EE M150L:
Introduction to Micromachining and MEMS Laboratory

Lecture:

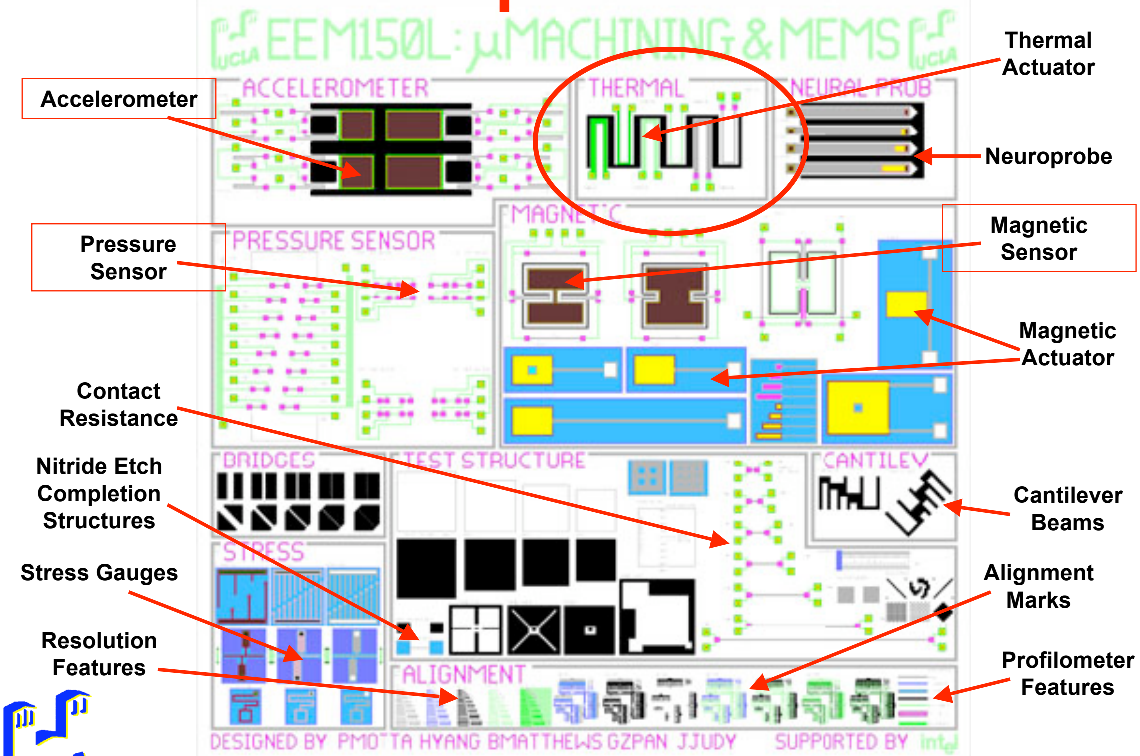
Thermal Expansion

Prof. Jack W. Judy

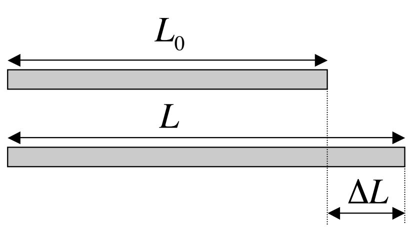
Today's Lecture

- Quiz 9
  - Thin-Film Stress
  - Wheatstone Bridges
- Device Physics
  - Thermal Expansion
  - Heat Generation
  - Temperature Distribution
  - Thermal Actuation
    - End Deflection of Plates and Beams

Thermal Expansion Devices



Linear Thermal Expansion



$$\Delta L = L - L_0 = \alpha \cdot L_0 \cdot \Delta T$$

L_0 = length before change in temperature [m]
L = length after change in temperature [m]
ΔL = change in length due to temperature [m]
ΔT = change temperature [K]
α = thermal expansion coefficient [1/K]

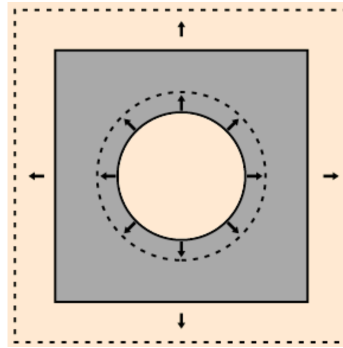
Relative change: $\frac{\Delta L}{L} = \alpha \cdot \Delta T$ **same as strain** $\varepsilon = \frac{\Delta L}{L}$

constraining leads to stress: $\sigma = \varepsilon \cdot E = \frac{\Delta L}{L} \cdot E = \alpha \cdot \Delta T \cdot E$

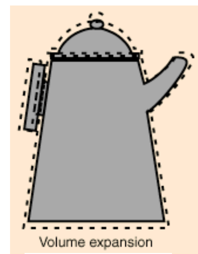
Material [units]	Atomic Mass [g/mol]	Density [kg/m3]	Elastic Modulus [GPa]	Poissons Ratio [-]	Electrical Resistivity [μΩ-cm]	Melting Point [K]	Boiling Point [K]	Thermal Conductivity [M/m-K]	Thermal Expansion Coefficient [10-6/K]
Al	26.981538	2700	130	0.35	2.65	933	2792	235	23.1
Cu	63.546	8920	130	0.34	1.7	1358	3200	400	16.5
Au	196.96655	19300	78	0.44	2.2	1337	3129	320	14.2
Ni	58.6934	8908	200	0.31	7	1728	3186	91	13.4
Fe	55.845	7874	211	0.29	9.7	1811	3134	80	11.8
Pt	195.078	21090	168	0.38	10.6	2041	4098	72	8.8
Ti	47.867	4507	116	0.32	40	1941	3560	22	8.6
Cr	51.9961	7140	279	0.21	12.7	2180	2944	94	4.9
SiC	40.0962	3200	448	-	depends	3100	-	490	4.2
SixNy	42.0922	3200	300	0.24	insulator	2173	-	16	2.8
Si	28.0855	2330	200	-	depends	1687	3173	150	2.6
SiO2	60.0843	2533	73	-	insulator	1883	2863	1.38	0.55

Expansion of Areas and Volumes

- **Question:**
  - **Will a hole in a material get larger or smaller when temperature increases?**
- **Answer:**
  - **Depends if the material around the hole is constrained or free to expand**
  - **Free to expand: *the hole expands***
    - **Consider if you had no hole and instead drew a circle on the block, the circle of material would expand**
    - **Even without the material in the circle, the material surrounding it would expand and would result in a larger hole**



$$\frac{\Delta A}{A} = 2 \cdot \alpha \cdot \Delta T$$



$$\frac{\Delta V}{V} = 3 \cdot \alpha \cdot \Delta T$$

Given: $L = L_0 \cdot (1 + \alpha \cdot \Delta T)$ **then** $A = L^2 = L_0^2 \cdot (1 + \alpha \cdot \Delta T)^2$

and $A = L_0^2 \cdot (1 + 2 \cdot \alpha \cdot \Delta T + \alpha^2 \cdot \Delta T^2) \approx L_0^2 \cdot (1 + 2 \cdot \alpha \cdot \Delta T)$

Similarly, volume expansion: $V \approx L_0^3 \cdot (1 + 3 \cdot \alpha \cdot \Delta T)$

Sorting the Material Data

Stiffness:

Material [units]	Atomic Mass [g/mol]	Density [kg/m3]	Elastic Modulus [GPa]	Poissons Ratio [-]	Electrical Resistivity [μΩ-cm]	Melting Point [K]	Boiling Point [K]	Thermal Conductivity [M/m-K]	Thermal Expansion Coefficient [10-6/K]
SiC	40.0962	3200	448	-	depends	3100	-	490	4.2
SixNy	42.0922	3200	300	0.24	insulator	2173	-	16	2.8
Cr	51.9961	7140	279	0.21	12.7	2180	2944	94	4.9
Fe	55.845	7874	211	0.29	9.7	1811	3134	80	11.8
Ni	58.6934	8908	200	0.31	7	1728	3186	91	13.4
Si	28.0855	2330	200	-	depends	1687	3173	150	2.6
Pt	195.078	21090	168	0.38	10.6	2041	4098	72	8.8
Al	26.981538	2700	130	0.35	2.65	933	2792	235	23.1
Cu	63.546	8920	130	0.34	1.7	1358	3200	400	16.5
Ti	47.867	4507	116	0.32	40	1941	3560	22	8.6
Au	196.96655	19300	78	0.44	2.2	1337	3129	320	14.2
SiO2	60.0843	2533	73	-	insulator	1883	2863	1.38	0.55

Melting Point:

Material [units]	Atomic Mass [g/mol]	Density [kg/m3]	Elastic Modulus [GPa]	Poissons Ratio [-]	Electrical Resistivity [μΩ-cm]	Melting Point [K]	Boiling Point [K]	Thermal Conductivity [M/m-K]	Thermal Expansion Coefficient [10-6/K]
SiC	40.0962	3200	448	-	depends	3100	-	490	4.2
Cr	51.9961	7140	279	0.21	12.7	2180	2944	94	4.9
SixNy	42.0922	3200	300	0.24	insulator	2173	-	16	2.8
Pt	195.078	21090	168	0.38	10.6	2041	4098	72	8.8
Ti	47.867	4507	116	0.32	40	1941	3560	22	8.6
SiO2	60.0843	2533	73	-	insulator	1883	2863	1.38	0.55
Fe	55.845	7874	211	0.29	9.7	1811	3134	80	11.8
Ni	58.6934	8908	200	0.31	7	1728	3186	91	13.4
Si	28.0855	2330	200	-	depends	1687	3173	150	2.6
Cu	63.546	8920	130	0.34	1.7	1358	3200	400	16.5
Au	196.96655	19300	78	0.44	2.2	1337	3129	320	14.2
Al	26.981538	2700	130	0.35	2.65	933	2792	235	23.1

Heat Generation and Distribution

- **Thermal expansion is often caused by on-chip means with heat generation = power dissipation**

$P = I^2 \cdot R$ **Heat Generation / Volume:** $q''' = \frac{P_{dissipated}}{V_{resistor}} = \frac{I^2 \cdot R}{L \cdot A}$

- **The heat generated in resistor is distributed by conduction, convection, and radiation**
  - **Radiation**
    - **only significant at very high T > 700 ºC (white hot)**
  - **Convection**
    - **typically negligible for individual micromachined devices**
  - **<u>Conduction</u>**
    - **dominant heat-loss mechanism for micromachined devices**

Heat Distribution: Conduction

- **Governed by diffusion laws:**

$$\frac{\partial}{\partial x}\left(\sigma_{th} \cdot \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\sigma_{th} \cdot \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\sigma_{th} \cdot \frac{\partial T}{\partial z}\right) + q''' = \rho \cdot C_p \cdot \frac{\partial T}{\partial t}$$

$\sigma_{th} =$ thermal conductivity [W/m·K]
$\rho =$ density [kg/m^3]
$C_p =$ specific heat [J/kg·K]

- **Assumptions**
  - **one dimensional** $\frac{\partial T}{\partial y} = \frac{\partial T}{\partial z} = 0$
  - **steady state** $\frac{\partial T}{\partial t} = 0$
  - **constant heat generation** $q''' =$ constant
  - **resulting 1-D equation:**

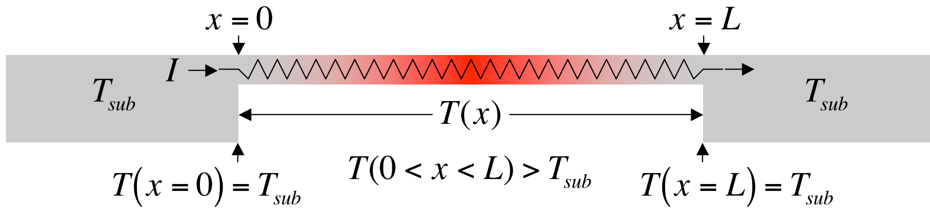
$$\frac{\partial}{\partial x}\left(\sigma_{th} \cdot \frac{\partial T}{\partial x}\right) + q''' = 0$$

Solution:

$$T(x) = -\frac{q'''}{2 \cdot \sigma_{th}} \cdot x^2 + B \cdot x + C$$

1-D Heat Conduction Example (1)

- **Resistive bridge**



$$T(x) = -\frac{q'''}{2 \cdot \sigma_{th}} \cdot x^2 + B \cdot x + C$$

- **apply boundary conditions**

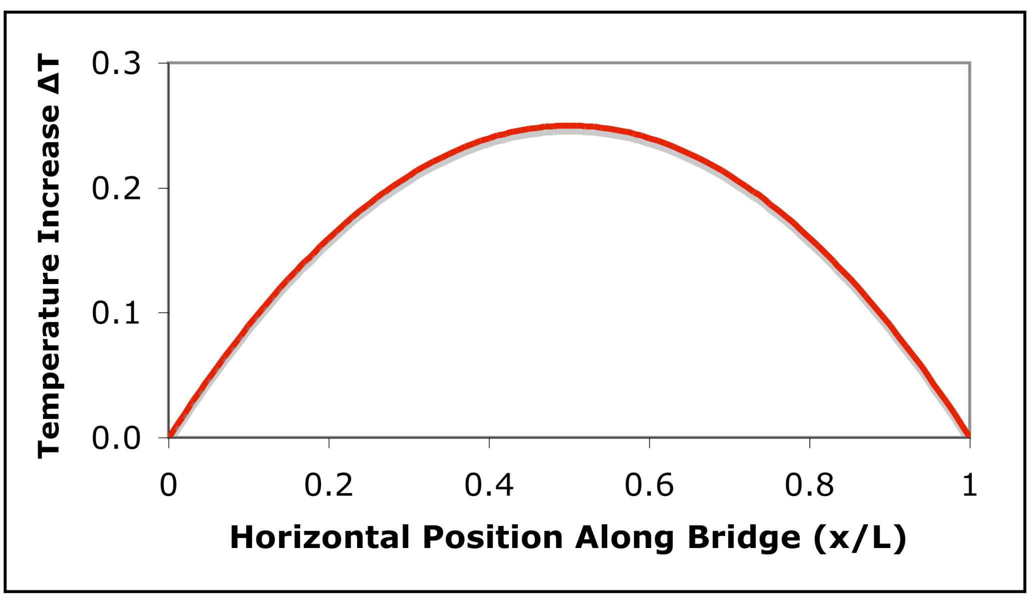
$$T(x=0) = T_{sub} \rightarrow C = T_{sub}$$

$$T(x=L) = T_{sub} \rightarrow B = \frac{q''' \cdot L}{2 \cdot \sigma_{th}}$$

$$T(x) = -\frac{q'''}{2 \cdot \sigma_{th}} \cdot x^2 + \frac{q''' \cdot L}{2 \cdot \sigma_{th}} \cdot x + T_{sub}$$

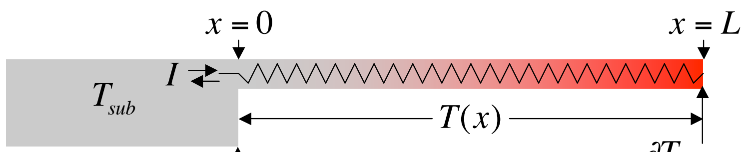
$$\Delta T(x) = -\frac{q'''}{2 \cdot \sigma_{th}} \cdot x^2 + \frac{q''' \cdot L}{2 \cdot \sigma_{th}} \cdot x$$

Temperature Distribution along Bridge



1-D Heat Conduction Example (2)

- **Resistive cantilever**



$T(x=0) = T_{sub}$ $\quad T(0 < x < L) > T_{sub}$ $\quad \frac{\partial T}{\partial x}(x = L) = 0$

$$\boxed{T(x) = -\frac{q'''}{2 \cdot \sigma_{th}} \cdot x^2 + B \cdot x + C}$$

– **apply boundary conditions**

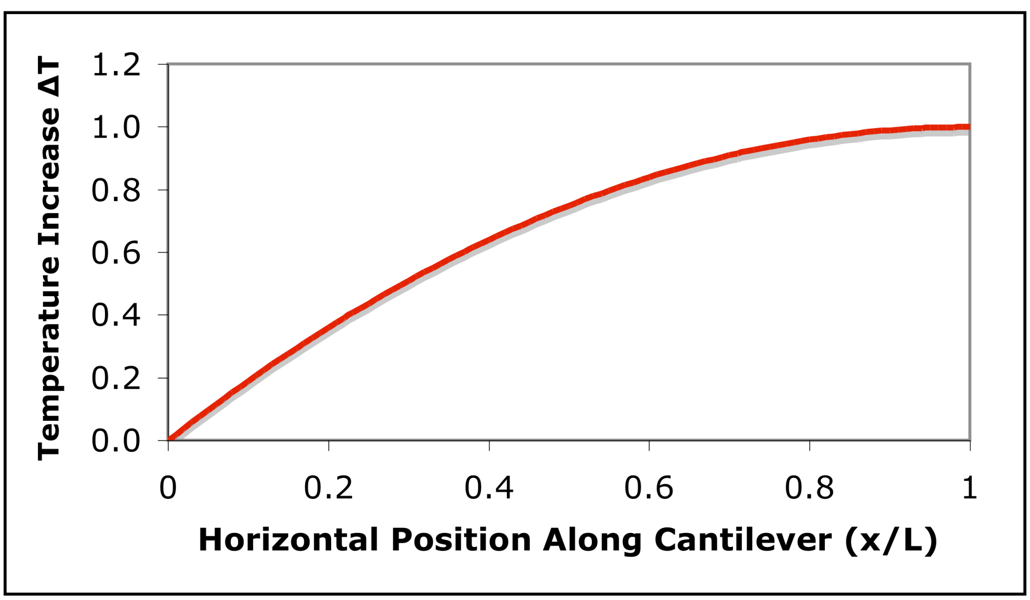
$$T(x=0) = T_{sub} \rightarrow C = T_{sub}$$

$$\frac{\partial T}{\partial x}(x = L) = 0 \rightarrow B = \frac{q''' \cdot L}{\sigma_{th}}$$

$$\rightarrow T(x) = -\frac{q'''}{2 \cdot \sigma_{th}} \cdot x^2 + \frac{q''' \cdot L}{\sigma_{th}} \cdot x + T_{sub}$$

$$\boxed{\Delta T(x) = -\frac{q'''}{2 \cdot \sigma_{th}} \cdot x^2 + \frac{q''' \cdot L}{\sigma_{th}} \cdot x}$$

Temperature Distribution along Cantilever

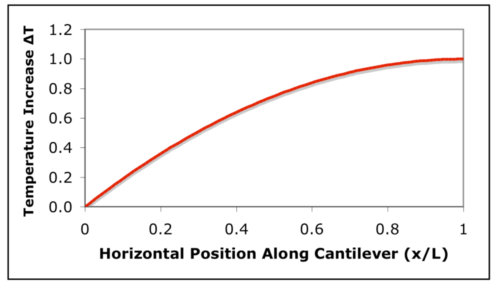


Determining Thermal Expansion

- Given that for this 1-D case we have

$$\Delta L = L \cdot \alpha \cdot \Delta T$$

- Need to integrate over the total length



$$\Delta L = \int_0^L \alpha \cdot \Delta T(x) \cdot \partial x$$

- Total Strain

$$\varepsilon = \frac{\Delta L}{L} = \int_0^L \frac{\alpha \cdot \Delta T(x)}{L} \cdot \partial x = \frac{\alpha}{L} \int_0^L \Delta T(x) \cdot \partial x$$

Bimorph Cantilever Curvature

- If cantilever is made of two layers with different thermal expansion coefficients
  - temperature change causes strain mismatch
  - strain mismatch leads to curvature
  - curvature leads to end deflection
- Simplified Solution
  - assume ΔT is uniform in vertical direction

$$\varepsilon_{mismatch} = \frac{\alpha_1 - \alpha_2}{L} \int_0^L \Delta T(x) \cdot \partial x$$

  - convert to stress (assume $E_1 = E_2 = E_s, \nu_1 = \nu_2 = \nu_s$) *"substrate" = thicker film*

Plate: $\sigma_{mismatch} = \varepsilon_{mismatch} \cdot \frac{E_s}{(1-\nu_s)}$ Beam: $\sigma_{mismatch} = \varepsilon_{mismatch} \cdot \frac{E_s}{(1-\nu_s^2)}$

Bimorph Cantilever Deflection

- **Use thermal stress mismatch with Stoney's equation (approximation)**

Plate: $R = \left(\frac{E}{1-\nu}\right)_s \cdot \frac{t_s^2}{6 \cdot t_f \cdot \sigma_{mismatch}}$ **Beam:** $R = \left(\frac{E}{1-\nu^2}\right)_s \cdot \frac{t_s^2}{6 \cdot t_f \cdot \sigma_{mismatch}}$

- **Assume curvature is spherical (plate) or circular (beam)**

Plate: $K = \frac{1}{R} = \frac{d^2 u_y}{dr^2}$ **Beam:** $K = \frac{1}{R} = \frac{d^2 u_y}{dx^2}$

- **Determine deflection for both cases**

Plate: $u_y = K\frac{r^2}{2}$ **Beam:** $u_y = K\frac{x^2}{2}$

Next Lecture

- **Lecture:**
  - Summary of Device Physics
  - Lab Report 2
  - Review of lecture material

EE M150L Lecture 9: *Thermal Expansion* Jack W. Judy © 2004