

Iterative Decoder Design for Joint Source-Channel LDPC Coding

Ksenija Lakovic, *Member, IEEE*, Tao Tian, and John Villasenor

Abstract — We consider joint source-channel coding with variable length codes (VLCs) and LDPC codes. We propose an iterative source-channel decoding system, which involves message passing between a soft-input-soft-output VLC decoder and an LDPC decoder. The proposed system utilizes the residual redundancy of the variable length code during the iterative decoding process, and is expected to exhibit decoding convergence at a lower SNR relative to the system in which source and channel decoding are performed separately. We present computer simulation results that confirm the expected gain of the proposed system.

Keywords — joint source-channel coding, LDPC codes, variable length codes

I. INTRODUCTION

STANDARD communication systems typically involve separate source and channel encoding and decoding. This is justified by the Shannon's separation theorem, which states that the two-stage system can approach optimum performance bounds as closely as desired, provided that both codes are allowed sufficiently large lengths. In practical systems, however, the code design is constrained in terms of complexity or delay. Practical source and channel codes are consequently suboptimal, and their performance can be improved by utilizing joint source-channel coding.

Since variable length source codes are applied in many data compression standards, extensive work has been done in the previous years to construct joint source-channel decoders that exploit the suboptimality of these codes. Early research in this area considered non-iterative decoding with Huffman codes and convolutional codes (CCs) [1]-[3]. More recently, Bauer and Hagenauer [4] introduced an iterative source-channel decoding system, which involves reversible variable length codes (RVLCs) [5] in combination with convolutional codes. As shown in [4], iterative decoding with soft-input-soft-output RVLC and CC decoders yields superior results at a low SNR, compared to non-iterative source-channel decoding.

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Additional improvements can be obtained by optimized VLC design, as presented in [6].

Iterative decoding systems for Huffman coded Markov sources in combination with convolutional codes were proposed in [7-8]. While Kliewer and Thobaben [8] proposed a system that utilizes soft-input-soft-output (SISO) constituent decoders similar to the decoders in [4], Guyader *et al.* [7] introduced a Bayesian-network based decoding system. Both [7] and [8] reported substantial gains associated with iterative decoding, while further improvements were presented in [9-10]. The application of iterative source-channel decoding in image communication systems was considered in [11-13].

In [14-15], we considered joint-source channel coding with variable length codes (VLCs) and turbo codes, and proposed iterative VLC-turbo decoders that simultaneously exploit the residual redundancy of VLCs and the good convergence properties of turbo codes. We showed that VLC-turbo decoders significantly outperform VLC-CC decoders, and that they perform very close to the theoretical limit. Further analysis on VLC-turbo systems can be found in [16-17].

Low-density parity-check (LDPC) codes, introduced by Gallager in 1960s, have recently regained attention [18-19] as the capacity-approaching channel coding alternative to turbo codes. Consequently, joint source-channel coding with LDPC codes represents an interesting alternative to joint source-channel turbo coding. In this paper we consider variable length codes in combination with LDPC codes. We propose an iterative joint source-channel decoding system, which involves a soft-input-soft-output VLC decoder in combination with an LDPC decoder. We compare the performance of this system to the system that involves separate VLC-LDPC decoding. Additionally, we discuss VLC-LDPC coding relative to VLC-turbo coding.

II. ITERATIVE SOURCE-CHANNEL DECODING WITH VLCs AND LDPC CODES

The considered system is shown in Fig. 1. Let $\underline{u}=(u_1, \dots, u_J)$ denote a sequence of J symbols at the output of an M -ary i.i.d. information source, with probability mass function $p_U=\{p^1, \dots, p^M\}$. This sequence is first encoded by a variable length code into a binary sequence $\underline{w}=(w_1, \dots, w_K)$, which is further encoded for error-protection into $\underline{x}=(x_1, \dots, x_N)$, and transmitted through a communication channel.

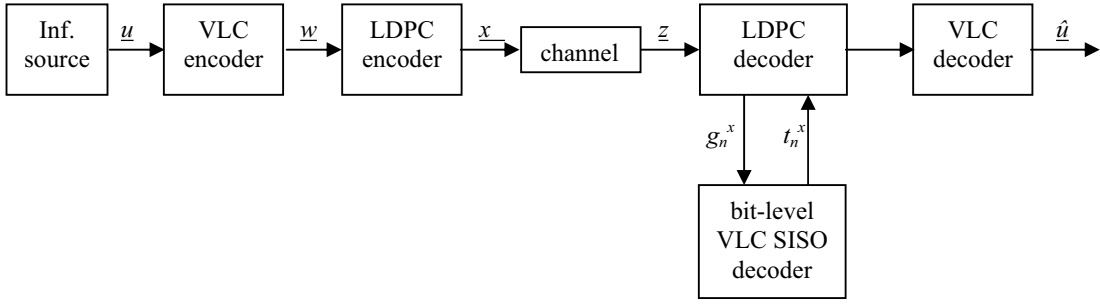


FIGURE 1: SYSTEM OVERVIEW

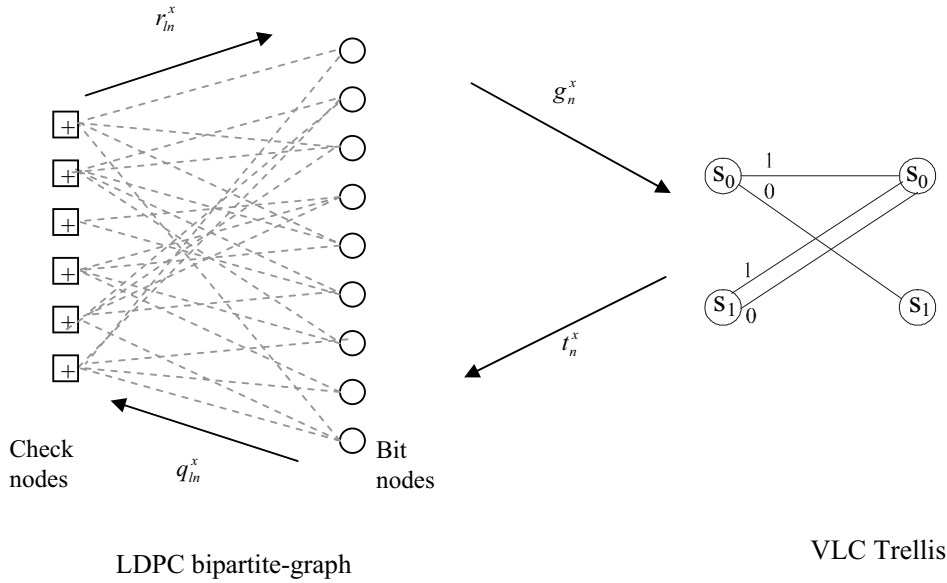


FIGURE 2: MESSAGE PASSING BETWEEN THE LDPC DECODER AND THE VLC DECODER

A standard approach to source-channel decoding involves LDPC decoding using the received bits $z=(z_1, \dots, z_N)$, followed by the VLC decoding. We propose an algorithm that involves iterative processing between the LDPC decoder and the bit-based soft-input soft-output VLC decoder [4], as detailed below.

Following the approach in [18], we assume that the LDPC decoder performs a message-passing algorithm over the graph associated with the code, which is shown in Fig. 2. Assuming that the parity check matrix of the LDPC code is denoted with $H=[h_{ln}]$, and that it consists of L columns and N rows, the associated graph consists of N bit nodes $b_n, n=1 \dots N$, and L check nodes $c_l, l=1 \dots L$. The check and the bit nodes are connected as defined by the 1's in the parity check matrix. Similarly to [18], $N(l)=\{n:h_{ln}=1\}$ denotes the set of indices of the bit nodes that participate in check node c_l , while $L(n)=\{l:h_{ln}=1\}$ denotes the set of indices of the check nodes that

participate in bit node b_n . Set $N(l)$ with bit n excluded is denoted with $N(l)\setminus n$, while $L(n)\setminus l$ denotes set $L(n)$ with bit l excluded.

Iterative source-channel decoding between the LDPC decoder and the VLC decoder involves the processing of four different types of messages:

- 1) messages r_m^x , which are passed from the LDPC check nodes to the LDPC bit nodes;
- 2) messages g_n^x , which are passed from the LDPC bit nodes to the VLC decoder;
- 3) messages t_n^x , which are passed from the VLC decoder to the LDPC bit nodes;
- 4) messages q_m^x , which are passed from the LDPC bit nodes to the LDPC check nodes.

Iterative processing starts with the initialization of the *a priori* probabilities $p_n^0=P(x_n=0)$ and $p_n^1=P(x_n=1)$. For

every (l, n) such that $h_{ln} = 1$, the messages q_{ln}^x are initialized as $q_{ln}^x = p_n^x$, $x \in \{0, 1\}$. After initialization, iterative source-channel decoding proceeds as follows:

1) The computation of messages r_{ln}^x is performed as in [9] (the so-called ‘horizontal step’ of LDPC decoding):

$$r_{ln}^x = \sum_{\{x_{n'}, n' \in N(l) \setminus n\}} P(z_l | x_n = x, \{x_{n'} : n' \in N(l) \setminus n\}) \times \prod_{n' \in N(l) \setminus n} q_{ln'}^{x_{n'}}$$

The quantity q_{ln}^x denotes the probability that bit n of \underline{x} has the value x , $x \in \{0, 1\}$, given the information obtained via the checks other than check l . The quantity r_{ln}^x denotes the probability of check l being satisfied if bit n of \underline{x} is considered fixed at x and the other bits have a separable distribution given by the probabilities $\{q_{ln'}^x : n' \in N(l) \setminus n\}$.

2) Instead of proceeding directly to the ‘vertical step’ of LDPC decoding, the proposed iterative source-channel decoding algorithm performs the computation of messages g_n^x . The message g_n^x represent the probability that bit n has the value x , $x \in \{0, 1\}$, obtained at a given iteration from all the bits different from n (the so-called ‘extrinsic information’). It is computed as:

$$g_n^x = \lambda_n \prod_{l \in L(n)} r_{ln}^{x_{n'}}$$

where λ_n is a normalization factor such that $g_n^0 + g_n^1 = 1$.

3) The extrinsic messages t_n^x are computed using the bit-based VLC trellis shown in Fig. 2. This trellis was first introduced in [21], and first used in [4] to perform iterative source-channel decoding with RVLCs and convolutional codes.

Each branch of this trellis, denoted as e , is associated with a bit value $w(e)$, and the starting and ending states denoted as $s^S(e)$ and $s^E(e)$, respectively. The *a priori* probability of branch e , denoted as $a(e)$, is computed using the source symbol probabilities, as $a(e) = P(e | s^S(e))$. The extrinsic messages t_n^x are calculated using the BCJR algorithm, as described by the following equations:

$$\begin{aligned} \alpha_n(s) &= \sum_{e: s^E(e)=s} \alpha_{n-1}(s^S(e)) \cdot a(e) g_n^{w(e)} \\ \beta_{n-1}(s) &= \sum_{e: s^S(e)=s} \beta_n[s^E(e)] \cdot a(e) g_n^{w(e)} \\ t_n^x &= \mu_n \sum_{e: w(e)=x} \alpha_{n-1}[s^S(e)] \cdot a(e) \beta_n[s^E(e)] \end{aligned}$$

where $n=1 \dots K$, $x \in \{0, 1\}$, and μ_n is a normalization factor such that $t_n^0 + t_n^1 = 1$. The initial values are $\alpha_0(0) = \beta_K(0) = 1$, while $\alpha_0(s) = \beta_K(s) = 0$, $s \neq 0$.

4) In the ‘vertical step’ of LDPC decoding, the extrinsic messages t_n^x received from the VLC decoder are used as the updated *a priori* probabilities of the systematic bits, i.e. the *a priori* probabilities are modified as:

$$p_n^x = t_n^x, \quad n=1 \dots K, \quad x \in \{0, 1\}.$$

Using these updated probabilities p_n^x , the ‘vertical step’ of LDPC decoding is performed as in [18], by computing the messages q_{ln}^x as:

$$q_{ln}^x = \nu_{ln}^x p_n^x \prod_{l' \in L(n) \setminus l} r_{ln'}^{x_{l'}}$$

where the normalization factors ν_{ln} are chosen so that $q_{ln}^0 + q_{ln}^1 = 1$.

At this point, the algorithm repeats from step 1.

III. SIMULATION RESULTS

Simulations were performed for the source $U = \{A, B, C\}$, with $p_U = \{0.7, 0.21, 0.09\}$, and the Huffman code $\{1, 00, 01\}$, as in [14]. The (10000, 5000) LDPC code was constructed using the approach discussed in [20]. The encoded bits were sent through a binary input AWGN channel. We considered two decoding scenarios. The first scenario involves the proposed iterative LDPC-VLC decoder, as shown in Fig. 1. The second scenario involves standard separate source-channel decoding (there is no message passing between the LDPC decoder and the bit-level SISO VLC decoder). Simulation results are presented in Fig. 3. The symbol error rate is evaluated in terms of the Levenshtein distance, as in [4], [14]. Results demonstrate that the proposed iterative VLC-LDPC decoder outperforms the separate VLC-LDPC decoder by about 0.3dB.

Additionally, the performance is compared against the VLC-turbo decoding systems discussed in [14]. For the given block length of 10000 bits, VLC-LDPC and VLC-turbo coding yield similar results. This is expected considering the discussion in [19]. LDPC codes are expected to be more advantageous for longer block lengths.

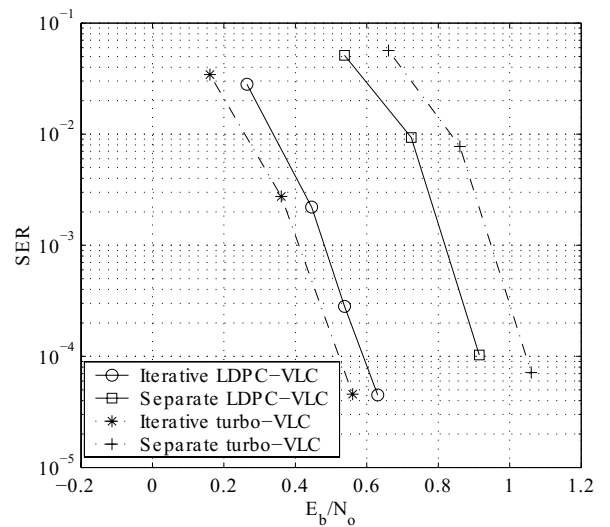


FIGURE 3. SIMULATION RESULTS

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