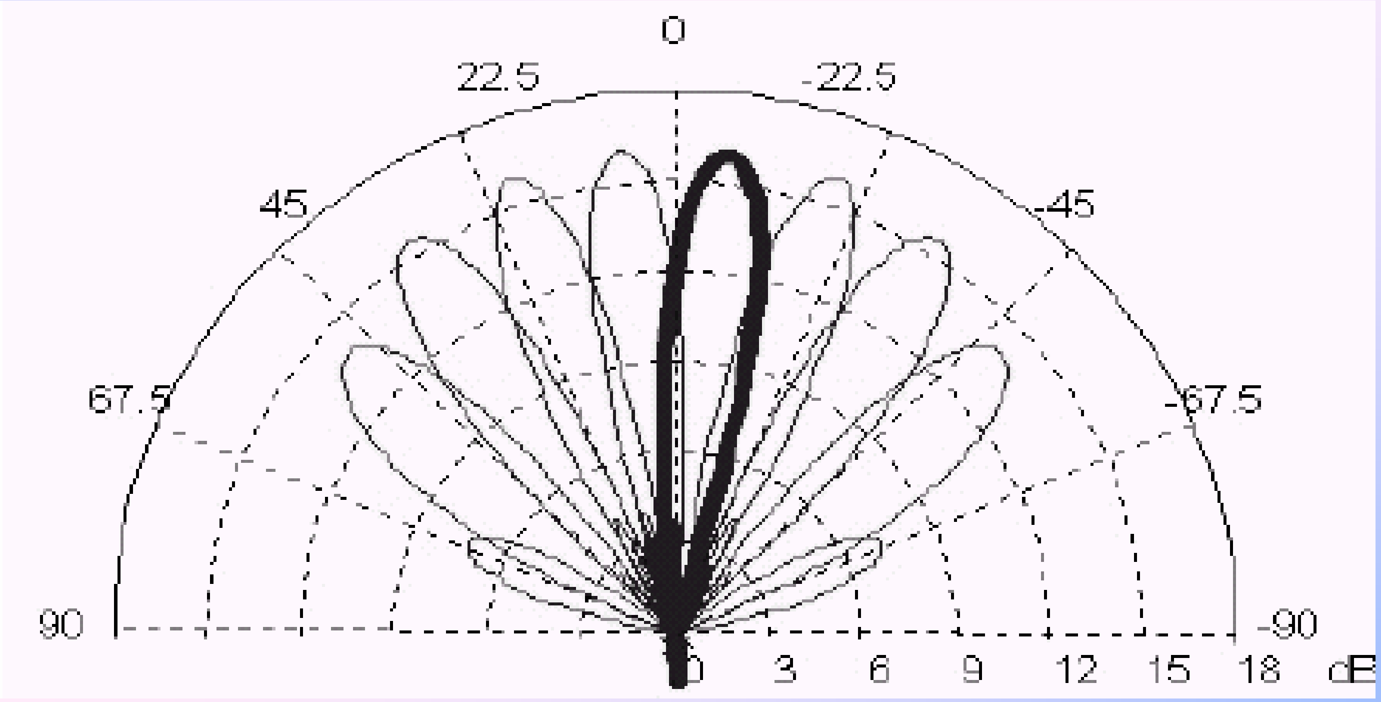
Phase progression



		Antenna Ports							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Tx/Rx ports	**4R**	0	157.5	315	112.5	270	67.5	225	22.5
	3R	0	112.5	225	337.5	90	202.5	315	67.5
	2R	0	67.5	135	202.5	270	337.5	45	12.5
	1R	0	22.5	45	67.5	90	112.5	135	157.5
	1L	0	-22.5	-45	-67.5	-90	-112.5	-135	-157.5
	2L	0	-67.5	-135	-202.5	-270	-337.5	-45	-12.5
	3L	0	-112.5	-225	-337.5	-90	-202.5	-315	-67.5
	4L	0	-157.5	-315	-112.5	-270	-67.5	-225	-22.5

Values are degrees

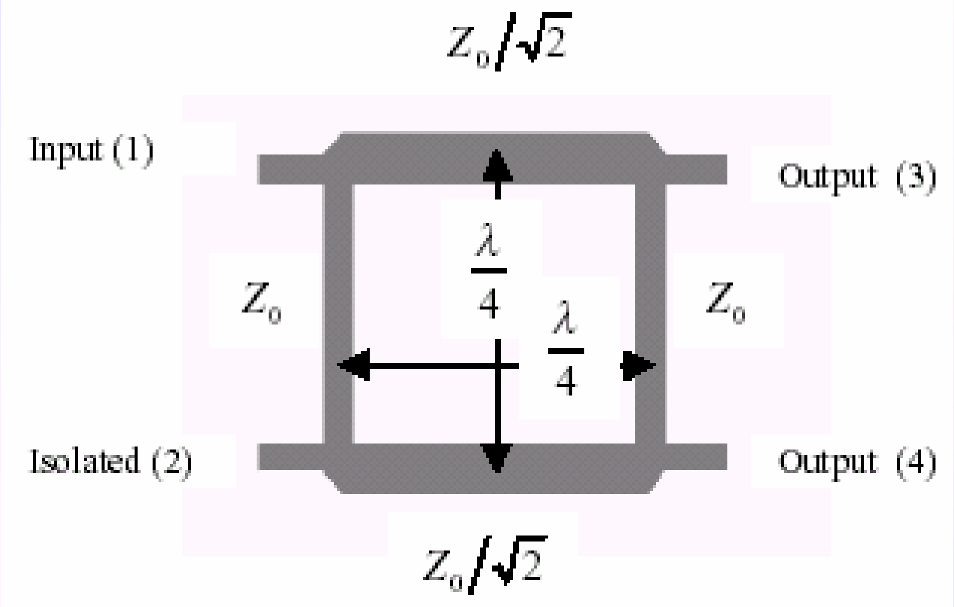
Beam pattern for 8x8 Butler matrix



Components of Butler matrix

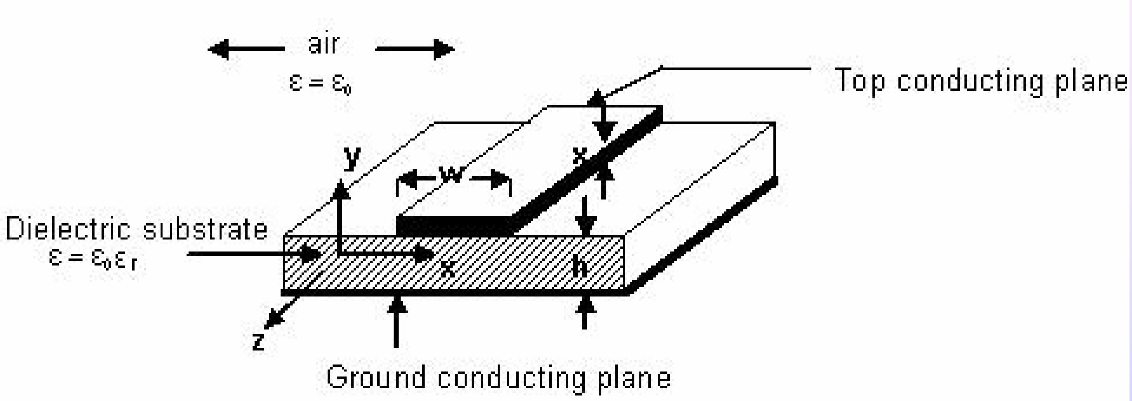
- Qudrature hybrid
- Fixed phase shifter
- Helical antenna

Qudrature Hybrid



Branch line coupler

Microstrip Lines



Basic Structure

Design Of Hybrid (Branchline Coupler)

Frequency = 1GHz Wavelength (in air) = 30 cm.

Substrate used for PCB manufacturing is FR-4 Glass Epoxy.

For FR-4 board, Relative dielectric constant (εr) = 4.4

The height of the dielectric (d) = 1.6 mm

Characteristic impedance Zo = 50 Ω

$$Zo1 = Zo/\sqrt{2} = 35.35\ \Omega$$

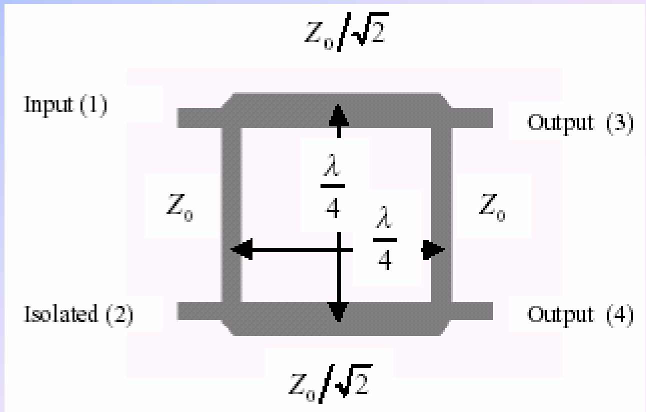
Formulae:

1. Effective Dielectric constant(εe) = $\frac{\varepsilon r+1}{2} + \frac{\varepsilon r-1}{2}(1 + 12\, d / W)^{-1/2}$

2 . $W/d = 2/\pi[B - 1 - \ln(2B - 1) + (\varepsilon r - 1)/2\varepsilon r\, \{\ln(B - 1) + 0.39 - 0.61/\varepsilon r\}]$. …W/d > 2

where $B = \frac{377\pi}{2Zo(\varepsilon r)^{1/2}}$

Calculations



For 50 Ω

B = 5.646312

W/d = 1.91335

W = 3.06136 mm

ε_{eff} = 3.33024

λmicrostrip = λo/√εeff = 16.43886 cm

Length of track = λmicrostrip/4 = 4.1097 cm

For 35.35 Ω

B = 7.98629

W/d = 3.26475

W = 5.2236

ε_{eff} =3.48619

λmicrostrip = λo/√εeff = 16.0674 cm

Length of track = λmicrostrip/4 = 4.0168 cm

Fixed Phase shifter

Semi rigid cable

- **Outer Conductor (Copper)**
  - minimizes the power loss
  - maximizes the mechanical integrity
  - provides the desired interface with connections

- **Center conductor(Silver plated copper)**
  - acts as primary signal carrier
  - provides excellent high frequency conductivity

- **Dielectric material (Polytetrafluoroethylene)**
  - maintains the spacing and geometry of the cable
  - assures mechanical integrity during forming and bending or under pressure

Design of phase shifter

L1→ a1

L2→ a2

Differential measurement

L1 provides phase shift of a1°

L2 provides phase shift of a2 °

(L1-L2) provides phase shift of (a1-a2) °

L1 = 10 cm, a1 = 97.2°

L2 = 15 cm, a2 = 18.5°

(15-10) cm → (18.5-97.2) °

5 cm → -78.7°

1 cm → -15.7°

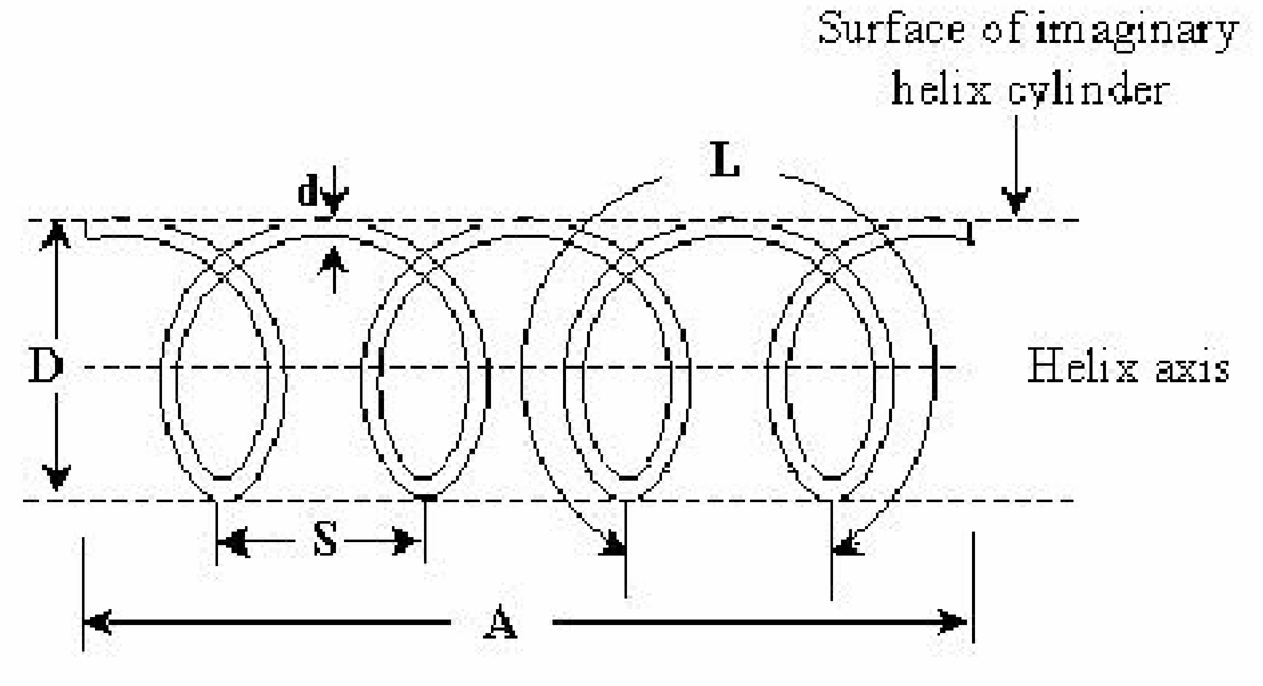
22.87 cm → 360°

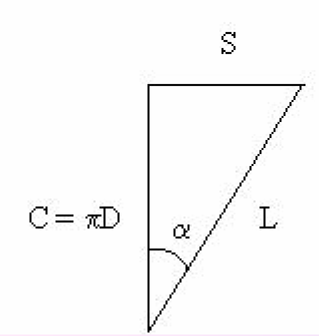
λc- wavelength in cable = 22.8 cm

λa- wavelength in air = 30.0 cm

Velocity factor = $\lambda c/\lambda a$ = 76%.

Helical antenna





Helical geometry

D = 110 mm

d = 2 mm

L = 353.5 mm

S = 75 mm

A = 450 mm

C = πD = 345.5mm

α= arc tan $S/\pi D$ = 12.24°

N = number of turns = 6

$1.2 \geq C_\lambda \geq 0.8$,

$14^o \geq \alpha \geq 12^o$ and $n \geq 4$

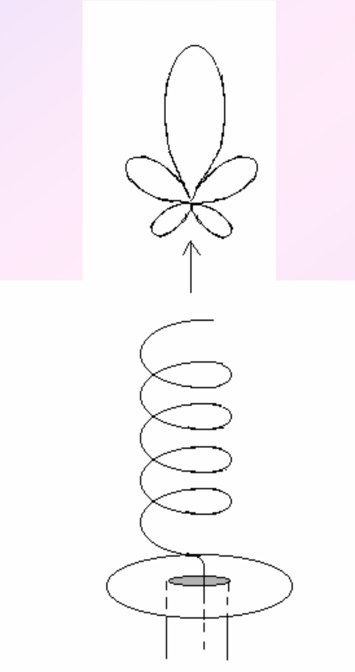
Transmission And Radiation Modes Of Helix

1. Normal mode-

   The field radiated by the antenna is maximum in a plane normal to the helix and minimum along its axis.

2. Axial mode

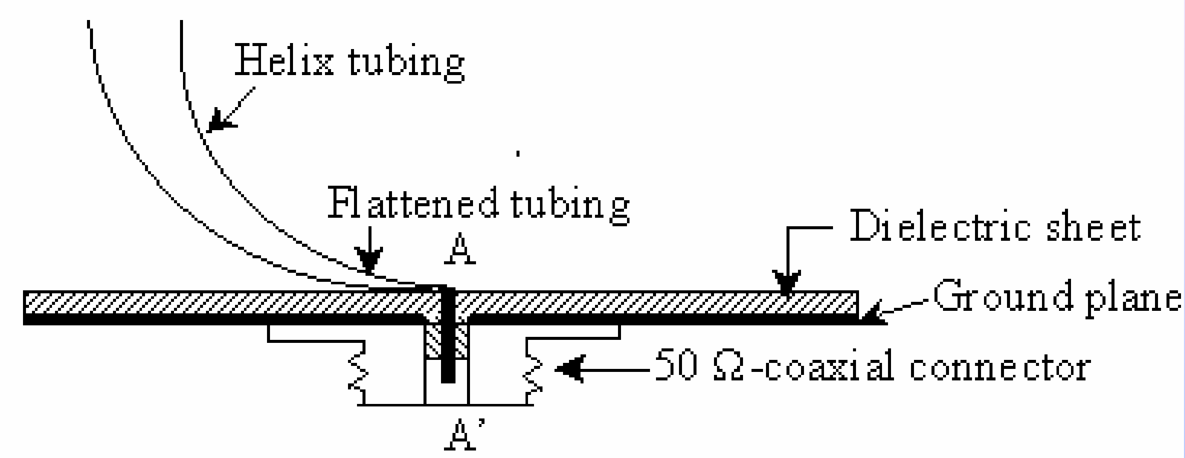
   The field radiated by the antenna is maximum along its axis.
   Power Beam Width (HPBW) = $\frac{52}{c\lambda\sqrt{ns\lambda}} = 36^{o}$



Impedance matching

With axial feed the terminal impedance (resistive) is given by

$R = 140C\lambda \quad (\Omega)$



Gradually tapered transition from helix to coaxial line

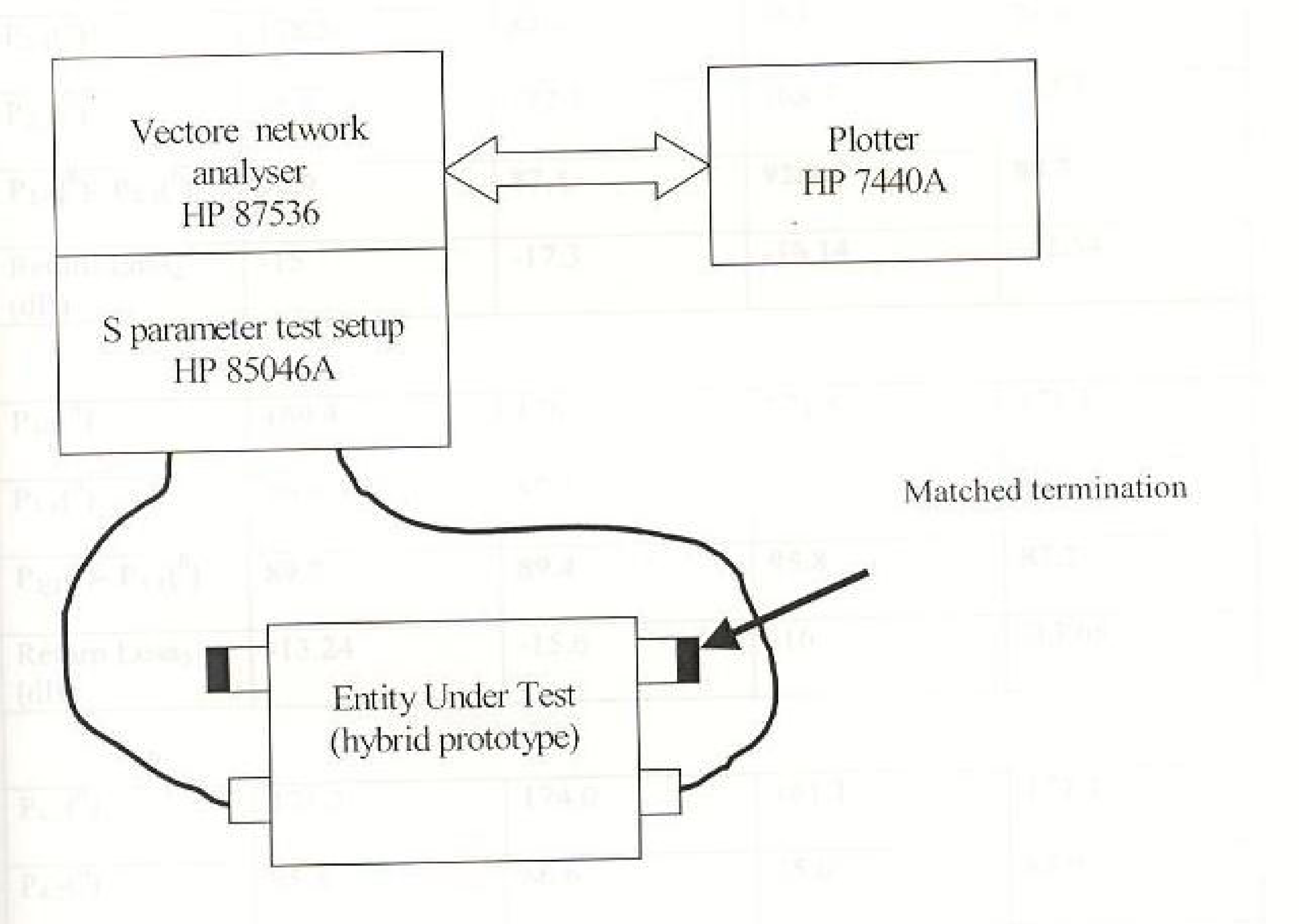
Testing and Results

Testing of Phase shifter

Testing of 4 x 4 Butler matrix

Beam formation of 4 x 4 Butler matrix

Testing of Hybrid



TESTING OF HYBRIDS ON ONE CARD:

Ports	**Hybrid A**	**Hybrid B**	**Hybrid C**	**Hybrid D**
$P_{1/4}(^0)$	-116.6	175.4	161.5	177
$P_{1/3}(^0)$	151.8	84.9	73.7	86.6
$\mathbf{P_{1/4}(^0)- P_{1/3}(^0)}$	**91.6**	**90.5**	**87.8**	**90.4**
Return Loss$_1$ (dB)	-16.2	-17.48	-15.6	-16.16
$P_{2/4}(^0)$	178.5	84.9	75.9	78.8
$P_{2/3}(^0)$	85.6	172.3	168.7	167.5
$\mathbf{P_{2/4}(^0)- P_{2/3}(^0)}$	**92.9**	**87.4**	**92.8**	**88.7**
Return Loss$_2$ (dB)	-15	-17.3	-16.14	-12.54
$P_{3/2}(^0)$	169.4	176.7	171.8	171.3
$P_{3/1}(^0)$	79.7	87.3	76	84.1
$\mathbf{P_{3/2}(^0)- P_{3/1}(^0)}$	**89.7**	**89.4**	**95.8**	**87.2**
Return Loss$_3$ (dB)	-13.24	-15.6	-16	-13.65
$P_{4/1}(^0)$	171.7	174.0	161.1	174.4
$P_{4/2}(^0)$	85.8	86.6	75.6	83.9
$\mathbf{P_{4/1}(^0)- P_{4/2}(^0)}$	**85.9**	**87.4**	**85.5**	**90.5**
Return Loss$_4$ (dB)	-15.23	-16.42	-14.2	-17.24

READINGS FOR 4x4 BUTLER MATRIX

THEORETICAL MATRIX

$$\begin{bmatrix} 2R \\ 1R \\ 1L \\ 2L \end{bmatrix} = \begin{pmatrix} 135 & 0 & 225 & 90 \\ 180 & 135 & 90 & 45 \\ 45 & 90 & 135 & 180 \\ 90 & 225 & 0 & 135 \end{pmatrix} * \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix}$$

THEORETICAL PROGRESSIVE PHASE SHIFT

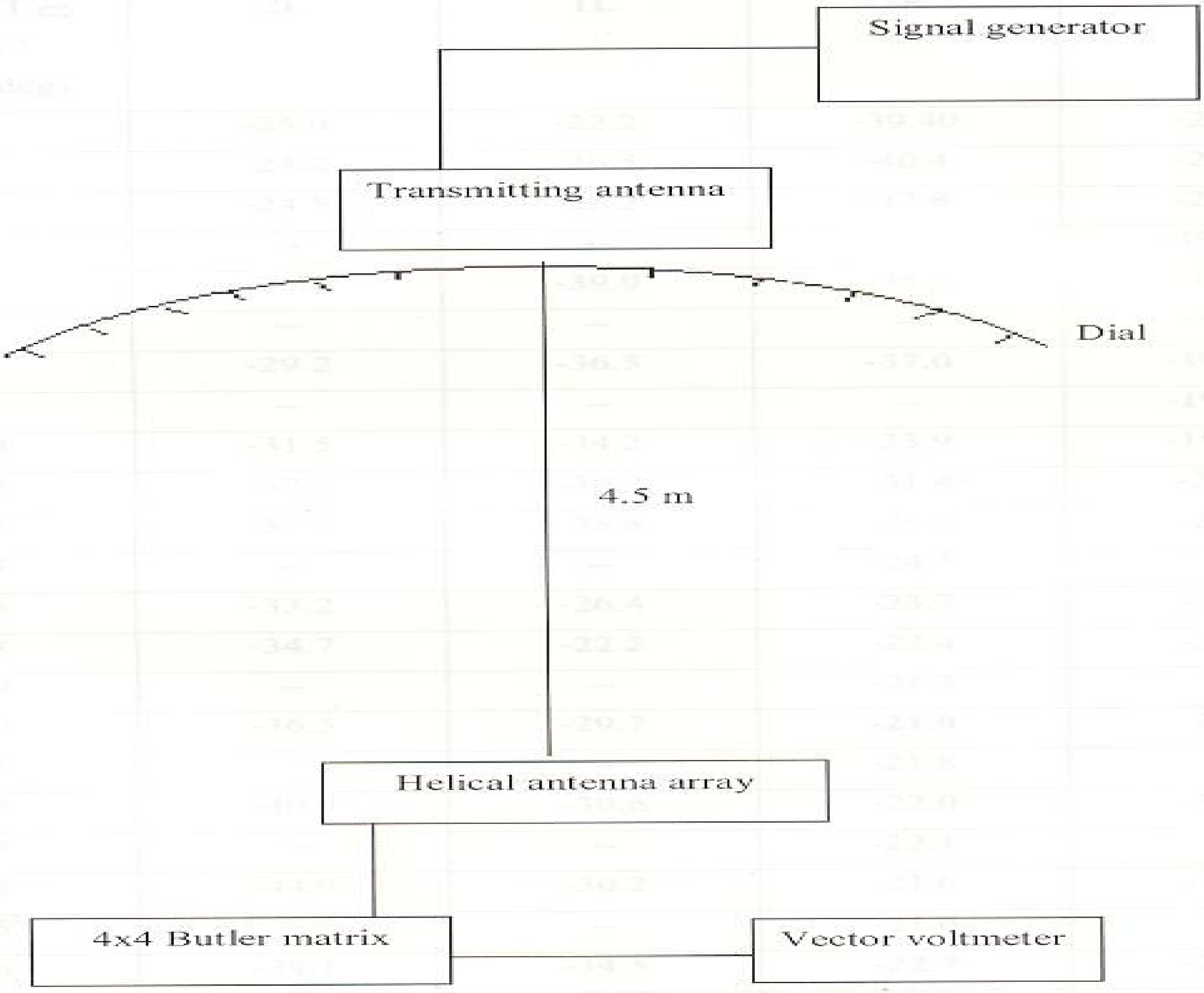
$$\begin{bmatrix} 2R \\ 1R \\ 1L \\ 2L \end{bmatrix} = \begin{pmatrix} 0 & -135 & 90 & -45 \\ 0 & -45 & -90 & -135 \\ 0 & 45 & 90 & 135 \\ 0 & 135 & -90 & 45 \end{pmatrix} * \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix}$$

OBSERVED READINGS

$$\begin{bmatrix} 2R \\ 1R \\ 1L \\ 2L \end{bmatrix} = \begin{pmatrix} 77.5 & -155.3 & -12.3 & 109.5 \\ 32.5 & 65.3 & 118.7 & 157.8 \\ 170.8 & 116.2 & 81.4 & 64.1 \\ 125.4 & -21.6 & -144.8 & 64.1 \end{pmatrix} * \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix}$$

$$\begin{bmatrix} 2R \\ 1R \\ 1L \\ 2L \end{bmatrix} = \begin{pmatrix} 0 & 127.2 & -89.8 & 32 \\ 0 & 33 & 86.4 & 119.5 \\ 0 & -54.6 & -89.4 & -148.3 \\ 0 & -147 & 89.8 & -61.3 \end{pmatrix} * \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix}$$

SETUP FOR BEAM PATTERN OF 4X4 BUTLER MATRIX:



TESTING FOR 4X4 BUTLER MATRIX:

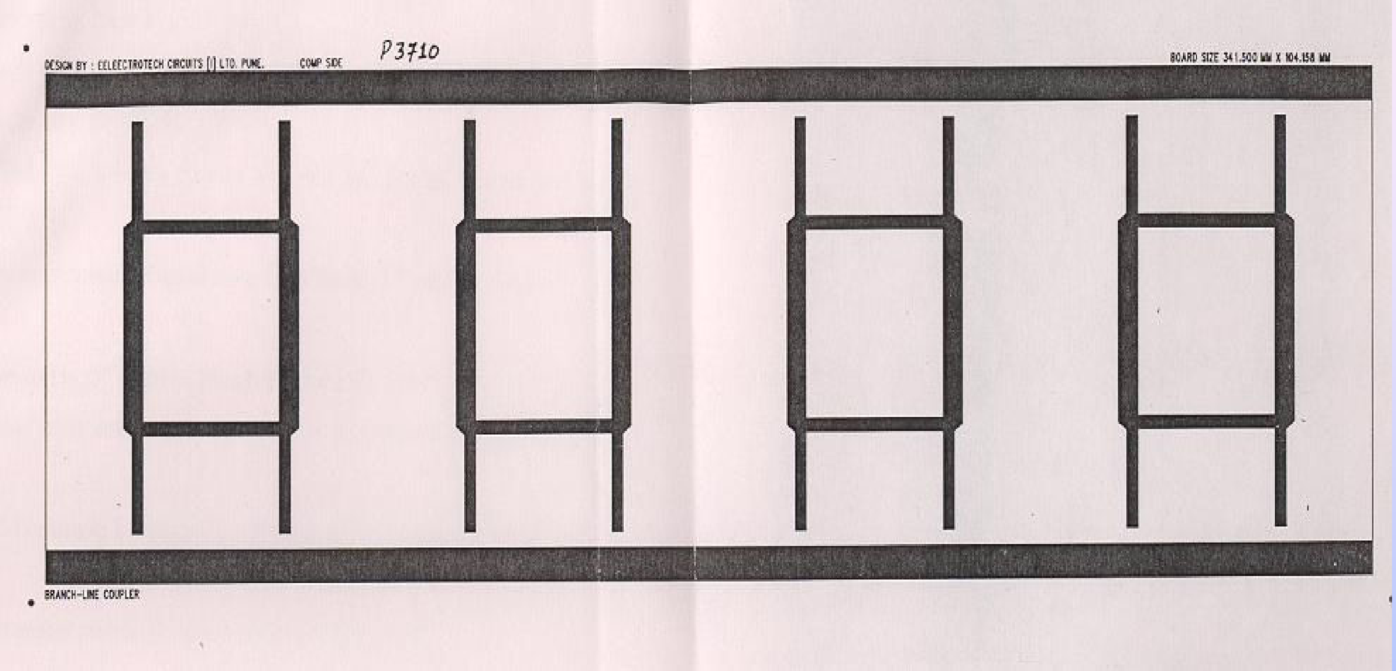
Input Frequency = 1 GHZ

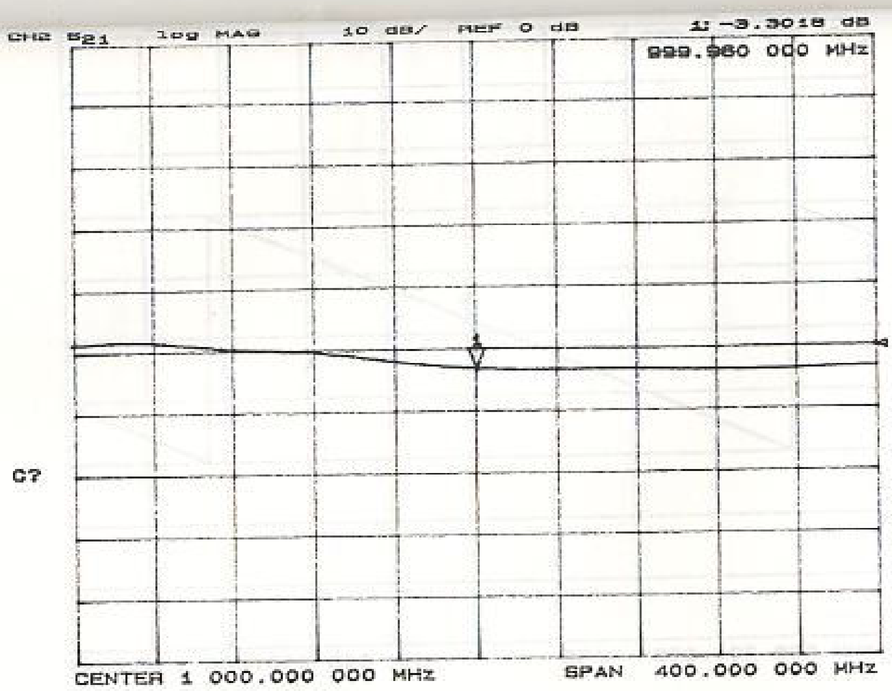
Input Power = +13dBm   Output power in dBm

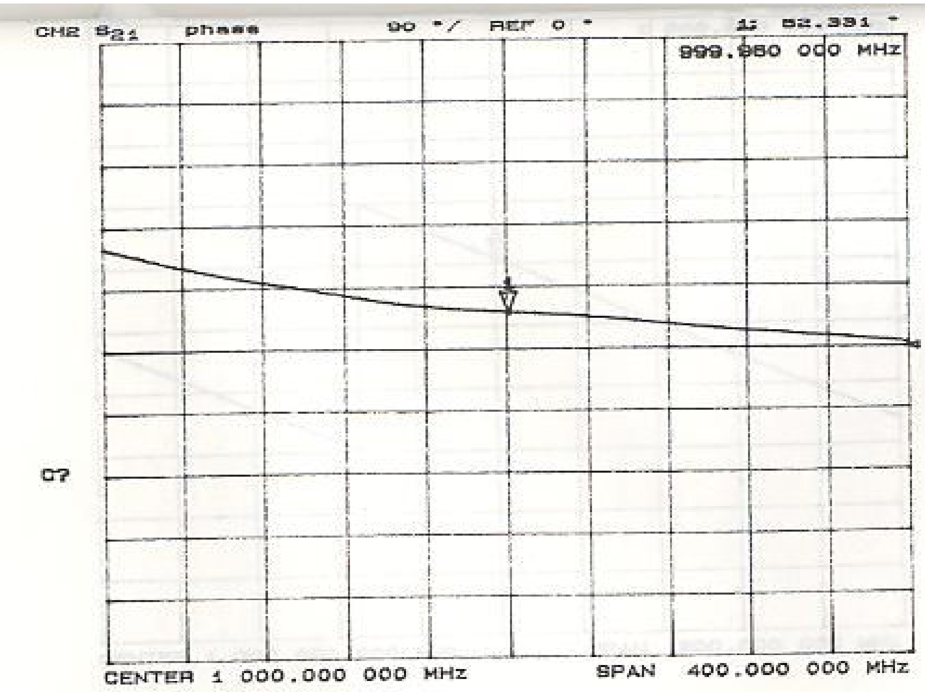
PORT ⇨ ANGLE ⇩ (deg)	2L	1L	2R	1R
0	-25.0	-22.2	-39.40	-24.9
-2	-24.2	-26.5	-40.4	-22.2
-4	-24.5	-28.2	-37.8	-20.9
-5	--	--	--	-19.91
-6	-26.6	-39.9	-38.2	-19.3
-7	--	--	--	-19.33
-8	-29.2	-36.5	-37.0	-19.27
-9	--	--	--	**-19.09**
-10	-31.5	-34.2	-33.9	-19.39
-12	-32.8	-30.2	-31.4	-21.2
-14	-37.5	-38.8	-28.8	-22.4
-15	--	--	-24.3	--
-16	-33.2	-26.4	-23.7	-25.1
-18	-34.7	-22.2	-22.4	-29.6
-19	--	--	-21.3	--
-20	-36.5	-29.7	**-21.0**	-37.3
-21	--	--	-21.8	--
-22	-40.9	-30.6	-22.0	-37.3
-23	--	--	-22.1	--
-24	-44.9	-30.2	-21.6	-32.5
-25	--	--	-21.6	--
-26	-34.1	-34.5	-22.7	-29.5
-28	-31.2	-39.5	-24.5	-29.3
-30	-28.9	-43.7	-26.3	-28.3
-32	-28.4	-44.7	-30.0	-29.5

-34	-26.6	-37.9	-33.0	-30.6
2	-27.5	-20.5	-48.2	-30.1
4	-31.2	-19.5	-32.7	-31.6
5	--	-20.5	--	--
6	-40.2	-20.7	-29.9	-31.5
7	--	-20.6	--	--
8	-33.9	-20.9	-28.6	-32.4
9	--	-21.6	--	--
10	-29.7	-22.4	-26.2	-33.3
11	-26.3	--	--	--
12	-23.7	-23.8	-25.5	-35.7
14	-23.5	-25.7	-27.1	-35.2
16	-22.4	-27.4	-26.9	-37.5
18	-22.2	-30.6	-32.9	-36.0
20	-22.6	-35.2	-37.4	-33.2
21	-23.2	--	--	--
22	-22.5	-36.4	-42.1	-29.7
23	-23.2	--	--	--
24	-24.5	-36.7	-36.2	-28.3
26	-26.2	-37.2	-32.4	-29.3
28	-29.4	-40.3	-28.4	-29.4
30	-30.2	-38.5	-28.0	-30.5
32	-32.7	-34.8	-26.8	-33.4
34	-34.2	-33.5	-26.6	-36.1

PCB Layout


DESIGN BY : EELECTROTECH CIRCUITS (I) LTD. PUNE.
COMP SIDE
P3710
BOARD SIZE 341.500 MM X 104.158 MM
BRANCH-LINE COUPLER


CH2 S21 log MAG 10 dB/ REF 0 dB 1: -3.3018 dB
999.960 000 MHz
C?
CENTER 1 000.000 000 MHz
SPAN 400.000 000 MHz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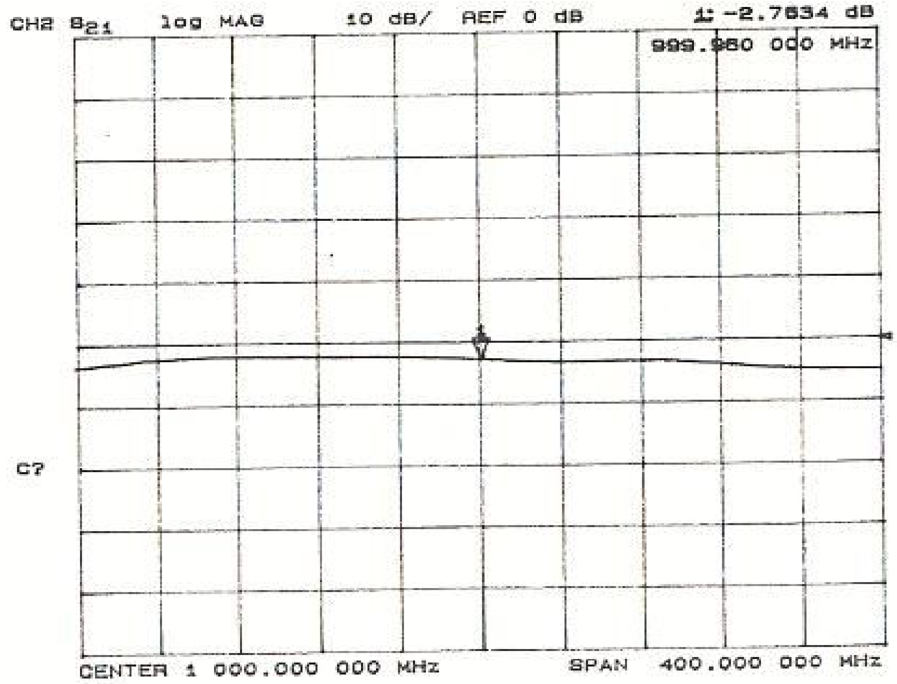
999.960 000 MHz
C?
CENTER 1 000.000 000 MHz
SPAN 400.000 000 MHz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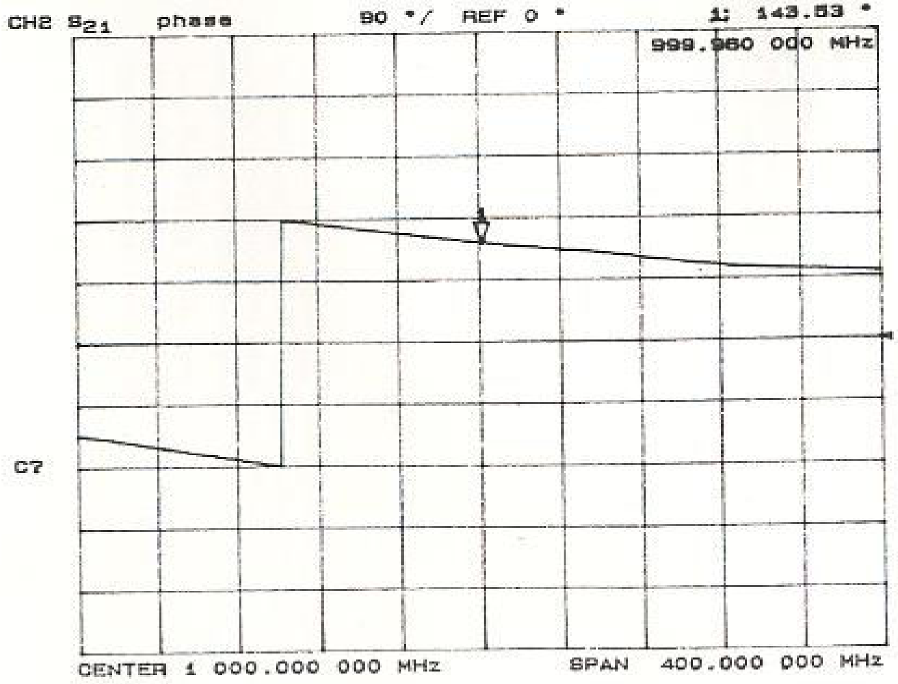
999.960 000 MHz
C?
CENTER 1 000.000 000 MHz
SPAN 400.000 000 MHz


CH2 S21 phase 90 °/ REF 0 ° 1: 143.53 °
999.960 000 MHz
C?
CENTER 1 000.000 000 MHz
SPAN 400.000 000 MHz

Applications

- -tracking of radio sources
- -direction finding

Future scope

- -Adaptive array and smart antenna
- - Digitization

LIST OF REFERENCES:

[1] Y. T. Lo and S. W. Lee, Antenna handbook, theory, applications and design, Van Nostrand Reinhold company, New York, 1988.

[2] Richard C. Johnson and Henry Jasik, Antenna Engineering Handbook, 2nd edition, McGraw Hill book company, New york, 1961.

[3] Constantine A. Balanis, Antenna theory, analysis and design, Harper and Row publishers, New York, 1981.

[4] Richard C. Johnson, Antenna Engineering Handbook, 3rd edition, McGraw Hill book company Inc.

[5] John D. Kraus, Antennas, 2nd edition, Tata McGraw Hill, 1997.

[6] R. C. Hansen, Microwave scanning antennas, VOL III, Education Academic Press Inc.-1969.

[7] David M. Pozer, Microwave Engineering, 2nd Edition, John Wiley & Sons Inc. 1998.

[8] Vijay Jain, Project report on branch line coupler at ISRO, 1991

THANK YOU