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Feedback Stabilization of One-Link Flexible Robot Arms: An Infinite-Dimensional System Approach

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Abstract: This research concerns the design of a controller for a flexible robot arm, which is modelled as a flexible beam clamped to a motor at the one end and free at the other end. A mass is also attached to the free end of the beam. To reduce the vibration of the tip mass, we apply a feedback through the angular acceleration of the motor. The proposed control law is a linear combination of the tip deflection and a linear functional of the beam deflection. We then prove that the closed-loop system is asymptotically stable.

Keywords: flexible beam, infinite-dimensional, semigroup

1. Introduction

In this paper, we consider a flexible robot arm, which is modelled as a flexible beam clamped to a motor at the one end and free at the other end. A mass is also attached to the free end of the beam. The behavior of the system can be described by an Euler-Bernoulli partial differential equation, together with appropriate initial and boundary conditions. Thus, the system is infinite-dimensional.

In general, we can formulate a linear infinite-dimensional control system into an abstract Cauchy problem on a Banach space (or Hilbert space) Z

$$\dot{z}(t) = Az(t) + Bu(t), \quad t \geq 0, \quad z(0) = z_0 \in D(A) \qquad (1)$$

where A is a closed operator with $D(A)$ dense in Z. The solution of this problem is

$$z(t) = T(t)z_0 + \int_0^t T(t-s)u(s)ds \qquad (2)$$

where $T(t)$ is a C_0-*semigroup* of bounded operator on Z.

One way to design a controller for an infinite-dimensional control system is that we find a finite-dimensional model, and then design the controller for this approximated model. However, neglecting the high frequency dynamics by using an approximated model may lead to a "spillover" effect, which can destroy the stability of the original system. One of the papers describing about this effect is Bontsema and Curtain [1]. They show that spillover can only occur if the approximation error exceeds the robustness margin of the controller. Therefore, the controller design for the original system using the infinite-dimensional system approach is an alternative way that will be considered here.

In previous works about flexible robot arms using infinite-dimensional models, there are many ways to prove the (asymptotic or exponential) stability of the system. For example, Guo [2] used the spectral determined growth condition. Chen et. al. [3] and Morgül [4, 5] used the energy multiplier method, while Luo et. al. [6] employed the frequency domain approach. However, in the above-mentioned works, the effects of the tip mass or the motion of the motor were not included in the mathematical model. Therefore, in this work, we will consider them simultaneously and propose a control law to stabilize the system. Here we apply a feedback through the angular acceleration of the motor to reduce the vibration of the tip mass. The proposed control law is a linear combination of the tip deflection and a linear functional of the beam deflection. We then prove that the closed-loop system is asymptotically stable. The remainder of this paper is organized as follows.

In section 2, we consider a flexible beam system. The equations of motion can be represented by partial differential equations with boundary conditions, which are examined in [6, 7]. We then proposed the control law and formulate the closed-loop system equation into standard form. In section 3, we first investigate the properties of the infinitesimal generator. We prove that it generates a contraction semigroup and its spectrum consists of only isolated eigenvalues. Then the asymptotic stability proof is obtained from the spectrum analysis by showing that the real parts of the eigenvalues are less than zero. Finally, the conclusion is given in section 4.

2. System Equations

Consider a flexible beam in Fig 1, where $w(t)$ is the deflection of the beam and $\theta(t)$ is the motor angle. The

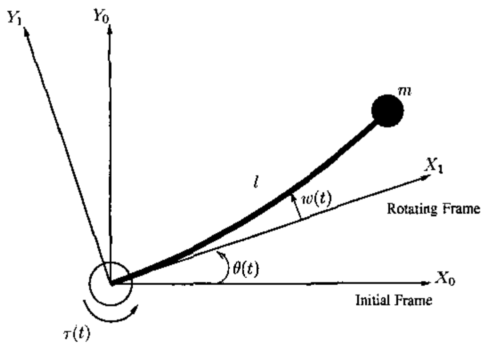

Fig 1: Flexible beam

equations of motion for this system are given by

$$\ddot{w}(x,t) + \frac{EI}{\rho} w''''(x,t) = -x\ddot{\theta}(t) \quad 0 < x < l\ ,\ t > 0, \tag{3}$$

$$w(0,t) = w'(0,t) = w''(l,t) = 0, \tag{4}$$

$$m\left[\ddot{w}(x,t) + l\ddot{\theta}(t)\right] = EIw'''(l,t), \tag{5}$$

$$I_{\mathrm{H}}\ddot{\theta}(t) = \tau(t) + EIw''(0,t). \tag{6}$$

where the constants EI, ρ, m and l are the parameters of the system.

We apply the feedback control law

$$\tau(t) = -EIw''(0,t) + KI_{\mathrm{H}}\left(\rho\langle\dot{w}, x\rangle + ml\dot{w}(l,t)\right) \tag{7}$$

where $K > 0$ is a constant. Substituting (7) into (6), we get the closed-loop system

$$\ddot{w}(x,t) + \frac{EI}{\rho} w''''(x,t) = -xK\left(\rho\langle\dot{w}, x\rangle + ml\dot{w}(l,t)\right), \tag{8}$$

$$w(0,t) = w'(0,t) = w''(l,t) = 0, \tag{9}$$

$$m\ddot{w}(x,t) + mlK\left(\rho\langle\dot{w}, x\rangle + ml\dot{w}(l,t)\right) = EIw'''(l,t). \tag{10}$$

Let us introduce a Hilbert space

$$H_0^2(0,l) = \left\{u \in H^2(0,l) \mid u(0) = u'(0) = 0\right\} \tag{11}$$

with a norm $\|u\|_{H_0^2} = \|u''\|_{L_2}$ and consider the Hilbert space $\mathcal{H} = H_0^2(0,l) \oplus L_2(0,l) \oplus \mathbb{C}$ with an inner product,

$$\langle u, v\rangle = EI\langle u_1'', v_1''\rangle_H + \rho\langle u_2, v_2\rangle_H + m\langle u_3, v_3\rangle_{\mathbb{C}}. \tag{12}$$

We can write (8)-(10) in the form $\dot{u} = \mathcal{A}u$ where,

$$u(t) = \begin{bmatrix} w(\cdot,t) & \dot{w}(\cdot,t) & \dot{w}(l,t)\end{bmatrix}^T \in \mathcal{H}$$

$$\mathcal{A} = \begin{bmatrix} 0 & I & 0 \\ -\frac{EI}{\rho}\frac{\partial^4}{\partial x^4} & -Kx\rho\langle\cdot, x\rangle & -Kxml \\ \frac{EI}{m}\frac{\partial^3}{\partial x^3}\big|_{x=l} & -Kl\rho\langle\cdot, x\rangle & -Klml \end{bmatrix} \tag{13}$$

$$D(\mathcal{A}) = \{(u_1, u_2, u_3) \in H^4(0,l) \oplus H_0^2(0,l) \oplus \mathbb{C} \mid u_1(0) = u_1'(0) = u_1''(l) = 0, u_2(l) = u_3\} \tag{14}$$

Note that $\mathcal{A}$ is an unbounded operator on this Hilbert space $\mathcal{H}$. In the next section, we will show that $\mathcal{A}$ generates a C_0 semigroup.

3. Main Results

In this section, we will describe the main results of the paper as follows:

The first result is to show that $\mathcal{A}$ in (13) is an infinitesimal generator of a contraction semigroup by applying the following theorem.

Theorem 3.1 [8)] Let A be a closed operator with $D(A)$ dense in Z. If

$$\mathrm{Re}\langle Az, z\rangle \leq \omega\|z\|^2 \qquad \forall z \in D(A) \tag{15}$$

$$\mathrm{Re}\langle A^*z, z\rangle \leq \omega\|z\|^2 \qquad \forall z \in D(A^*) \tag{16}$$

then A is the infinitesimal generator of a C_0 semigroup $T(t)$ satisfying $\|T(t)\| \leq e^{\omega t}$.

Lemma 3.2 The operator $\mathcal{A}$ in (13) generates a contraction semigroup.

Proof. We can prove that $\mathcal{A}$ is invertible and its inverse $\mathcal{A}^{-1} : \mathcal{H} \to \mathcal{H}$ is

$$\mathcal{A}^{-1}v = \begin{bmatrix} \frac{K}{EI}q_2(x)[\rho\langle v_1, x\rangle + mlv_1(l)] + Pv_2 + \frac{m}{EI}q_1(x)v_3 \\ v_1(x) \\ v_1(l) \end{bmatrix} \tag{17}$$

where

$$Pv_2 = -\frac{\rho}{EI}\int_0^x \int_0^{x_4} \int_{x_3}^l \int_{x_2}^l v_2(x_1)dx_1dx_2dx_3dx_4,$$

$$q_1(x) = \frac{x^3}{6} - \frac{lx^2}{2},$$

$$q_2(x) = \rho\left(\frac{l^2x^3}{12} - \frac{l^3x^2}{6} - \frac{x^5}{120}\right) + mlq_1(x).$$

We see that $\mathcal{A}^{-1}$ is a bounded linear operator on $\mathcal{H}$. From the Closed Graph theorem, $\mathcal{A}^{-1}$ is closed and so is $\mathcal{A}$. Next, by the definition of the adjoint operator, we have

$$\mathcal{A}^* = \begin{bmatrix} 0 & -I & 0 \\ \frac{EI}{\rho}\frac{\partial^4}{\partial x^4} & -Kx\rho\langle\cdot, x\rangle & -Kxml \\ -\frac{EI}{m}\frac{\partial^3}{\partial x^3}\big|_{x=l} & -K\rho l\langle\cdot, x\rangle & -Klml \end{bmatrix}, \tag{18}$$

$$D(\mathcal{A}^*) = \{(v_1, v_2, v_3) \in H^4(0,l) \oplus H_0^2(0,l) \oplus \mathbb{C} \mid v_2(0) = v_2'(0) = v_1''(l) = 0, v_3 = v_2(l)\}.$$

Consider

$$\begin{aligned}\langle \mathcal{A}u,u\rangle_{\mathcal{H}} &= EI\,\langle u_2'',u_1''\rangle + \rho\left\langle -\frac{EI}{\rho}u_1'''',u_2\right\rangle \\ &- \rho\,\langle Kx\,[\rho\,\langle u_2,x\rangle + mlu_3]\,,u_2\rangle \\ &+ m\left\langle -Kl\,[\rho\,\langle u_2,x\rangle + mlu_3] + \frac{EI}{m}u_1'''(l),u_3\right\rangle_{\mathbf{C}} \\ &= EI\overline{\langle u_1'',u_2''\rangle} - EI\,\langle u_1'',u_2''\rangle - EIu_1'''(l)\overline{u_2(l)} \\ &- K\,[\rho\,\langle u_2,x\rangle + mlu_3]\left(\rho\overline{\langle u_2,x\rangle} + ml\overline{u_3}\right) \\ &+ EIu_1'''(l)\overline{u_3} \\ &= EI\overline{\langle u_1'',u_2''\rangle} - EI\,\langle u_1'',u_2''\rangle \\ &- K\,|\rho\,\langle u_2,x\rangle + mlu_3|^2\end{aligned}$$

Therefore,

$$\operatorname{Re}\langle \mathcal{A}u,u\rangle_{\mathcal{H}} = -K\,|\rho\,\langle u_2,x\rangle + mlu_3|^2 \le 0. \quad (19)$$

Similarly, from the adjoint operator of $\mathcal{A}$ in (18)

$$\begin{aligned}\langle \mathcal{A}^*u,u\rangle_{\mathcal{H}} &= -EI\,\langle u_2'',u_1''\rangle + \rho\left\langle \frac{EI}{\rho}u_1'''',u_2\right\rangle \\ &- \rho\,\langle Kx\,[\rho\,\langle u_2,x\rangle + mlu_3]\,,u_2\rangle \\ &+ m\left\langle -Kl\,[\rho\,\langle u_2,x\rangle + mlu_3] - \frac{EI}{m}u_1'''(l),u_3\right\rangle_{\mathbf{C}} \\ &= -EI\overline{\langle u_1'',u_2''\rangle} + EI\,\langle u_1'',u_2''\rangle + EIu_1'''(l)u_2(l) \\ &- K\,[\rho\,\langle u_2,x\rangle + mlu_3]\left[\rho\overline{\langle u_2,x\rangle} + ml\overline{u_3}\right] \\ &- EIu_1'''(l)\overline{u_3} \\ &= -EI\overline{\langle u_1'',u_2''\rangle} + EI\,\langle u_1'',u_2''\rangle \\ &- K\,|\rho\,\langle u_2,x\rangle + mlu_3|^2\end{aligned}$$

Thus,

$$\operatorname{Re}\langle \mathcal{A}^*u,u\rangle_{\mathcal{H}} = -K\,|\rho\,\langle u_2,x\rangle + mlu_3|^2 \le 0. \quad (20)$$

Since $\mathcal{A}$ is closed with $D(\mathcal{A})$ dense in $\mathcal{H}$ and from (19)-(20), (15)-(16) is satisfied with $\omega = 0$. This shows that A generates the contraction semigroup, $\|T(t)\| \le 1$. □

Next, we will show that the spectrum of $\mathcal{A}$, indeed, consists of only isolated eigenvalues with finite multiplicity by applying the following theorem.

Theorem 3.3 [8] Let A be a closed linear operator with $0 \in \rho(A)$ and A^{-1} is compact. The spectrum of A consists of only isolated eigenvalues with finite multiplicity.

Lemma 3.4 $\mathcal{A}^{-1}$ is compact.

Proof. $\mathcal{A}^{-1} : \mathcal{H} \to \mathcal{H}$ can be written in the following form

$$\mathcal{A}^{-1} = \begin{bmatrix} T_1 & T_2 & T_3 \\ I & 0 & 0 \\ T_4 & 0 & 0 \end{bmatrix}$$

If all T_i's are compact operators, then $\mathcal{A}^{-1}$ is compact. We will prove the compactness property of each T_i as follows:

1. Consider $T_1 : H_0^2(0,l) \to H_0^2(0,l)$ defined by

$$T_1v = \frac{K}{EI}q_2(x)(\rho\,\langle v,x\rangle + mlv(l))$$

Let S_N be a bounded set of $v \in H_0^2(0,l)$ with $\|v\|_{H_0^2} \le N$. Then,

$$\begin{aligned}\|T_1v\|_{H_0^2} &= \frac{K}{EI}\|q_2(x)\|_{H_0^2}|\rho\,\langle v,x\rangle + mlv(l)| \\ &\le \frac{K}{EI}\|q_2(x)\|_{H_0^2}(\rho|\,\langle v,x\rangle\,| + ml|v(l)|) \\ &\le \frac{K}{EI}\|q_2(x)\|_{H_0^2}\left\{\rho l\sqrt{\frac{l}{3}}\|v\|_{L_2} + mlM_1\|v\|_{H_0^2}\right\} \quad (21) \\ &\le \frac{K}{EI}\|q_2(x)\|_{H_0^2}\left\{\rho l\sqrt{\frac{l}{3}}N' + mlM_1N\right\} \quad (22) \\ &\le M_2\end{aligned}$$

where (21) is obtained by using the Sobolev imbedding theorem [9] and the Cauchy-Schwarzt inequality. From the fact that $\|\cdot\|_{H^2} \sim \|\cdot\|_{H_0^2}$, we get (22). This shows that T_1v is uniformly bounded.

Since $q_2(x)$ is continuous, i.e., for all $x_0 \in (0,l)$ and $\epsilon_1 > 0$, there exists $\delta_1 > 0$ such that

$$|x - x_0| < \delta_1 \Rightarrow \|q_2(x) - q_2(x_0)\| < \epsilon_1,$$

we have

$$\begin{aligned}\|T_1v(x) - T_1v(x_0)\| &= \frac{K}{EI}|\rho\,\langle v,x\rangle + mlv(l)|\,\|q_2(x) - q_2(x_0)\| \\ &\le \frac{K}{EI}\left\{\rho l\sqrt{\frac{l}{3}}N' + mlM_1N\right\}\|q_2(x) - q_2(x_0)\|.\end{aligned}$$

Let $\epsilon = EI\epsilon_1/K(\rho l\sqrt{\frac{l}{3}}N' + mlM_1N)$, so

$$|x - x_0| < \delta_1 \Rightarrow \|T_1v(x) - T_1v(x_0)\| < \epsilon$$

Notice that δ_1 does not depend on the choice of $v \in S_N$, which implies that T_1v is equicontinuous. From Arzela's theorem, the image of T_1v is a precompact set. Therefore, T_1 is compact.

2. Consider $T_2 : L_2(0,l) \to H_0^2(0,l)$ defined by

$$T_2v = -\frac{\rho}{EI}\int_0^x\int_0^{x_4}\int_{x_3}^l\int_{x_2}^l v(x_1)dx_1dx_2dx_3dx_4$$

Let $f \in L_2(0,l)$ and let χ_S be the characteristic function of a set S. We know that $\chi_{(0,x)} \in L_2[0,l] \times L_2[0,l]$. Thus, the operator A defined by

$$Af = \int_0^x f(\tau)d\tau = \int_0^l \chi_{(0,x)}f(\tau)d\tau$$

is a compact operator from $L_2(0,l) \to L_2(0,l)$ and T_2 can be considered as the composition of the operator A

defined above. Since the compositions of compact operator are compact, we can conclude that T_2 is compact.

3. Consider $T_3 : \mathbb{C} \to H_0^2(0,l)$ defined by

$$T_3 v = \frac{m}{EI} q_1(x) v$$

As in the case of T_1, we can see that T_3 is compact.

4. $I : H_0^2(0,l) \to L_2(0,l)$ is a compact operator. This can be proved by the Hilbert-Schmidt Imbedding Theorem [9)]

5. $T_5 : H_0^2(0,l) \to \mathbb{C}$, $T_5 v = v(l)$

From the Sobolev Imbedding theorem [9)], T_5 is a bounded linear functional. Its image has finite dimensional range, so T_5 is compact.

According to all of the above, we can conclude that $\mathcal{A}^{-1}$ is compact. □

Lemma 3.5 The spectrum of $\mathcal{A}$ in (13) consists of only isolated eigenvalues with finite multiplicity.

Proof. By the definition of the resolvent set, $0 \in \rho(\mathcal{A})$. The proof is completed following Lemma 3.4 and Theorem 3.3. □

In what follows, we analyze the eigenvalues of $\mathcal{A}$ by showing that all these eigenvalues lie on the open-left half complex plane.

Lemma 3.6 For any $0 < K < \infty$, we have

$$\operatorname{Re} \lambda(\mathcal{A}) < 0.$$

Proof. Consider the eigenvalue problem

$$\mathcal{A}\phi(x) = \lambda\phi(x) \tag{23}$$

where λ and $\phi(x) = \begin{bmatrix}\phi_1(x) & \phi_2(x) & \phi_3\end{bmatrix}^T$ are an eigenvalue and the corresponding eigenvector of $\mathcal{A}$ respectively. From (13)-(14), we get the ordinary differential equation of $\phi_1(x)$ and the boundary conditions.

$$\phi_1''''(x) + \frac{\rho\lambda^2}{EI}\phi_1(x) = -\frac{\rho K\lambda}{EI}\left[\rho\langle\phi_1, x\rangle + ml\phi_1(l)\right]x \tag{24}$$

$$\phi_1(0) = \phi_1'(0) = \phi_1''(l) = 0 \tag{25}$$

$$\phi_1'''(l) = \frac{Kml\lambda}{EI}\left[\rho\langle\phi_1, x\rangle + ml\phi_1(l)\right] + \frac{m\lambda^2}{EI}\phi_1(l) \tag{26}$$

Take the inner product with ϕ_1 on both sides in (24), we have

$$\langle\phi_1'''', \phi_1\rangle + \frac{\rho\lambda^2}{EI}\langle\phi_1, \phi_1\rangle + \frac{\rho K\lambda}{EI}(\rho\langle\phi_1, x\rangle + ml\phi_1(l))\langle x, \phi_1\rangle = 0. \tag{27}$$

Since

$$\begin{aligned}
\langle\phi_1'''', \phi_1\rangle &= \int_0^l \phi_1''''\overline{\phi_1}dx \\
&= \phi_1'''\overline{\phi_1}\,|_0^l - \int_0^l \phi_1'''\overline{\phi_1'}dx \\
&= \phi_1'''(l)\overline{\phi_1(l)} - \phi_1''\overline{\phi_1'}\,|_0^l + \int_0^l \phi_1''\overline{\phi_1''}dx \\
&= \phi_1'''(l)\overline{\phi_1(l)} + \|\phi''\|^2 \\
&= \lambda\frac{\rho Kml}{EI}\langle\phi_1, x\rangle\overline{\phi_1(l)} + \lambda\frac{Km^2l^2}{EI}|\phi_1(l)|^2 \\
&\quad + \lambda^2\frac{m}{EI}|\phi_1(l)|^2 + \|\phi''\|^2,
\end{aligned} \tag{28}$$

and by substituting (28) in (27), we obtain

$$\begin{aligned}
&\lambda\rho Kml\langle\phi_1, x\rangle\overline{\phi_1(l)} + \lambda Km^2l^2|\phi_1(l)|^2 + \lambda^2 m|\phi_1(l)|^2 \\
&\quad + EI\|\phi''\|^2 + \rho\lambda^2\|\phi_1\|^2 + \lambda\rho^2 K|\langle\phi_1, x\rangle|^2 \\
&\quad + \lambda\rho Kml\phi_1(l)\langle x, \phi_1\rangle = 0.
\end{aligned} \tag{29}$$

$$\begin{aligned}
&\lambda^2\left\{m|\phi_1(l)|^2 + \rho\|\phi_1\|^2\right\} + EI\|\phi''\|^2 \\
&\quad + \lambda K\left\{\rho^2|\langle\phi_1, x\rangle|^2 + m^2l^2|\phi_1(l)|^2\right\} \\
&\quad + \lambda K\left\{2\rho ml\operatorname{Re}(\phi_1(l)\langle x, \phi_1\rangle)\right\} = 0.
\end{aligned} \tag{30}$$

$$\lambda^2\left\{m|\phi_1(l)|^2 + \rho\|\phi_1\|^2\right\} + \lambda K\left|\rho\langle\phi_1, x\rangle + ml\phi_1(l)\right|^2 + EI\|\phi''\|^2 = 0. \tag{31}$$

Let $\lambda = a + ib$, then (31) can be written as

$$\begin{aligned}
&(a^2 - b^2 + i2ab)\left\{m|\phi_1(l)|^2 + \rho\|\phi_1\|^2\right\} + EI\|\phi''\|^2 \\
&\quad + (a + ib)K\left|\rho\langle\phi_1, x\rangle + ml\phi_1(l)\right|^2 = 0,
\end{aligned} \tag{32}$$

which can be splitted into two equations as

$$\begin{aligned}
&(a^2 - b^2)(m|\phi_1(l)|^2 + \rho\|\phi_1\|^2) + EI\|\phi''\|^2 \\
&\quad + a\cdot K\left|\rho\langle\phi_1, x\rangle + ml\phi_1(l)\right|^2 = 0,
\end{aligned} \tag{33}$$

and

$$\begin{aligned}
&2ab(m|\phi_1(l)|^2 + \rho\|\phi_1\|^2) \\
&\quad + b\cdot K\left|\rho\langle\phi_1, x\rangle + ml\phi_1(l)\right|^2 = 0.
\end{aligned} \tag{34}$$

First, suppose $b = 0$ in (33), we have

$$a^2(m|\phi_1(l)|^2 + \rho\|\phi_1\|^2) + a\cdot K\left|\rho\langle\phi_1, x\rangle + ml\phi_1(l)\right|^2 + EI\|\phi''\|^2 = 0. \tag{35}$$

It can be shown that

$$\left|\rho\langle\phi_1, x\rangle + ml\phi_1(l)\right|$$

is not equal to zero. Then all coefficients of a are real number greater than zero. Therefore, all roots a of (35) are less than zero.

Next, when $b \neq 0$ in (34),

$$a = -\frac{K\left|\rho\langle\phi_1, x\rangle + ml\phi_1(l)\right|^2}{2(m|\phi_1(l)|^2 + \rho\|\phi_1\|^2)} < 0 \tag{36}$$

Therefore, $\operatorname{Re}(\lambda) < 0$. □

Finally, to prove the closed-loop stability we need the following theorem to show the asymptotic stability of the semigroup generated by $\mathcal{A}$.

Theorem 3.7 [10, 11] Let $T(t)$ be a uniformly bounded semigroup on a Banach space X with an infinitesimal generator A. Suppose that

1. $\sigma(A) \cap i\mathbb{R}$ is countable,
2. $\sigma_P(A^*) \cap i\mathbb{R} = \emptyset$,

then $T(t)$ is asymptotically stable.

Theorem 3.8 The semigroup generated by $\mathcal{A}$ is asymptotically stable.

Proof This follows from Lemmas 3.2, 3.5, 3.6 and Theorem 3.7.

4. Conclusion

In this work, we consider the design of a controller for a one-link flexible robot arm, modelled as an infinite-dimensional system, where the effects of the tip mass and the motion of the motor were included simultaneously in the mathematical model. The proposed control law is a linear combination of the tip deflection and a linear functional of the beam deflection. It was shown that (1) the infinitesimal generator of the closed-loop system generates a contraction semigroup, (2) the spectrum of the generator consists of only isolated eigenvalues with finite multiplicity, and (3) the real parts of these eigenvalues are less than zero. Therefore, the closed-loop system is asymptotically stable.

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