An Analysis of Range Difference Based Target Localization in Uniformly Distributed Sensor Field

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I. Introduction

Target localization is one of several applications that capitalizes on the sensor network framework. There have been a number of investigations for target localization and tracking in sensor network context. These ideas, however, were mostly experimented only in centralized processing scenarios. The study of the parameters that affect both the localization accuracy and the energy expenditure is significant for the decentralized system design. In this paper, we focus on Range Difference (RD) based target localization due to the feasibility of real-time performance. Our study will emphasize the analysis of the localization performance associated with decentralized processing. Assuming a densely and uniformly distributed sensor placement, we derive the Mean Square Error when the cluster is activated. The influential parameters which are the number of participating sensors, the distance between reference sensor and the actual target location, and the size of the activated clusters are explicitly presented in the expression. The derived expression is verified by simulation results. We also discuss the benefit earned from the analysis and suggest a solution for an optimization problem that would aid the large scale system design.

II. Localization Performance Analysis

Sensor field and clustering aspects are assumed as follows

- Sensors are uniformly distributed in the dense field
- One sensor is selected to be a cluster head
- The proximity of the cluster head to the actual target location is D_0
- The sensors located within the radio range R_c from the cluster head, hereafter called cluster members, participate the localization process
- The intervals of time series data received by cluster members are sent to the cluster head for TDOA estimates

The cluster members' coordinates are denoted as $\{(x_1, y_1), \ldots (x_N, y_N)\}$ and (x_0, y_0) denotes the cluster head location. Assuming the target is located at $\mathbf{z}_s = (x_s, y_s)$ generating real valued, continuous time, zero-mean, jointly WSS Gaussian random process, the differences of the distance between sensors i and j ($i, j = 0, \ldots, N$) and the source denoted by d_{ij} can be obtained by the basic relation: $d_{ij} = D_i - D_j$ where $D_i = \sqrt{(x_s - x_i)^2 + (y_s - y_i)^2}$. By using (x_0, y_0) as a location of reference sensor, we have $\hat{d}_{i0} = d_{i0} + n_i \quad i = 1 \ldots N$ where $n_i \sim N(0, \sigma_i^2)$ since the TDOA estimate obtained by generalized cross correlation with Gaussian data is asymptotically normally distributed in high SNR environment [1]. The Cramer Rao Bound (CRB) is attained as $\mathbf{C} = (\mathbf{H}^T \mathbf{Q}^{-1} \mathbf{H})^{-1}$ [3] where

$$\mathbf{H} = \begin{bmatrix} \frac{x_0 - x_s}{D_0} - \frac{x_1 - x_s}{D_1} & \frac{y_0 - y_s}{D_0} - \frac{y_1 - y_s}{D_1} \\ \vdots & \vdots \\ \frac{x_0 - x_s}{D_0} - \frac{x_N - x_s}{D_N} & \frac{y_0 - y_s}{D_0} - \frac{y_N - y_s}{D_N} \end{bmatrix} \tag{1}$$

and $\mathbf{Q} = \mathrm{diag}(\sigma_1^2, \ldots, \sigma_N^2)$. In this scenario we assume that the measurement noise has uniform variance within the cluster so that $\sigma_1^2 = \sigma_2^2 = \ldots \sigma_N^2 = \sigma_0^2$ is assumed.The performance of the estimator can be evaluated by Mean Square Error ($\mathbf{M}_{\hat{z}_s}$) of the estimate as $\mathbf{E}\{\|\hat{\mathbf{z}}_\mathbf{s} - \mathbf{z}_\mathbf{s}\|^2\} = \mathbf{E}\{(\hat{x}_s - x_s)^2 + (\hat{y}_s - y_s)^2\} \geq CRB(\hat{x}_s) + CRB(\hat{y}_s)$ CRB can be obtained under the assumption of high SNR environment [2]. We are assuming that such a condition is met and the localization performance is evaluated using this bound.By substituting $\mathbf{H}$ and $\mathbf{Q}$ and using the assumption of a large number of N to approximate the summation by the expectation over the uniformly random sensor locations, $\mathbf{M}_{\hat{z}_s}$ can be presented as follows

$$\mathbf{M}_{\hat{z}_s} = \begin{cases} \frac{\sigma_0^2}{N}\left(\frac{-128(\mathfrak{f}+\mathfrak{e})k^2+96\pi k+128(\mathfrak{f}-\mathfrak{e})}{-3\pi k^5+32(\mathfrak{f}+\mathfrak{e})k^4-12\pi k^3-32(\mathfrak{f}+\mathfrak{e})k^2+36\pi k+64(\mathfrak{f}-\mathfrak{e})}\right) \\ \frac{\sigma_0^2}{N}\left(\frac{-128(\mathfrak{f}+\mathfrak{e})k^5+96\pi k^4+128(\mathfrak{f}-\mathfrak{e})k^3}{-32(\mathfrak{f}+\mathfrak{e})k^3+24\pi k^2+32(\mathfrak{f}-\mathfrak{e})k-3\pi}\right) \end{cases} \tag{2}$$

In (2), $k = \frac{D_0}{R_c}$ where the upper and lower expression represent the case of $0 < k \leq 1$ and $k \geq 1$ respectively. $\mathfrak{f}$ and $\mathfrak{e}$ are the short version of $\mathfrak{f}(\frac{\pi}{2}, k)$ and $\mathfrak{e}(\frac{\pi}{2}, k)$ when $0 < k \leq 1$ and $\mathfrak{f}(\sin^{-1}(\frac{1}{k}), k)$ and $\mathfrak{e}(\sin^{-1}(\frac{1}{k}), k)$ when $k \geq 1$ where $\mathfrak{f}(\phi, k)$ and $\mathfrak{e}(\phi, k)$ represent the elliptic integrals of the first and second kind. It is noticeable from the expression that $\mathbf{M}_{\hat{z}_s}$ is proportional to σ_0^2 and $\frac{1}{N}$. $\mathbf{M}_{\hat{z}_s}$ is also a function of the ratio of D_0 and R_c. We plot the curves between k and $\mathbf{M}_{\hat{z}_s}$ in Figure 1(a) and 1(b) where σ_0^2 is 0.01 and $N = 50, 100, 150, 200$. The curves illustrate that $\mathbf{M}_{\hat{z}_s}$ increases with growing rate as k increases. We conducted monte carlo simulations with 1000 different topologies. 100 sensors are uniformly placed within the circle having radius $R_c = 5$. D_0 varies from 0 to 5 in the case the target is inside the cluster ($0 < k \leq 1$) and 5 to 10 in the case the target is outside the cluster ($k \geq 1$) and $\sigma_0^2 = 0.01$. Figure 2(a) and (b) illustrates the curve between $\mathbf{M}_{\hat{z}_s}$ and D_0 when the target is inside and

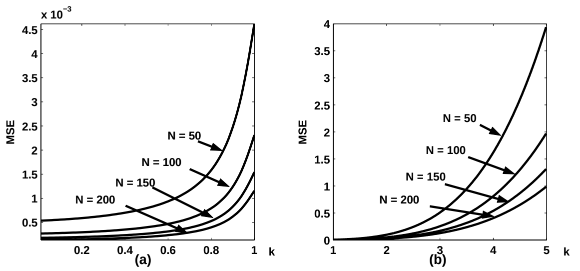

Fig. 1. $\mathbf{M}_{\hat{z}_s}$ increases with growing rate as k increases when $0 < k \leq 1$ (a) and $k \geq 1$ (b)

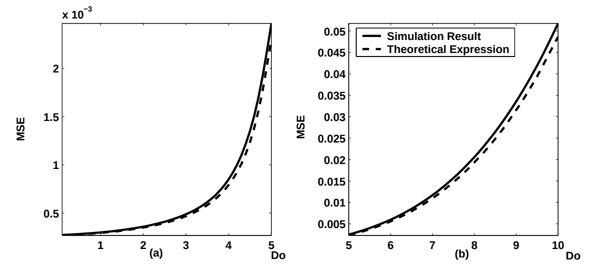

Fig. 2. $\mathbf{M}_{\hat{z}_s}$ with the variation of D_0 and R_c = 5 when the target is inside (a) and outside (b) the cluster

outside the cluster. The solid line represents the result from the simulation whereas the dashed line is the one obtained from theoretical expression. The results show that the derivation and the simulation are consistent although the small difference occurs due to the assumption of large number of sensors.

III. Discussion

We have learned that to obtain good performance of RD based localization, it requires not only a large number of sensors and small noise measurement variance but also a small ratio of the proximity of the actual target location to the reference sensor and the cluster radius. Therefore, the cluster head should be the sensor which is the most likely located closest to target. The derived expression is also useful when D_0 can be estimated based on the performance of detection scheme and the sensor topology. R_c can be properly designed to suit the requirement of the accuracy. The R_c selection causes a trade-off between communication overhead within the cluster and localization performance [4]. The optimization problem can be formulated as $OBJ(R_c) = w_1 \frac{E(R_c)}{E(R_c^0)} + w_2 \frac{\mathbf{M}_{\hat{z}_s}(Rc)}{\mathbf{M}_{\hat{z}_s}(R_c^0)}$ where $E(R_c^0)$ represents the normalized value which might be initially expected to be incurred by the designed cluster with radius R_c^0. w_1 and w_2 where $w_1 + w_2 = 1$ are designated by the weighting of the importance between the two factors. Considering a fixed sensor density ρ in the field, the number of sensors in the cluster is proportional to R_c^2 as $N = \rho(\pi R_c^2)$. Assuming the energy dissipated by transmission (E_{tx}) is proportional to $\sum_{i=1}^{N} R_i^2 \approx N\mathbf{E}\{R_i^2\}$ where R_i denotes the distance between each cluster member and the cluster head. Thus, $E_{tx} \propto R_c^4$. Let $f_{0<k\leq 1}(\frac{D_0}{R_c})$ and $f_{k\geq 1}(\frac{D_0}{R_c})$ represent rational function of k in (2). The optimal solution $\hat{R}_c$ is the following

$$\hat{R}_c = \text{arg min} \quad w_1\Big(\frac{R_c}{R_c^0}\Big)^4 + w_2\Big(\frac{R_c^0}{R_c}\Big)^2\Big(\frac{f_{0<k\leq 1/k\geq 1}(\frac{D_0}{R_c})}{f_{0<k\leq 1/k\geq 1}(\frac{D_0}{R_c^0})}\Big)$$

Either $f_{0<k\leq 1}(\frac{D_0}{R_c})$ or $f_{k\geq 1}(\frac{D_0}{R_c})$ is used depending on the value of $\frac{D_0}{R_c}$. By assuming D_0 with a normal distribution $\sim N(5, 0.1)$ and $R_c^0 = 6$, we conducted monte carlo simulation by using 3 different values of $w_1; 0.2, 0.5$ and 0.8. The results in Figure 3 show that the optimal values $\hat{R}_c$ depends on the selected weights. By assuming the prior information of estimated $D_0, \hat{D}_0 = 5$, we also include the result obtained

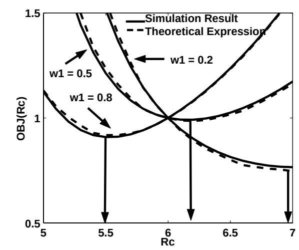

Fig. 3. $OBJ(R_c)$ for different values of w_1 with $R_c^0 = 6$ and $D_0 \sim N(5, 0.1)$ lead to different optimal solutions

by the derived expression presented by dashed line. We apply steepest descent method to estimate the optimal value of R_c and the solutions for these cases are 5.53, 6.15, and 7.03 respectively. These are consistent with the simulation results. The interpretation on the solution (In case $w_1 = w_2 = 0.5$, for example) is that the selection of $R_c = 6.15$ based on the the defined energy model is considered to be more efficient since it requires only 10.38% more transmission cost in order to achieve 13.23% less Mean Square Error in comparison with the normalized condition.

IV. Conclusion and Future work

In this paper, we derive Mean Square Error ($\mathbf{M}_{\hat{z}_s}$) of RD based localization associated with the decentralized processing in sensor network application. It was found that $\mathbf{M}_{\hat{z}_s}$ is inversely proportional to the number of participating sensors. $\mathbf{M}_{\hat{z}_s}$ decreases as the ratio of D_0 and R_c becomes smaller. Therefore, a larger cluster and a closer distance from the cluster head to the target is desirable. The knowledge was applied in a case example of optimizing between communication cost and estimation accuracy. Future work is to consider the variation of noise variance as a function of distance to the target so that the expression of MSE will be more practical.

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