

Spatial Fidelity And Estimation in Sensor Networks

Ameesh Pandya, Huiyu Luo, and Greg Pottie

Department of Electrical Engineering

University of California, Los Angeles

{ameesh, huiyu, pottie}@ee.ucla.edu

Abstract— We consider spatial fidelity in sensor networks and show that many problems in such networks are not well defined without it. We consider sensor network gathering data on multiple point sources, or a distributed source such as bandlimited or non-bandlimited field. The field could either be deterministic or random. For these networks, we derive the conditions leading to cooperation between the power constrained sensor nodes for the data fusion purposes and analyze the sensor density under spatial fidelity and cooperation. We also propose the heuristics for sampling criterion in case of distributed sensor network.

I. INTRODUCTION

Unlike ad hoc network nodes, sensor nodes are deployed to gather data from either point or distributed phenomena. This leads to a cluster of sensors having nearly the same information, leading to correlation among themselves. Individual nodes will have some combination of sensing, signal processing and communications capability and may self-organize for a variety of cooperative sensing and communication tasks, subject to resource constraints such as energy and bandwidth [1, 2].

For purposes of this paper, the sensor network problem is for some end user to extract information concerning some source or set of sources to some desired level of fidelity, subject to resource constraints. Fidelity encompasses such concepts as spatial or temporal resolution, misidentification probability or other accuracy measures, and network quality of service related measures such as latency from initial observation. Resource constraints can include signal processing cycles, energy consumption, and information rate.

Consider a bounded region, \mathcal{R} , with a point source, X sensed by a group of L sensors, $\{Y_i\}_{i=1}^L$. This can be mathematically modelled as:

$$Y_i = X + N_i \quad (1)$$

where, i ranges from $1, \dots, L$ and N_i is zero mean independent Gaussian noise with variance $\sigma_{N_i}^2$. If the data available at the sensors is encoded separately and decoded jointly, then (1) represents a well known CEO system [3–5]. Suppose \mathcal{R} consists of m uniformly distributed multiple point sources, $\{X_i\}_{i=1}^m$, then modelling the network as CEO system is not obvious due to the interference between the sources. The scenario where we can apply CEO result is when the region \mathcal{R} is sampled with infinite sensors i.e. number of sensors should be much greater than the number of sources. In such a case, we have M independent CEO systems. However, in practice, such a scenario is highly unlikely. Generally we have finite sensor density and the sources could be very near to each other, in some cases even

overlapping, to have any effective sensing or reproduction. Because of this reason, we consider the notion of spatial fidelity for sensor networks in this paper.

The spatial fidelity, for point sources, could be broadly defined as the distance separating the point sources in Euclidean space. The reproduction or detection decision depends on the spatial fidelity. The sensor networks, in general, are densely deployed and same source is observed by group of sensors rather than one. The data available at sensors is the noisy version of the actual source. Hence, sensors cooperate (data fusion) among themselves to enhance the detection probability or reproduction quality of the source at fusion center. We calculate the number of sensors required to cooperate and the total number of sensors needed to deploy as a function of spatial fidelity.

The source required to be sensed by sensors could also be distributed such as band-limited field ¹ [6, 7] or non-bandlimited [8]. The distributed process however could be modelled using correlated point sources. In practice, distributed continuous processes are, however, never fully observable. A typical approach in sensing is to sample the processes in time and space, in which the distributed phenomena are reasonably modelled as sets of correlated point sources.

The spatial fidelity criterion for distributed phenomena transforms into a problem of sampling resolution. In other words, it dictates the rate at which the field should be sampled with the sensors. Obviously, for better reproduction of the field the resolution should be as fine as possible which leads to denser sampling of the field. But this, however, is highly unlikely in a real time situation. Hence, based on the field gradient or change in the sensor readings, the sensors could be either turned on or off resulting in energy conservation. For example, sensors deployed on highly smooth field [9, 10] such as C^0 or C^1 need only a few sensors functioning. On the other hand, an abruptly or frequently changing field needs a large number of sensors to have the desired reproduction quality.

The rest of the paper is organized as follows: In Section II spatial fidelity for the point sources is considered. This section also calculates the required number of sensors for the given source density, spatial fidelity, and desired distortion in the final reproduction or detection. Section III extends the idea of spatial fidelity to distributed sources leading to heuristics for sampling criteria. The paper concludes in Section IV.

¹By band-limited field, it means that the power spectral density of the field is band-limited.

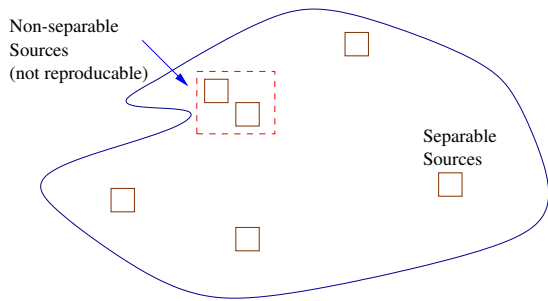


Fig. 1. Separable and non-separable point sources.

II. SPATIAL FIDELITY AND COLLABORATIVE PROCESSING FOR POINT PHENOMENON

Consider the point sources placed too close to each other to have any reliable distinction between the sources as in Figure 1. In some cases the sources may even overlap. Hence, there would be significant interference at a sensor, making difficult for any effective sensing and thus reproduction of either of the sources. Note that the sources and sensors considered in this paper are homogeneous i.e. of same nature. This show that the sensor network design should incorporate these situations. For this purpose the notion of spatial fidelity is considered.

A. Spatial Fidelity

Assume that the power decays with the distance according to an exponent $\alpha \geq 2$. In order to have any reliable sensing of the point sources there should be a minimum separation between sources in Euclidean space. This will either completely or partially suppress the interference from other sources. Based on this, we have the following definition of spatial fidelity:

Definition II.1—Spatial Fidelity for Point Phenomenon:

Spatial fidelity, δ_{ij} , is the minimum separation between any two homogenous sources i and j , $i \neq j$, in Euclidean space to attain the desired detection probability or reproduction quality. Thus,

$$\|X_i - X_j\| \geq \delta_{ij}, \quad \forall i, j (i \neq j). \quad (2)$$

where $\|\cdot\|$ denotes Euclidean norm. Hence, spatial fidelity, analogous to image processing, represents resolution. The value of δ is user defined and depends on the applications or the desired distortion in the reproduced source. However, we assume the same value of spatial fidelity for any two homogeneous sources, $\delta_{ij} = \delta$, $\forall i, j$, and $i \neq j$.

In sensor network applications, if the sources do not satisfy spatial fidelity constraints then they are considered to be non reproducible. For instance, if two heat sources are less than some distance away to have any reliable reading, the sources are considered to be indistinguishable. However, this does not hold for the mixture of heterogeneous sources and sensors.

B. Collaborative processing

The required number of sensors for the desired detection or reproduction at the fusion center depends on the number of sources. Since a cluster or group of sensors is involved in gathering data from the same source, the sensors can cooperate by

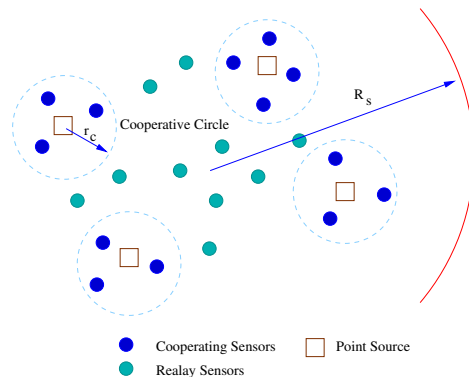


Fig. 2. Data Gathering system multiple point sources and their respective cooperative region.

locally fusing their data, hence limiting the number of transmitting sensors and also exploiting spatial correlation. Here, the problem of required sensor density for given spatial fidelity, number of sources, and desired distortion is considered. Also, the number of sensors needed for local fusion is also evaluated.

1) *Density of Locally Cooperating Sensors:* Consider a circular region of radius R_s containing m randomly located point sources. Also deployed are p uniformly distributed sensors as shown in Figure 2. Assume that the number of sources and sensors along with the spatial fidelity is known. For the purpose of this paper, assume that the number of sensors per unit area, ρ , remains constant. Mathematically, ρ , is defined as:

$$\rho = \frac{p}{\pi R_s^2}. \quad (3)$$

Since each source has a group of sensors observing it rather than one dedicated sensor, the multiple copies of the same information is available. However, based on the distance between source and sensor, the observation quality at each sensor will differ. For conserving resources, it is advisable to have only one sensor per source to transmitting the information. Ideally, the sensor with the best (or desired) signal-to-interference noise ratio (SINR) should be considered as the transmitting sensor. However, if no sensor meets the desired SINR then the nodes can cooperate locally. Local fusion is also encouraged for reliable detection [11].

For the purpose of local fusion, it could be shown that there exists a cooperating circular region of radius r_c around each source. The radius of this region certainly varies from source to source, but for the sake of simplicity assume it to be the same for all sources. Also, assume that the source localization has been already performed and hence the sensors are aware of the relative distance between them and any given source. This assumption is only for analytical tractability. Later, this assumption will be relaxed for calculating total number of sensors required to be deployed. The cooperating regions could either be non-overlapping or overlapping for any particular source. Consider a source X_m being observed by q sensors lying within a cooperative region as shown in Figure 3. The interference at the sensors is assumed only due to sources and not the communication interference from the other sensors. This is a valid assumption as the appropriate choice of medium access control

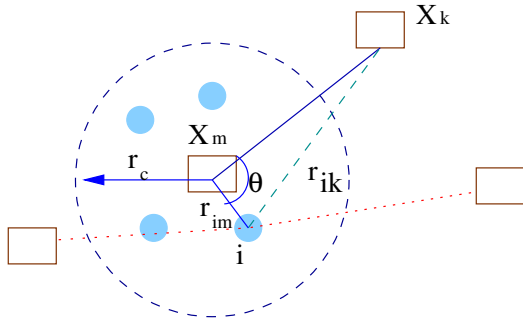


Fig. 3. Source-sensor positions for SINR calculations.

(MAC) layer protocol can always prevent the communication interference. The SINR at sensor i lying in the cooperative region of X_m is then given by:

$$\text{SINR}_i = \frac{P_{X_m} (r_{im}/r_0)^{-\alpha}}{\sigma_{N_i}^2 + \sum_{k=1}^{m-1} P_{X_k} (r_{ik}/r_0)^{-\alpha}} \quad (4)$$

where α is the path-loss coefficient, r_{ij} is the Euclidean distance between the sensor i and source j , r_0 is the constant of proportionality, P_{X_j} is the transmitted power by source X_j and $\sigma_{N_i}^2$ is the variance of zero-mean Gaussian noise, N_i , at sensor i . The denominator of (4) is the interference at sensor i :

$$I_i = \sigma_{N_i}^2 + \sum_{k=1}^{m-1} P_{X_k} (r_{ik}/r_0)^{-\alpha} \quad (5)$$

From geometry (Figure 3), the distance between the sensor i and source k , $k = 1, 2, \dots, m-1$, can be calculated to be:

$$r_{ik} = \sqrt{r_{X_m X_k}^2 + r_{im}^2 - 2r_{X_m X_k} r_{im} \cos \theta} \quad (6)$$

where $r_{X_m X_k}$ is the distance between the source X_m and X_k , and θ is the measure of $\angle i X_m X_k$. If the spatial fidelity constraint, δ , is satisfied by the source X_k with X_m , then $r_{X_m X_k} \geq \delta$. If the constraint is not being satisfied then both the sources X_m and X_k would not be considered for the reproduction at the fusion center. Hence, it is clear that the spatial fidelity will influence SINR $_i$. The larger the source separation, the higher the SINR at the sensor.

Firstly, consider the statistical approach. The probability that the sensor node will be in radius r_c is:

$$P_{r_c} = \frac{\pi r_c^2}{\pi R_s^2} = \frac{r_c^2}{R_s^2} \quad (7)$$

Using (3), the above equation can be rewritten as:

$$P_{r_c} = \frac{\rho \pi r_c^2}{p} \quad (8)$$

Therefore, the probability that the number of sensors, q , in the cooperative region of X_m is k could be calculated as:

$$\Pr(q = k) = \binom{p}{k} P_{r_c}^k (1 - P_{r_c})^{p-k} \quad (9)$$

If $p \rightarrow \infty$, then:

$$\lim_{p \rightarrow \infty} \Pr(q = k) = \frac{\lambda^k}{k!} e^{-\lambda} \quad (10)$$

where $\lambda = \rho \pi r_c^2$. Hence, asymptotically the number of sensors in the cooperative region follows a Poisson distribution with intensity $\rho \pi r_c^2$.

Suppose X_m to be a zero-mean Gaussian with the variance $\sigma_{X_m}^2$. Consider the q sensors observing X_m within the cooperative region. The observation available at each sensor could be modelled as:

$$Y_i = \gamma_i X_m + Z_i \quad (11)$$

where i ranges from $1, \dots, q$ and $\text{SINR}_i = \frac{\gamma_i^2 \sigma_{X_m}^2}{\sigma_{Z_i}^2}$. Here, Z_i also considers the interference from other sources. The interference, I_i , in (5) is certainly not Gaussian for a finite number of sources. But, if $m \rightarrow \infty$, then from the central limit theorem $\sum_{k=1}^{m-1} P_{X_k} (r_{ik}/r_0)^{-\alpha}$ tends to Gaussian. γ_i 's are the inhomogeneous coefficients, modelling the practical scenario of not being identical. Based on this, the mean squared estimate of X_m is given by [12]:

$$\hat{X}_m = \left[(\Gamma \sigma_{X_m}^2 \Gamma' + R_Z)^{-1} \Gamma \sigma_{X_m}^2 \right]' \mathbf{Y} \quad (12)$$

where \mathbf{Y} denotes column vector $\{Y_i\}_{i=1}^q$, Γ is a column vector $\{\gamma_i\}_{i=1}^q$ and R_Z denotes the covariance matrix of noise $\{Z_i\}_{i=1}^q$. Since Z_i 's are independent of each other R_Z is a diagonal matrix. If Z_i 's are also identical with variance σ_Z^2 , then the distortion in reproduction of source X_m is given by:

$$D = \frac{\sigma_{X_m}^2 \sigma_Z^2}{(\Gamma' \Gamma \sigma_{X_m}^2 + \sigma_Z^2)} \quad (13)$$

The distortion, D , clearly depends on SINR which is a function of δ . This implies that $D = f(\delta)$. Using this and (13), the number of sensors needed for cooperation can be calculated in terms of spatial fidelity. For instance, assume Γ to be a vector of identical elements γ . In this case, $\Gamma' \Gamma = q \gamma^2$. Hence from (13),

$$q = \frac{\sigma_Z^2}{\gamma^2 \sigma_{X_m}^2} \left[\frac{\sigma_{X_m}^2}{D} - 1 \right]^+ \quad (14)$$

where $x^+ = \max(0, x)$.

The above considered the case of one source having q sensors locally cooperating. If m sources are present satisfying the fidelity constraint, then the sensor network will have a total of $m q$ sensors locally cooperating for those m sources within their assigned cooperative region.

2) *Calculating Sensor Density:* The analysis for calculating the sensor density involving m point sources is highly complicated and to some extent non-tractable. For instance, assuming the localization of sources as in the previous discussion results in a non-practical scenario and even that does not help in simplifying things. This is because calculating the cooperating sensors could be done but evaluating the total number of relays is highly traffic dependent. Even the statistical analysis is equally difficult. For example, consider the computation of $\Pr(\text{distance between any two sources} \geq \delta)$. Although the sensor nodes are

i.i.d. in location, the Euclidean distance between them is dependent. Hence, calculating this probability for m sources itself is complicated. Hence, simulation is performed to evaluate the sensor density for a given spatial fidelity, number of sources, and desired mean squared distortion.

The simulation setup assumes a unit area circular region with m Gaussian point sources randomly deployed. For the purpose of the simulations, the area of the circular region is kept constant.

Firstly, the simulation is carried out to calculate the probability of the sources that do not satisfy spatial fidelity criteria. That is, the number of sources are calculated that are spaced at distance less than δ . For this purpose, m is varied from 10 to 200. The simulation for each value of m is executed for 1000 iterations and the value of probability is calculated averaging over those values. The probability plot for different values of δ is shown in Figure 4.

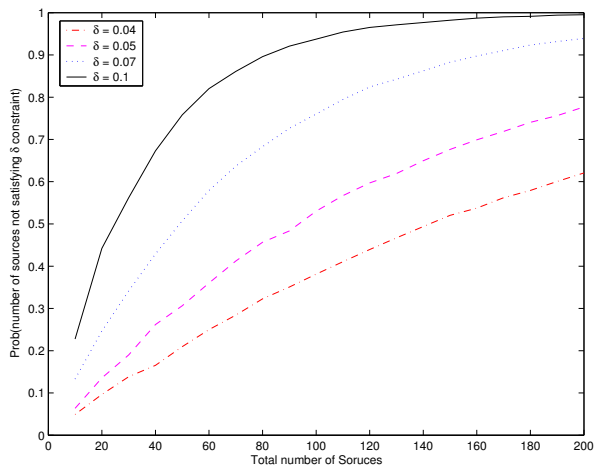


Fig. 4. Probability plot for the number of sources not satisfying spatial fidelity constraints.

With the increase in number of sources, the spatial separation between them will certainly reduce as the area of the region is constant. Hence at higher node density, the resolution should be finer for the desired reproduction. As seen in Figure 4, at higher source density, the probability of sources disobeying spatial fidelity constraint, δ , increases.

Next, simulation for the sensor density is considered. For this purpose the number of sources, m , is fixed. The goal in this simulation is to know the mean squared distortion in the estimates. This will give the relation between the required number of sensors and distortion. The transmission power is assumed to decay in second order with the distance. The sensors observing the source are locally fused (assuming coherent sources) until the desired signal strength is obtained. The simulation plot is given in Figure 5. Similar to the probability simulation, each point in the plot is the ensemble average over 1000 iterations. For any particular δ , with the increase in number of sensors, the distortion should decrease. This is obvious as the dense deployment of sensors provides better signal strength at the sensor along with high spatial correlation and sensor diversity. Now, as the δ increases, then the number of sources that satisfy the spatial fidelity constraint decreases. Hence, the total number

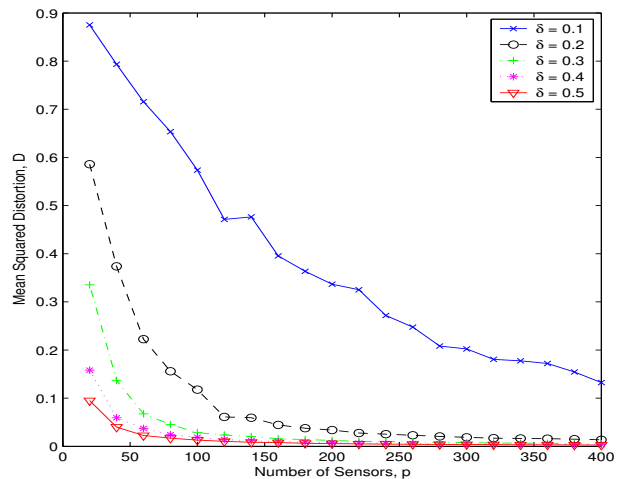


Fig. 5. Simulation Plot depicting the relation between the required number of sensors and mean squared distortion for different values of δ . The path-loss coefficient is assumed to be 2.

of sources to consider for reproduction decreases. This leads to the deployment of fewer sensors for higher δ as seen from Figure 5.

III. SPATIAL FIDELITY FOR DISTRIBUTED PHENOMENON

This section considers the sensing of distributed phenomena such as field. The sampling of these fields is one of the most challenging areas of research. In an ideal case, sampling above the Nyquist rate avoids aliasing effect. The assumption of super Nyquist sampling also holds for this paper. However, determining whether the field is under sampled, over sampled, or critically sampled still poses an interesting problem. These sampling issues have been considered in [6, 13]. Here, sampling of the phenomenon is considered in a different context. Note that while collectively sensors are observing a distributed source, each of them is collecting data from a point source. All together, these highly correlated point sources form the distributed phenomena.

Section II discussed the notion of spatial fidelity for point phenomena. We now extend that definition for distributed phenomena. The idea behind the spatial fidelity, however, remains the same. That is, it represents resolution of the field.

Definition III.1—Spatial Fidelity for Distributed Phenomenon: Spatial fidelity, δ , for the distributed phenomenon represents the cut-off or sampling rate. It is governed by the gradient or the change in the amplitude of the field with respect to time. Consider the distributed field as in Figure 6. Suppose the field is sampled at every δ_0 units. The spatial sampling frequency of the field is then $\frac{1}{\delta_0}$. If change in the amplitude is represented by ΔX , then the slope of a curve within a sample is given by (Figure 6):

$$\zeta = \lim_{\Delta l \rightarrow 0} \frac{\Delta X}{\Delta l} = \frac{\partial X}{\partial l} = \frac{\partial X}{\delta}$$

However, this frequency may not result into true reproduction of the field. Consider the curve EF . It abruptly changes from E to F and hence, it is desirable to have more samples (or higher

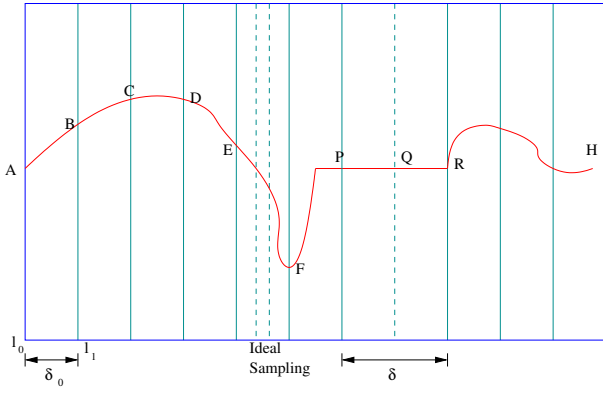


Fig. 6. Distributed field with sampling interval, δ_0 (sampling frequency = $1/\delta_0$). The dashed lines between points E and F represent an ideal situation.

sampling rate) for the better reproduction of the field. This situation is an example of under sampling. In contrast, the region PR is smooth and flat. For this region, the sampling rate could be low, that is the sample at point Q is not required. With the readings at P and R , the region could be accurately reproduced. This situation corresponds to over sampling. The critically sampled region is AB in Figure 6. All these contribute to energy cost in sensor networks. Hence, the sampling should be adaptive. This adaptive nature of sampling is represented by spatial fidelity. To summarize, let δ_0 be the user specified spatial fidelity criterion. If i and j represent two points on the field, then the following three conditions for spatial fidelity, δ , could be possible:

Under Sampling

$$\left| \frac{\Delta X}{\delta} \right| > \epsilon \quad \text{and} \quad \delta < \delta_0 \quad (15)$$

Critical Sampling

$$\left| \frac{\Delta X}{\delta} \right| = \epsilon \quad \text{and} \quad \delta = \delta_0 \quad (16)$$

Over Sampling

$$\left| \frac{\Delta X}{\delta} \right| < \epsilon \quad \text{and} \quad \delta > \delta_0 \quad (17)$$

where, ϵ is any positive quantity. ϵ also accounts for the allowable tolerance in the gradient. The above three conditions define spatial fidelity for distributed sources. Note, that the above sampling criteria is proposed as heuristics.

In most cases, the user provides the lower bound on the spatial fidelity. If the lower bound is δ_0 , then $\delta \geq \delta_0$. With this constraint then we can reproduce the field with the desired distortion. The error in reconstruction may occur from the situations such as the region between C and D in Figure 6. The readings at both these points are the same but there is change in the readings in between which cannot be captured. However, this error does not propagate and is limited to local region.

Since the distributed source could be observed as the collection of correlated point sources, the source-sensor relations derived for point sources could also be extended for distributed phenomena.

IV. CONCLUSIONS

For sensor networks, we believe that the main objective is information extraction to some level of fidelity. The notion of spatial fidelity is considered to characterize the sensor network. In a broad sense, spatial fidelity is the desired separation between the point sources. For a distributed phenomenon, it translates into desired and/or adaptive sampling frequency. This is analogous to the resolution of pixels in image processing. For reliability, the data is considered to be fused locally. Based on the spatial fidelity and SINR available at the sensors, required number of sensors for local cooperation is calculated. Further, the relation between number of sensors and desired distortion is evaluated, assuming the knowledge of sources and spatial fidelity. Spatial fidelity also lead to the heuristics for sampling criterion in case of distributed phenomenon.

REFERENCES

- [1] G Pottie and W Kaiser. Wireless integrated network sensors. *Communications of the ACM*, 43(5):51–58, 2000.
- [2] D Estrin, L Girod, G Pottie, and M Srivastava. Instrumenting the world with wireless sensor networks. In *Proceedings of International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Salt Lake City, UT, June 2001.
- [3] T Berger, Z Zhang, and H Vishwanathan. The CEO problem. *IEEE Transactions on Information Theory*, 42(3):887–902, May 1996.
- [4] H Viswanathan and T Berger. The quadratic gaussian ceo problem. *IEEE Transactions on Information Theory*, 43(5), September 1997.
- [5] Y Oohama. The rate-distortion function for the quadratic gaussian CEO problem. *IEEE Transactions on Information Theory*, 44(3):1057–1070, May 1998.
- [6] P Ishwar, A Kumar, and K Ramchandran. Distributed sampling for dense sensor networks: A bit-conservation principle. In *Proceedings of the International Workshop on Information Processing in Sensor Networks*, April 2003.
- [7] A Scaglione and S Servetto. On the interdependence of routing and data compression in multi-hop sensor networks. In *Proceedings of the 8th annual international conference on Mobile computing and networking*, pages 140–147. ACM Press, 2002.
- [8] D Marco, E Duarte-Melo, M Liu, and D Neuhoff. On the many-to-one transport capacity of a dense wireless sensor network and the compressibility of its data. In *Proceedings of the International Workshop on Information Processing in Sensor Networks (IPSN)*, April 2003.
- [9] R Nowak and U Mitra. Boundary estimation in sensor networks: Theory and methods. In *2nd International Workshop on Information Processing in Sensor Networks*, volume 20, Palo Alto, CA, 2003.
- [10] R Willett, A Martin, and R Nowak. Backcasting: adaptive sampling for sensor networks. In *Proceedings of the third international symposium on Information processing in sensor networks*, pages 124–133, Berkeley, CA, 2004. ACM Press.
- [11] P Varshney. *Distributed Detection and Data Fusion*. New York: Springer-Verlag, 1997.
- [12] A Sayed. *Fundamentals of Adaptive Filtering*. Wiley-IEEE Computer Society Press, 2003.
- [13] A Kumar, P Ishwar, and K Ramchandran. On distributed sampling of smooth non-bandlimited fields. In *Proceedings of the third international symposium on Information processing in sensor networks*, pages 89–98, 2004.