

Rate-Compatible Low-Density Parity-Check Codes

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Abstract — Rate-compatible coding is appropriate for communication systems that experience a range of operating SNRs but seek to adhere to a single underlying codec structure. This paper constructs rate-compatible low-density parity-check (LDPC) codes by carefully selecting degree distributions, followed by a combination of information nulling and parity puncturing. Techniques that suppress error floors are included as part of the construction methodology.

Compared to punctured convolutional codes and turbo codes, LDPC codes enjoy more freedom in puncturing pattern selection, thus allowing for an almost continuous spectrum of code rates and more robustness to catastrophic events.

A density evolution algorithm was developed in [1] to find asymptotically good puncturing profiles for LDPC codes with rate $0.5 \leq R \leq 0.9$. We combine this technique and information nulling to achieve close-to-capacity performance with low error floors across $0.1 \leq R \leq 0.9$.

Fig. 1 shows how the proposed method achieves low rate 0.2 and high rate 0.8 from a length- 10^4 mother code that has rate $R_0 = 0.5$. Information bits are on the left side (white area) and parity bits on the right side (shaded area).

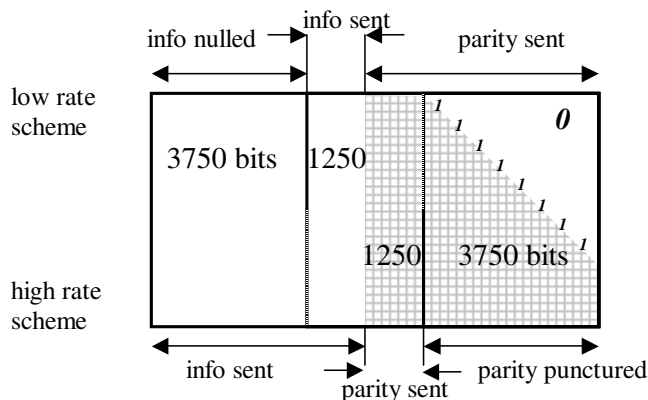


Fig. 1: Information nulling and parity puncturing.

We used Gaussian-approximated density evolution to find the node-wise degree distribution $\tilde{\lambda}_i^{(R)}$ for $0.1 \leq R < 0.5$. We borrowed the node-wise degree distribution for $0.5 \leq R \leq 0.9$ from [1]. $\tilde{\lambda}_i^{(R)}$ is further normalized to

$$\tilde{L}_i^{(R)} = \begin{cases} \frac{1-R_0}{1-R} \tilde{\lambda}_i^{(R)}, & \text{if } 0 \leq R \leq R_0, \\ \tilde{\lambda}_i^{(R_0)} (1 - \pi_i^{(R)}), & \text{if } R_0 < R \leq 1, \end{cases} \quad (1)$$

where $\pi_i^{(R)}$ represents the fraction of punctured degree- i variables (see [1]). \tilde{L}_i is plotted in Fig. 2.

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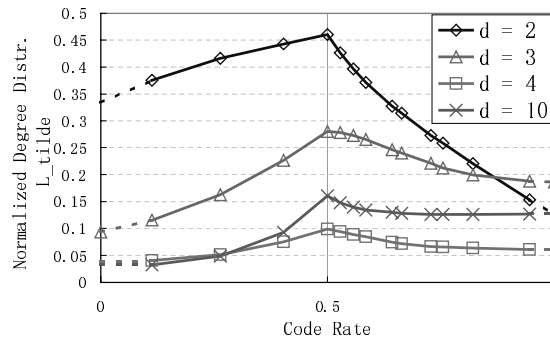


Fig. 2: Normalized node-wise degree distribution \tilde{L}_i .

The curves in Fig. 2 are extrapolated to code rate 0 and 1 according to $\tilde{L}_i^{(0)} + \tilde{L}_i^{(1)} = \tilde{L}_i^{(R_0)}$. A best-effort algorithm assigns column degrees progressively with minimum discrepancy from the distribution in Fig. 2.

The lower triangular structure in Fig. 1 can suppress error floors. Any column (variable) subset of the triangular portion is not a stopping set, because the leftmost column of this subset has at least a neighbor (the upmost “1” of this column) that is singly connected to this set.

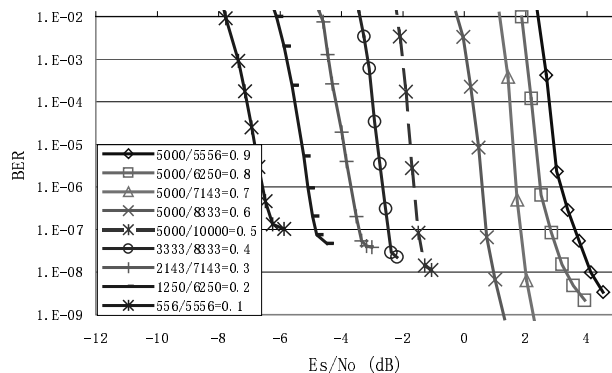


Fig. 3: E_s/N_0 simulation results for AWGN channels.

The ACE algorithm ([2]) was applied during the construction of the underlying mother parity matrix in order to further suppress error floors. This is particular useful in the lower rate regime. Throughout the construction $d_{ACE} = 9$, while a progressive set of η_{ACE} values of 6,5,4 and 3 was used for rates 0.1, 0.2, 0.3, and 0.4 respectively.

REFERENCES

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- [2] T. Tian, C. Jones, J.D. Villasenor, and R.D. Wesel, “Construction of irregular LDPC codes with low error floors,” *Proc. Int. Conf. Commun.*, vol. 5, pp. 3125–3129, May, 2003.