Markov Structures in Turbo Decoding

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Abstract — **We describe techniques for modifying a decoder for turbo codes to incorporate a hidden Markov source model or a hidden Markov channel model. This allows the receiver to utilize (and under some conditions, estimate) the statistical characteristics of the source or channel during the decoding process, and leads to significantly improved performance relative to systems in which the Markov properties are not exploited. When applied for Markov channels, the methods presented here can allow decoding at levels which are beyond the capacity of a corresponding interleaved and assumed memoryless channel in which the Markov structure is not utilized.**

I. Introduction

Parallel concatenated codes, or "turbo" codes [1], represent one of the most significant advances in channel coding in recent years. Most of the work on turbo codes has focused on i.i.d. sources in combination with AWGN channels, and more recently, fading channels. Sources and channels with memory are quite common in practice, thus motivating the research on exploiting this structure in the context of turbo codes. In a turbo encoder, two or more constituent convolutional coders are used, with interleavers used to randomize the relative order of source bits prior to the input the convolutional encoders. The decoder for turbo codes consists of one constituent decoder corresponding to each encoder. The constituent decoders perform iterative processing and exchange of information (called "extrinsic" information in the turbo code literature) about the reliability of the input bits as the decoding progresses.

The usual interpretation of iterative decoding in turbo codes is to consider the extrinsic information at the output of a given constituent decoder as *a priori* information at the input to the next constituent decoder. Memory in the source or channel, however, constitutes an additional source of *a priori* information that can also be utilized during decoding. We show how to modify the decoding process to take advantage of this *a priori* information for the two cases: 1) the source follows a hidden Markov model and the channel is AWGN, and 2) the pattern of errors introduced by the channel is generated by a binary-input, binary-output finite state Markov model but the source is i.i.d. Due to space limitations, the concepts underlying the decoding modifications to handle Markov structures are described qualitatively below. A more detailed explanation of some of the specific modifications can be found in [2] and [3].

The concept of using *a priori* information from the source to improve the channel decoding performance has been used in [4, 5] and elsewhere. Our work contrasts with this work in that it treats hidden Markov source models (as opposed to non-hidden Markov sources), and in particular, that it addresses turbo codes. Turbo decoding for i.i.d. sources generating unequally likely bits is mentioned in [6].

Issues of capacity and coding for binary-input, binary-output finite state Markov channels are addressed in [7, 8] and references therein. Exploiting the increased capacity such channels offer (relative to the capacity of the corresponding interleaved channel) has proven to be challenging in practice. We show that when turbo codes are used, this higher capacity can in fact be approached, without the need to perform state estimation that can be vulnerable to errors.

II. Decoder modifications

To exploit the properties of the hidden Markov model representing the source or the channel, the trellises used in decoding can be replaced with expanded "super-trellises" that jointly describe both the Markov model as well as the convolutional coder. As shown in Figure 1, the number of states in the joint super-trellis is a function of the number of states in the hidden Markov model and the number of states in the convolutional coder. Each one of the branches in the joint super-trellis will have an associated *a priori* transition probability that is a function of the parameters of the hidden Markov model.

Expanded trellises have been used previously for combined demodulation-convolutional decoding for ISI channels [9]. However, in contrast with ISI channels in which the path through the super-trellis is a deterministic function of the input sequence, in turbo decoders modified for hidden Markov sources or channels the path is probabilistic. When the source follows a hidden Markov model and the channel is AWGN, the state of the source and the state of convolutional code (which define the state in the supertrellis) are related through the input bit generated by the source. However, when the source is i.i.d. and the channel has Markov structure, the state of the Markov channel and the state of the convolutional code are independent. Although both source and channel Markov models can be handled by expansion of the decoding trellises and the form of the resulting equations are the same, there are important differences in the details of some of the terms.

Once the expanded trellises have been constructed, it is necessary to use the *a priori* probability that the Markov model imposes on each branch in the decoding process. Each constituent decoder must use the extrinsic information available from the other decoder(s) as well as the *a priori* information (from the channel or source) to obtain a new estimation for the transition probability in each one of its trellis branches. Assuming that the constituent interleavers fully randomize the input bits, we show how the *a priori* information from the channel or source should be used to calculate the extrinsic information in each constituent decoder. We show that the transition probability that each constituent decoder should use in a given branch is the product of the extrinsic probabilities from all the other decoders and the *a priori* probability associated with that branch.

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III. Results

Figure 2 shows the decoded bit error rate as a function of iteration number for several different values of E_b/N_0 (where the noise is Gaussian) for an input sequence generated using a hidden Markov source with 2 states S_j, $0 \leq j \leq 1$. The parameters of the model are $P_t(S_0|S_0) = .1$ and $P_t(S_1|S_1) = .15$, where $P_t(S_k|S_l)$ is the probability of transition from state S_l to S_k and $P_o(0|S_0) = .95$, $P_o(1|S_1) = .925$, with $P_o(i|S_j)$ representing the probability of producing output bit i, $i \in \{0,1\}$, when the source is in state S_j. This hidden Markov source was input to a rate 1/3 encoder that included a systematic bit and two identical recursive 8-state convolutional encoders with generator matrix $G(D) = \frac{1+D+D^2+D^3}{1+D^2+D^3}$ and an interleaver with length 16384. Each simulation consisted of at least 10 million bits. Using this source, the theoretical (unconstrained) limit for E_b/N_0 using rate 1/3 codes is approximately -2.2 dB. The modified decoder performs within about 1.2 dB of this limit, showing convergence at $E_b/N_0 = -1.0$ dB. As shown in the figure, a decoder without the modifications described here fails to converge even at an E_b/N_0 of .1 dB.

While the decoding iterations will converge most quickly if the receiver has perfect knowledge of the hidden Markov source model, at the cost of increasing the number of iterations it is also possible to perform estimation of the model parameters during the decoding of each block. This leads to good performance even when the model parameters used to initialize decoding are quite different from the true ones.

For the case of finite state Markov channels it is necessary not only to modify the decoding equations, but also to consider the channel properties in choosing the encoder structure. A general encoder for rate 1/2 codes is shown in Figure 3. We have obtained empirical encoder design rules as a function of the rate of the code and the channel statistics. As might be expected, we have found that for a given rate constraint the number of (punctured) constituent encoders should increase when the channel becomes more bursty. For a large set of channels and rates we have used this framework to demonstrate reliable communication at noise levels that are well beyond the capacity of a corresponding interleaved and assumed memoryless channel in which the Markov structure is not utilized. As far as we are aware, this represents the first demonstration of communication over a binary Markov channel above the capacity of the corresponding interleaved channel.

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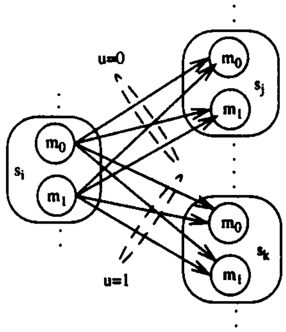

Fig. 1: Joint decoder trellis describing the encoder (states s) and the hidden Markov source (states m). u is the information bit.

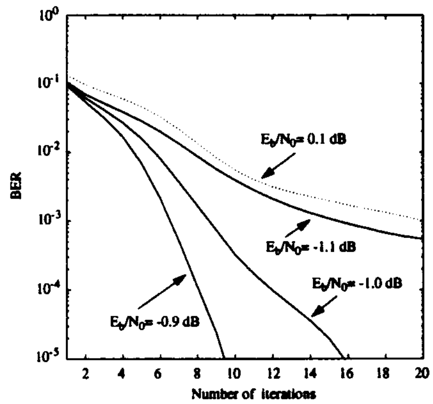

Fig. 2: BER as a function of the iteration number for the hidden Markov source and channel decoder (solid lines) described in the text. The dotted line illustrated the performance when an unmodified decoder (that does not exploit source Markov properties) is used.

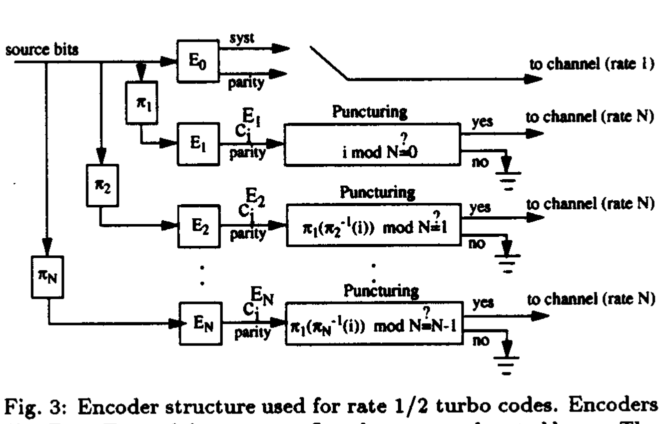

Fig. 3: Encoder structure used for rate 1/2 turbo codes. Encoders E_0, E_1 .. E_N each have rate 1. Interleavers are denoted by π_l. The puncturing functions are built as indicated in the figure.